Digital Image Processing

Lecture # 2
Connected Component Analysis

Spatial & Gray Level Resolution

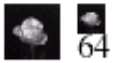
Spatial Resolution

1024

512

256

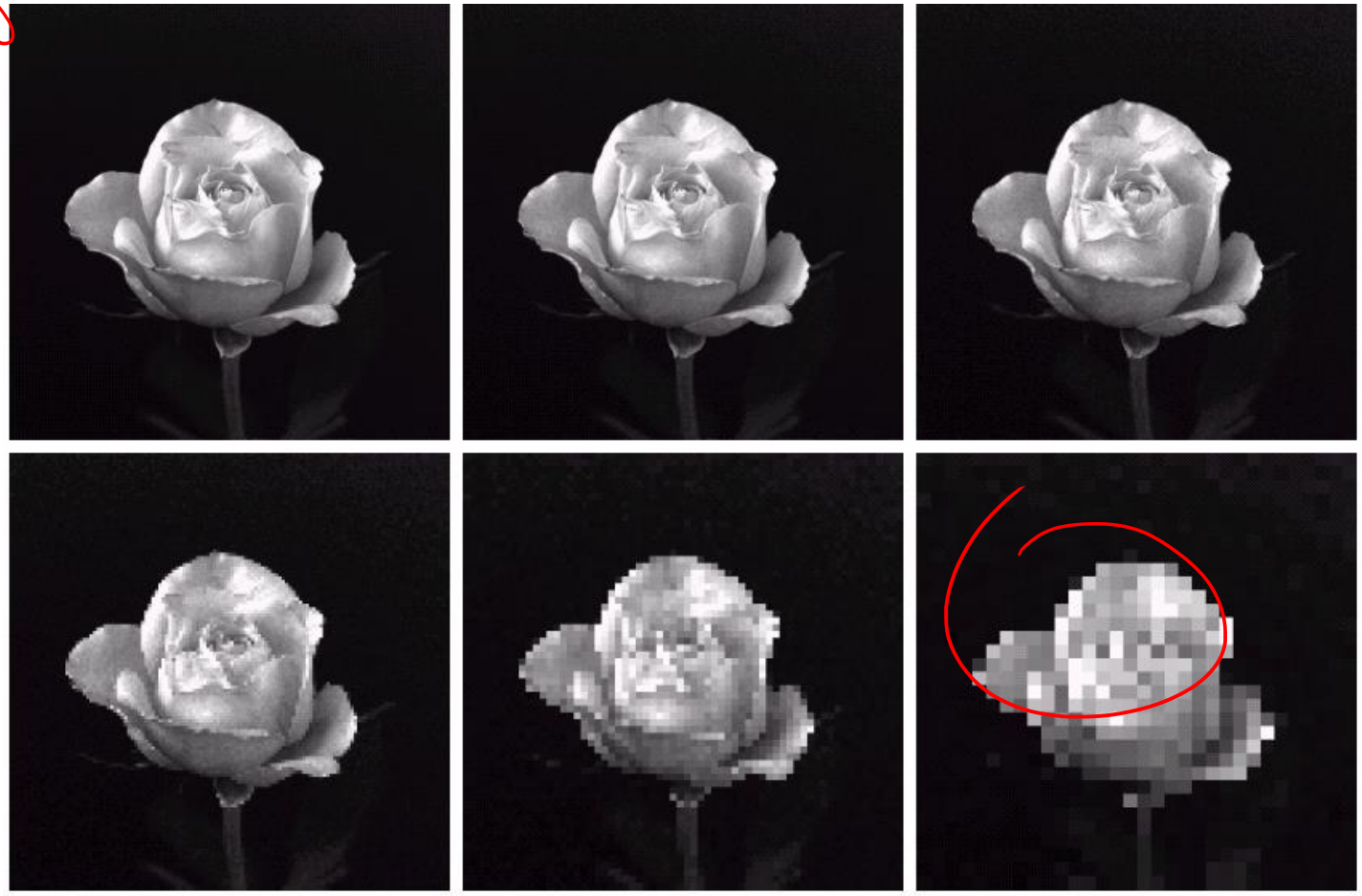
128



64

32

Spatial Resolution



Intensity Level Resolution

- *Intensity level resolution* refers to the number of intensity levels used to represent the image
  - The more intensity levels used, the finer the level of detail in an image
  - Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

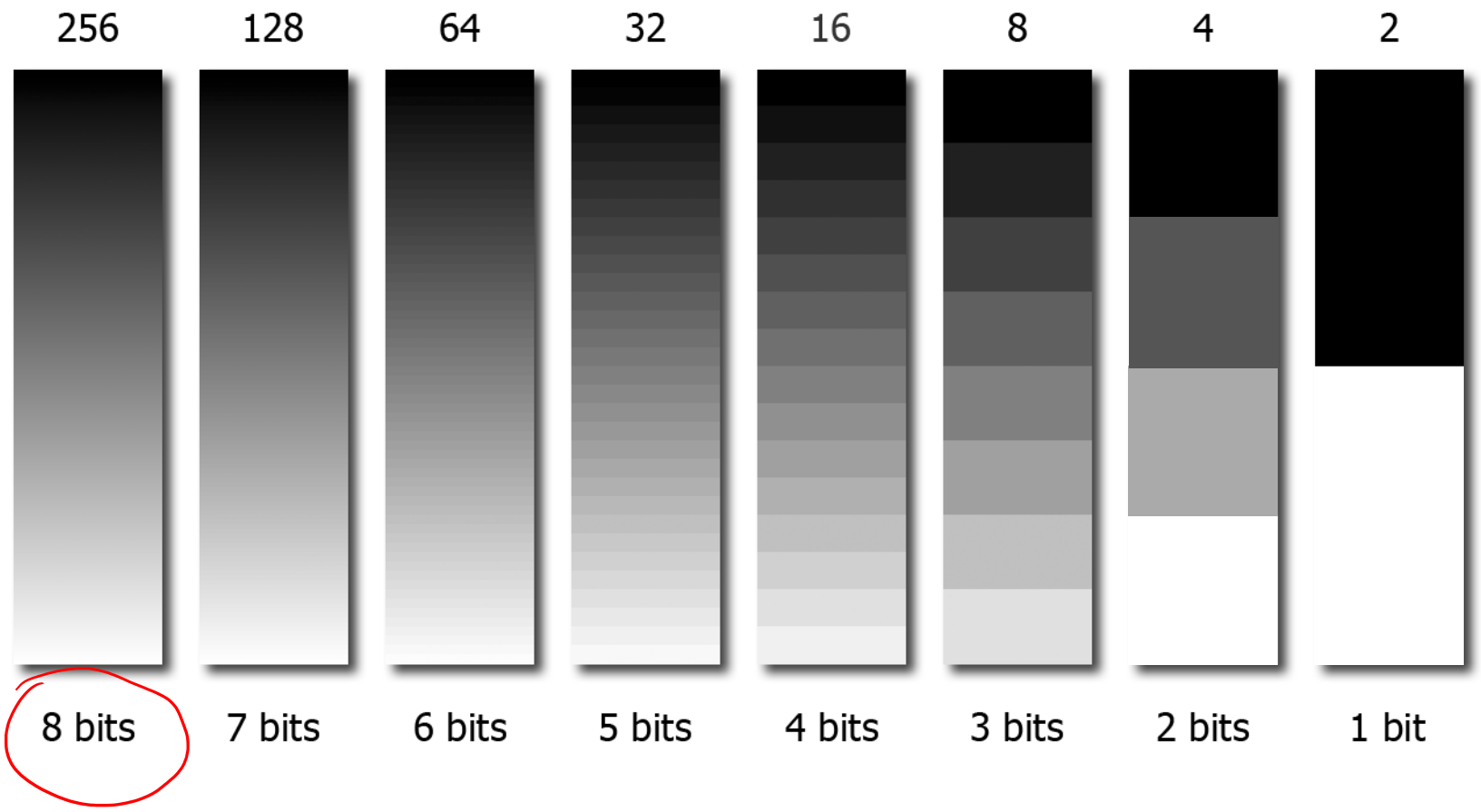
Intensity Level Resolution

Number of Bits	Number of Intensity Levels	Examples
1	2	0, 1
2	4	00, 01, 10, 11
4	16	0000, 0101, 1111
8	256	00110011, 01010101
16	65,536	1010101010101010

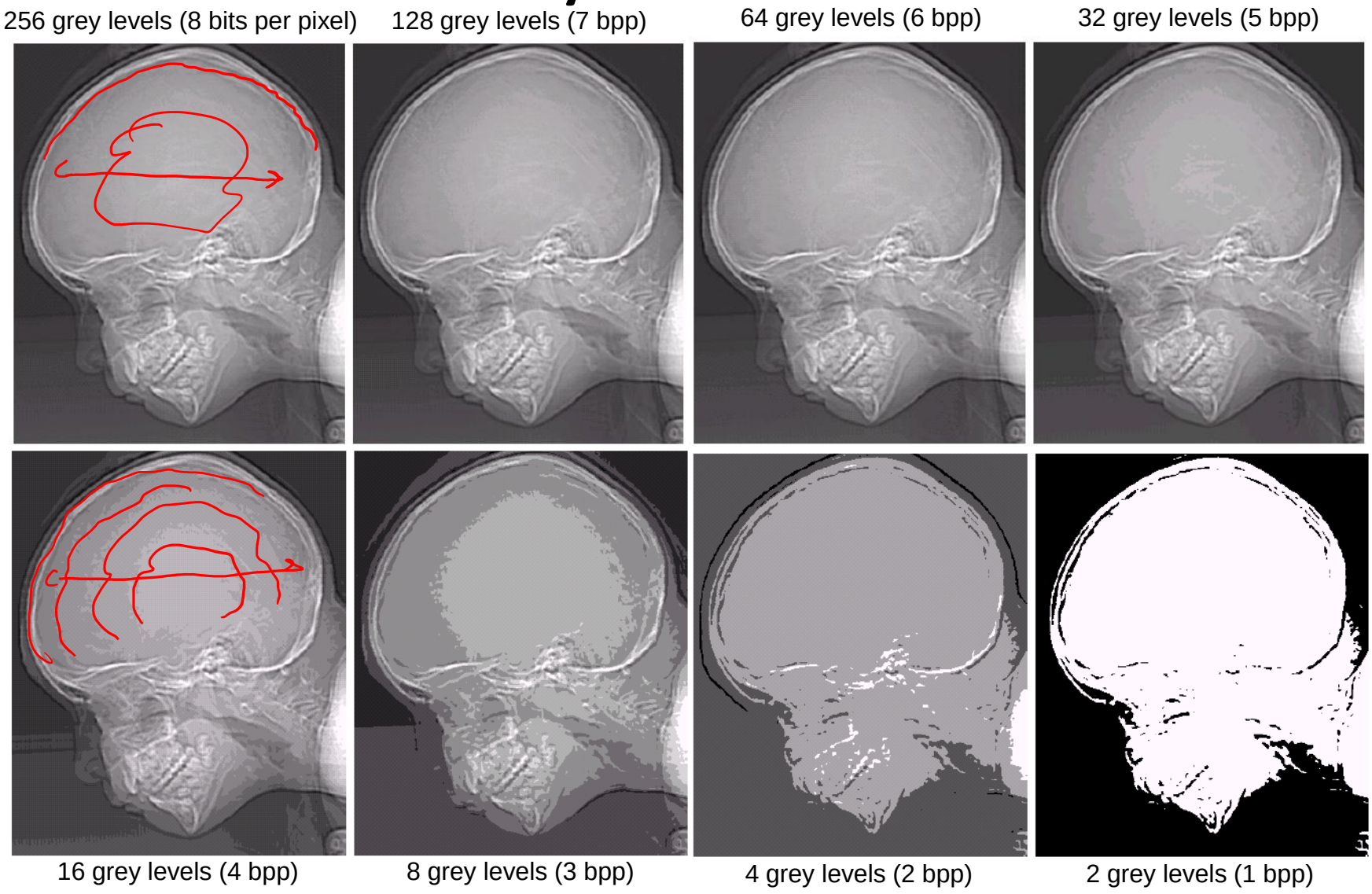
Intensity Level Resolution

M x N x K

false contours/edge



Intensity Level Resolution



Resolution: How much is enough?

- How many samples and gray levels are required for a good approximation?
  - Quality of an image depends on number of pixels and gray-level number
  - The more these parameters are increased, the closer the digitized array approximates the original image
  - But: Storage & processing requirements increase rapidly as a function of N, M, and k

Resolution: How much is enough?

- Depends on what is in the image and what you would like to do with it

Image Resizing

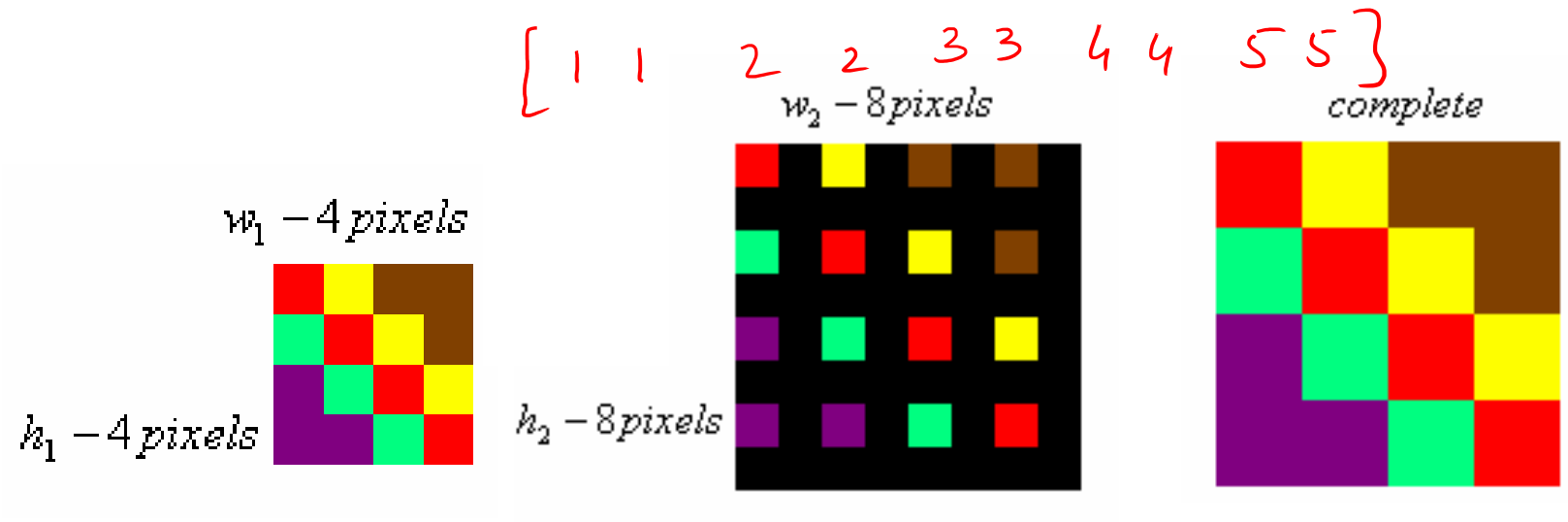
- Upsampling → Interpolation
- Down Sampling → Decimation

- Pixel replication

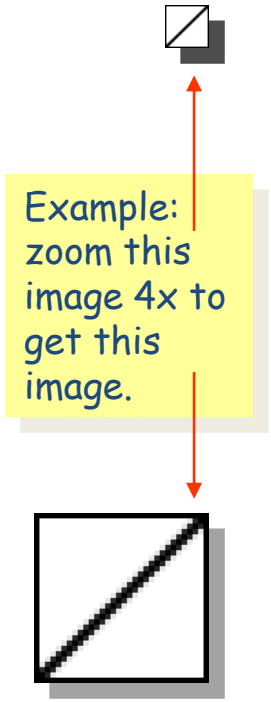
[1 2 3 4 5]

[1 1 2 2 3 3 4 4 5 5] (One step)

[1 1 1 2 2 2 3 3 3 4 4 4 5 5 5] (Two step)

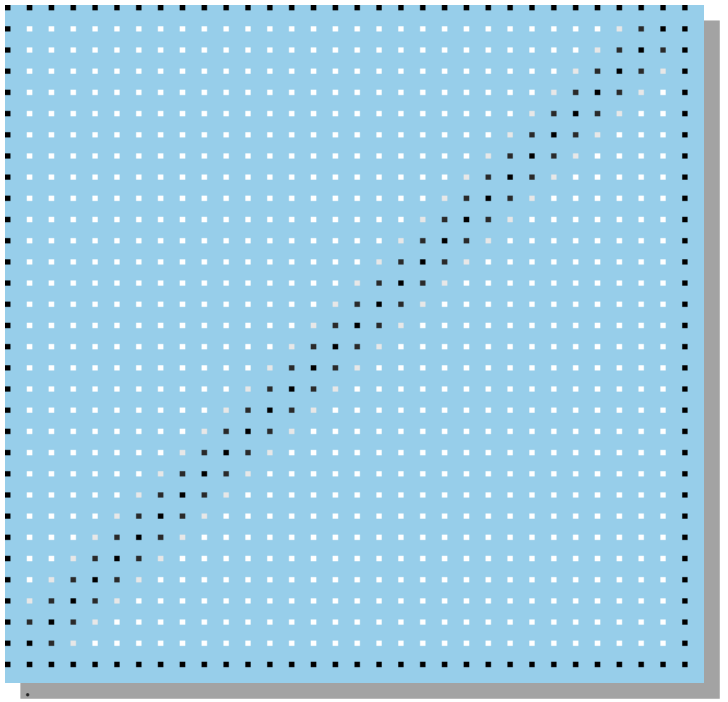


Enlarging an Image



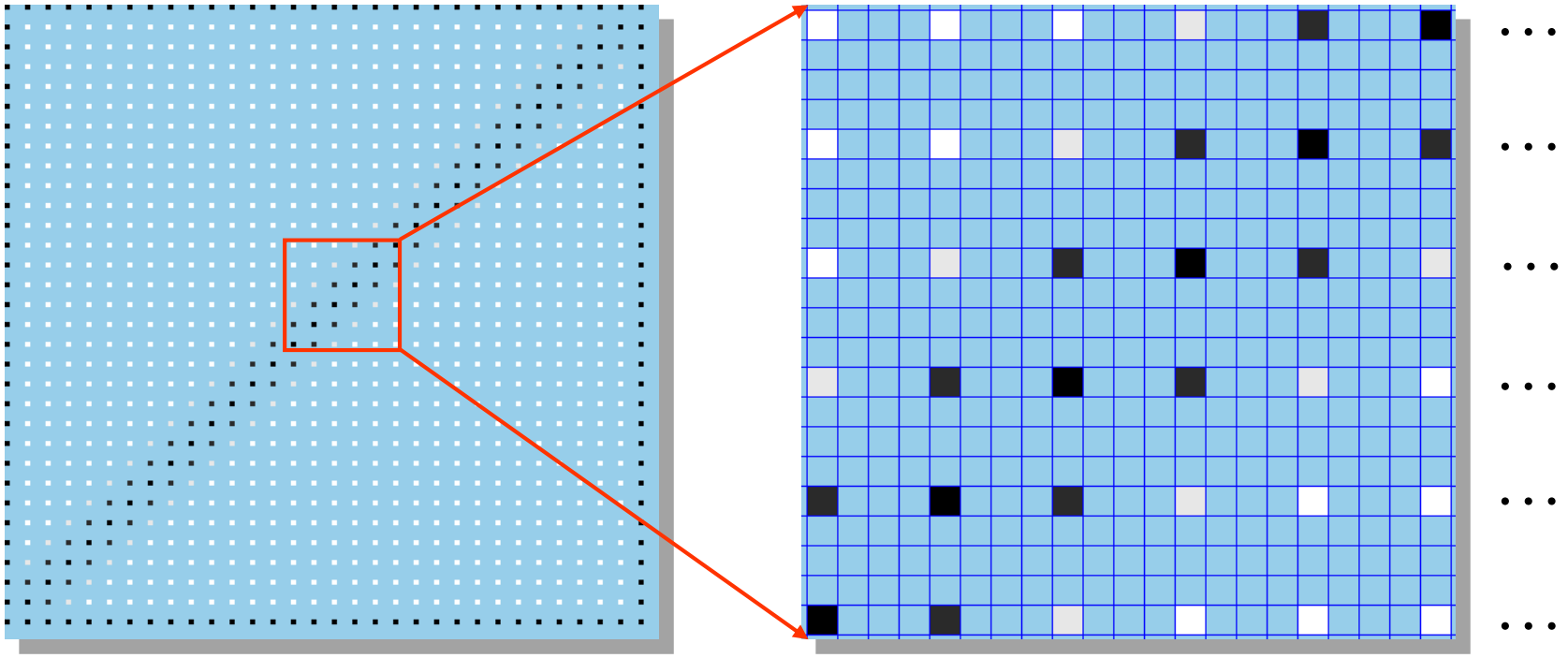
Example: zoom this image 4x to get this image.

Start with a blank image 4 times the linear dimensions of the original.



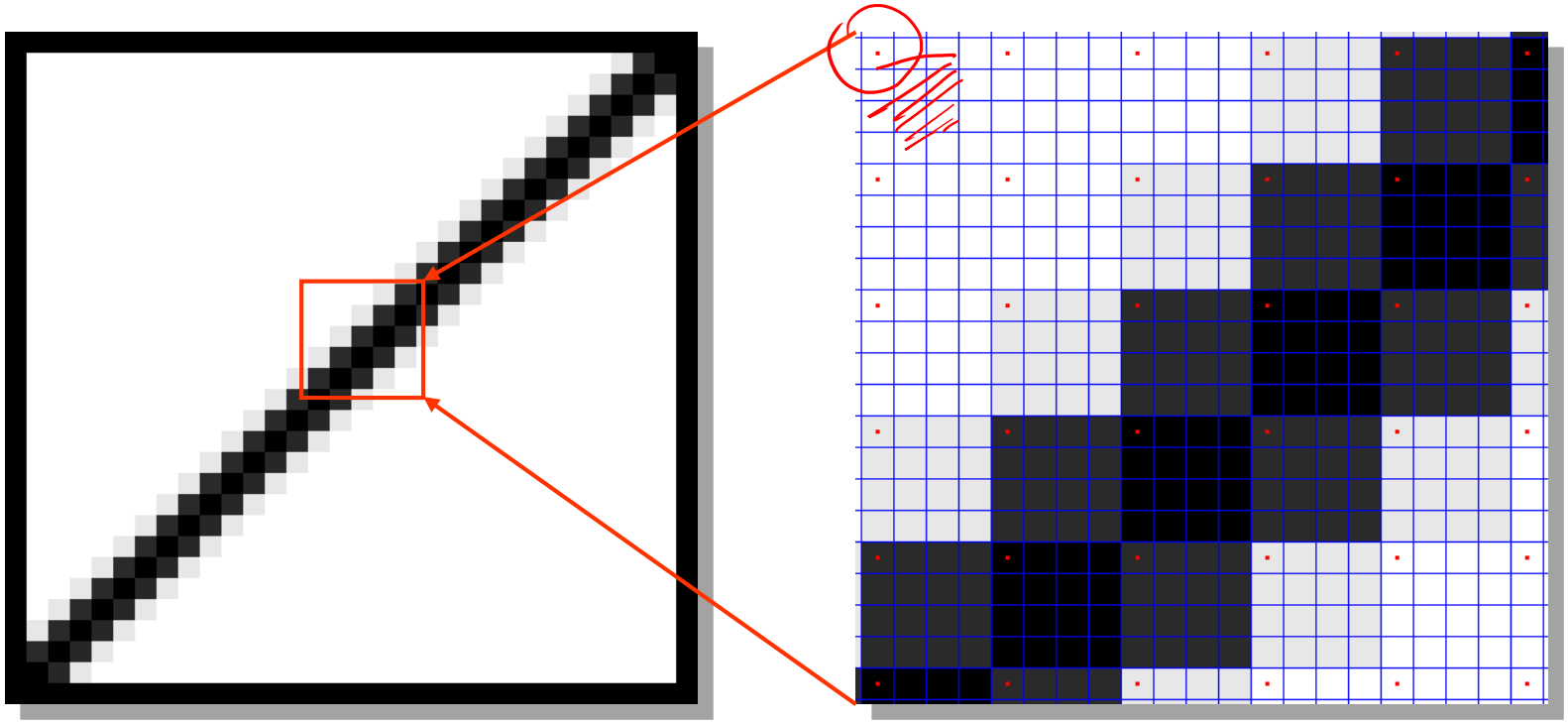
Fill in every 4th pixel in every 4th row with the original pixel values.

Enlarging an Image



Detail showing every 4th pixel in every 4th row with the original pixel values.

Enlarging an Image



Replicate the values

Image Interpolation

- Nearest neighbour interpolation
  - Simple but produces undesired artefacts
- Bilinear Interpolation
  - Contribution from 4 neighbors
- Bicubic Interpolation
  - Contribution from 16 neighbors

3	9
7	8

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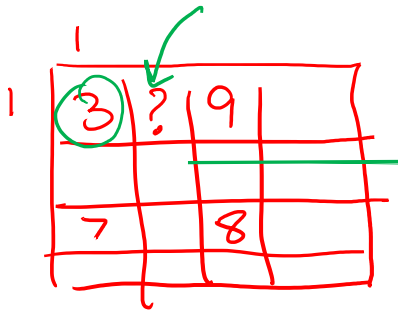
$$Ax + By + Cxy + D = f(x, y) \leftarrow$$



$$f(1,2) = A + 2B + 2C + D \Leftarrow$$

$$f(1,1) = A + B + C + D$$

$$3 = A + B + C + D \quad (1)$$

$$9 = A + 3B + 3C + D \quad (2)$$

$$7 = 3A + B + 3C + D \quad (3)$$

$$8 = 3A + 3B + 9C + D \quad (4)$$

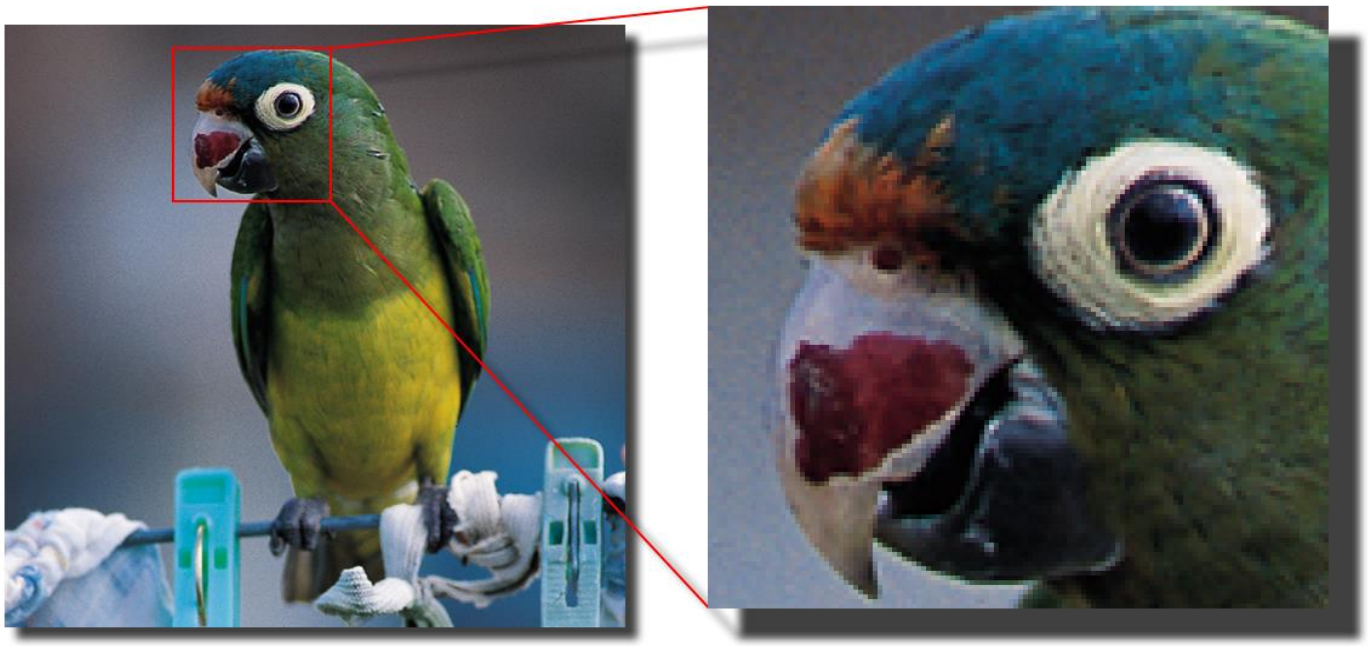
Interpolation: Comparison

We'll enlarge this image by a factor of 4 …

… via bilinear interpolation and compare it to a nearest neighbor enlargement.

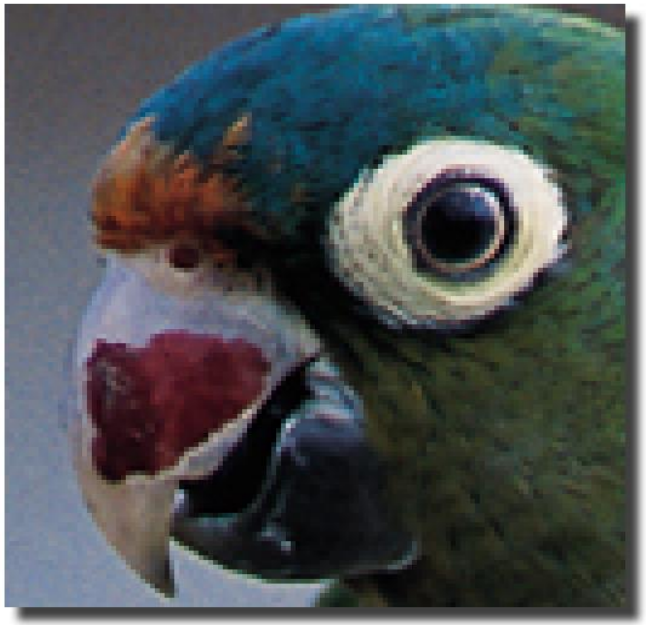
Interpolation: Comparison

Original Image



To better see what happens, we'll look at the parrot's eye.

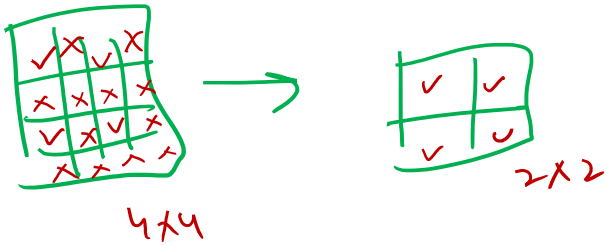
Interpolation: Comparison

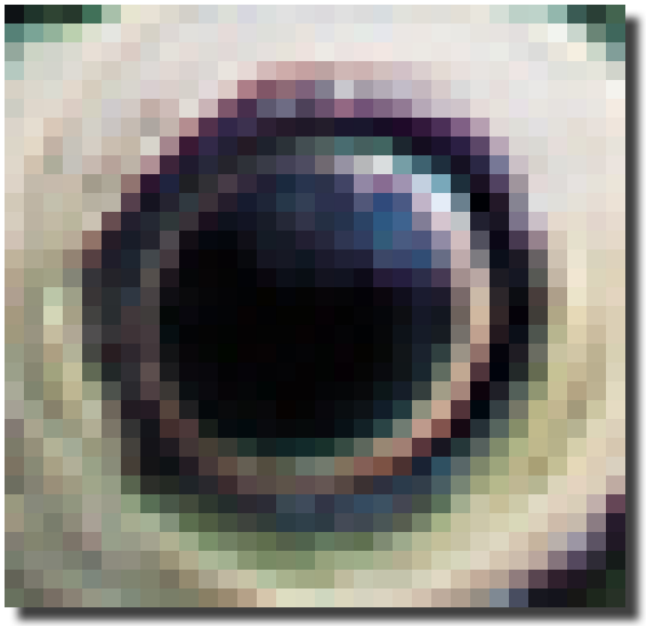


Pixel replication

Bilinear interpolation

Interpolation: Comparison



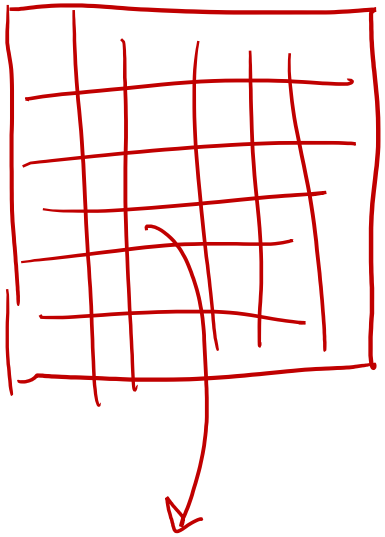


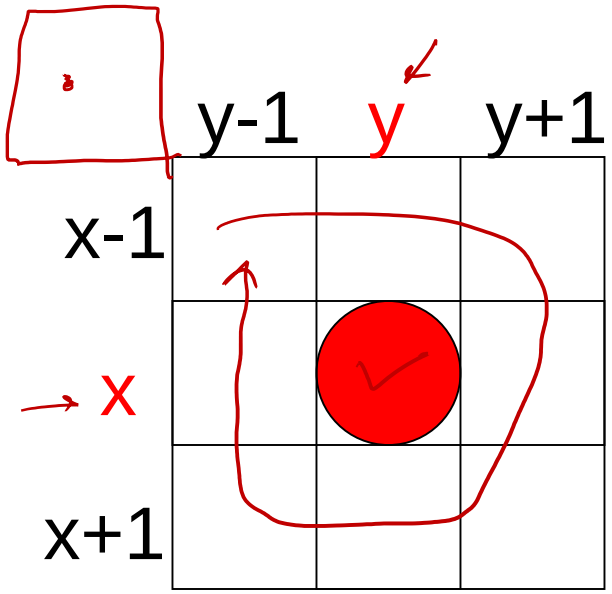
Pixel replication

Bilinear interpolation

Relationships between pixels

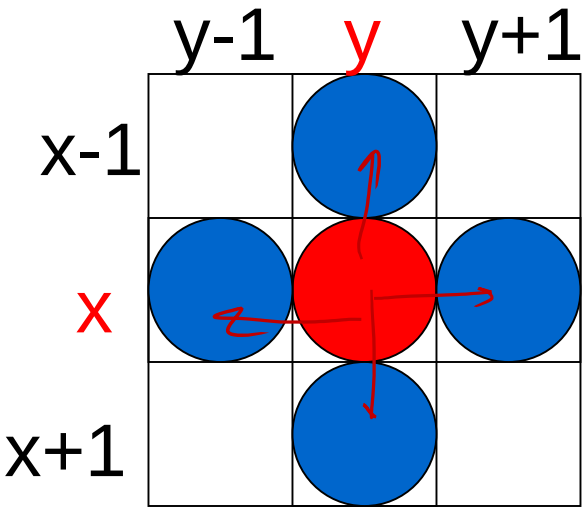
- Neighbors of pixel are the pixels that are adjacent pixels of an identified pixel





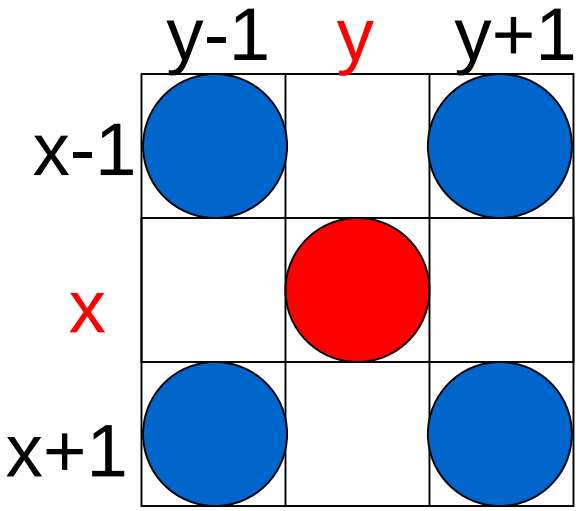
4- Neighbors of a Pixel –$N_4(p)$

What are the coordinates of each of the blue pixels



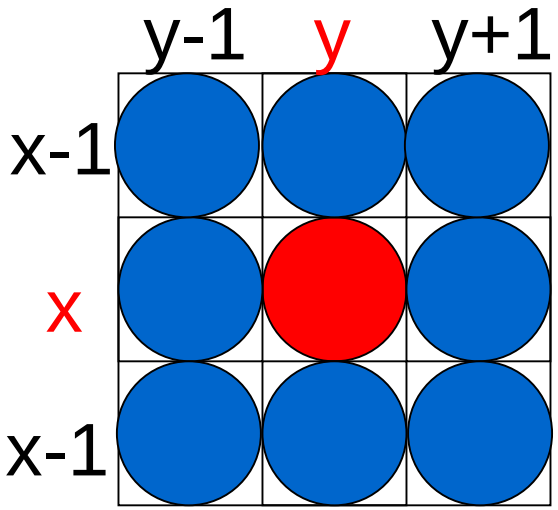
(x-1,y), (x+1,y), (x, y-1), (x, y+1)

Diagonal Neighbors of a Pixel –$N_D(p)$



(x-1,y-1), (x+1,y-1), (x-1, y+1), (x+1, y+1)

8- Neighbors of a Pixel –$N_8(p)$



$$N_8(p) = N_4(p) \cup N_D(p)$$

(x-1,y), (x+1,y), (x, y-1), (x, y+1)

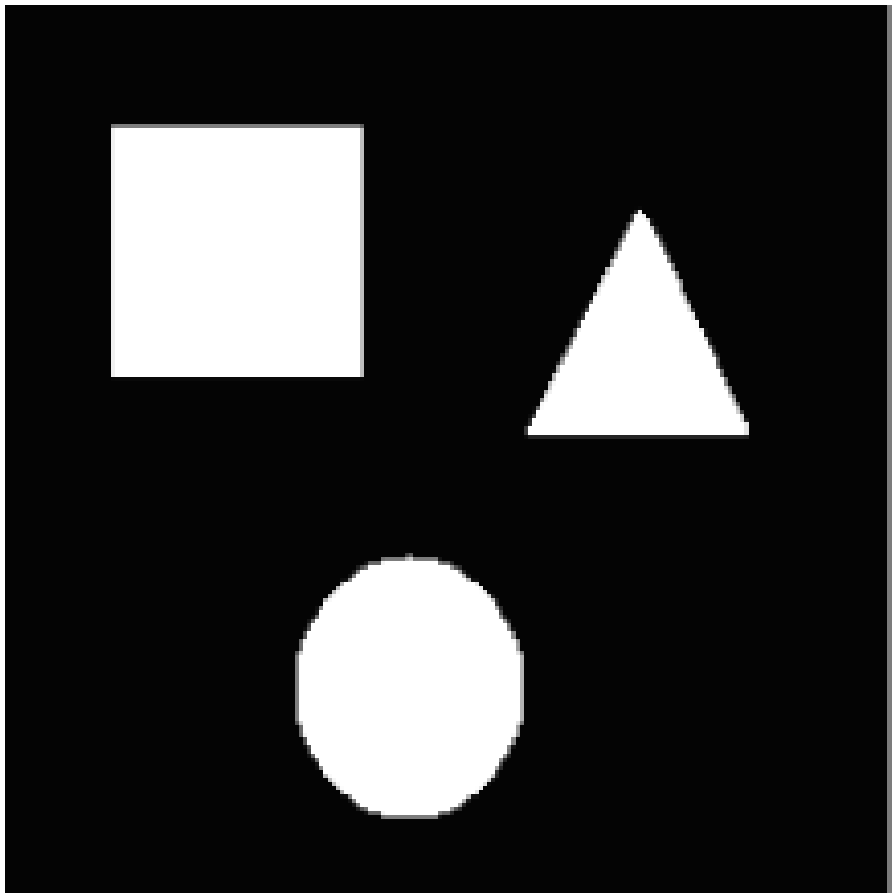
(x-1,y-1), (x+1,y-1), (x-1, y+1), (x+1, y+1)

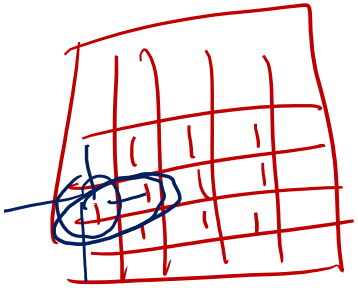
Determine different regions in the image

CCA

/

Connected Component analysis





$V = \{1\}$

Connectivity

- Establishing boundaries of objects and components in an image
- Group the same region by assumption that the pixels being the same color or equal intensity
- Two pixels p & q are connected if
  - *They are adjacent in some sense*
  - *If their gray levels satisfy a specified criterion of similarity*

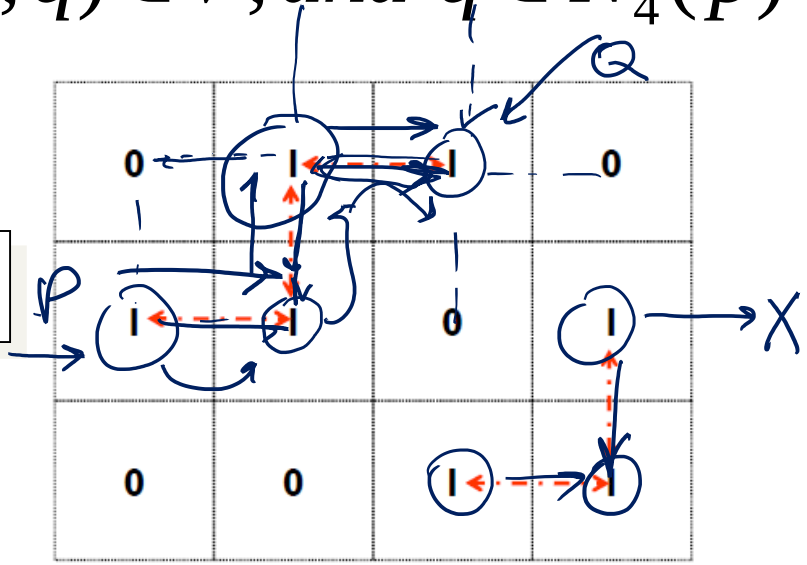
Connectivity

V: Set of gray levels used to define the criterion of similarity

4-connectivity

If gray level $(p,q) \in V, and\ q \in N_4(p)$

Set of gray levels V = {1}



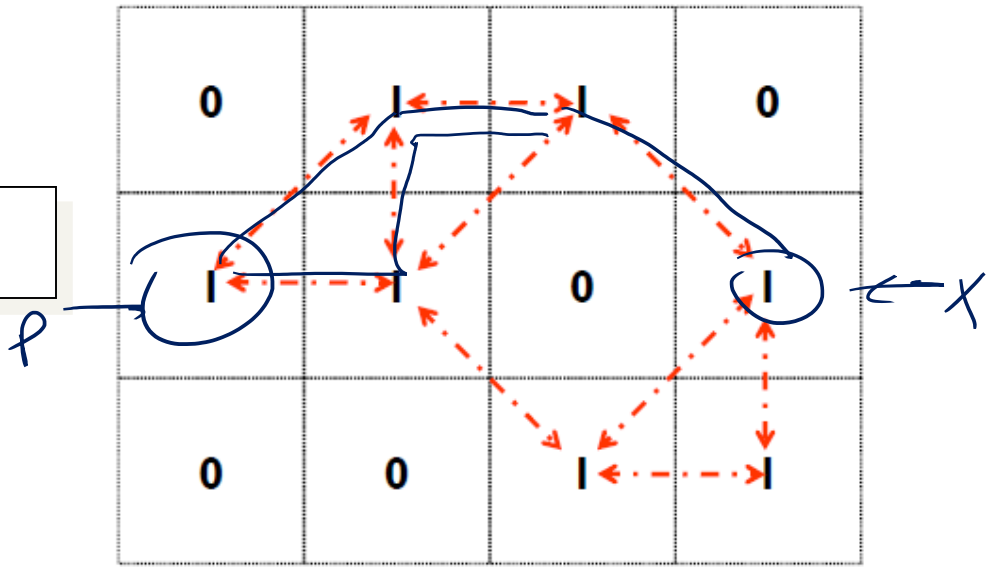
Connectivity

V: Set of gray levels used to define the criterion of similarity

8-connectivity

If gray level $(p,q) \in V, and\ q \in N_8(p)$

Set of gray levels V = {1}



Connectivity

V: Set of gray levels used to define the criterion of similarity

m-connectivity (Mixed Connectivity)

If gray level

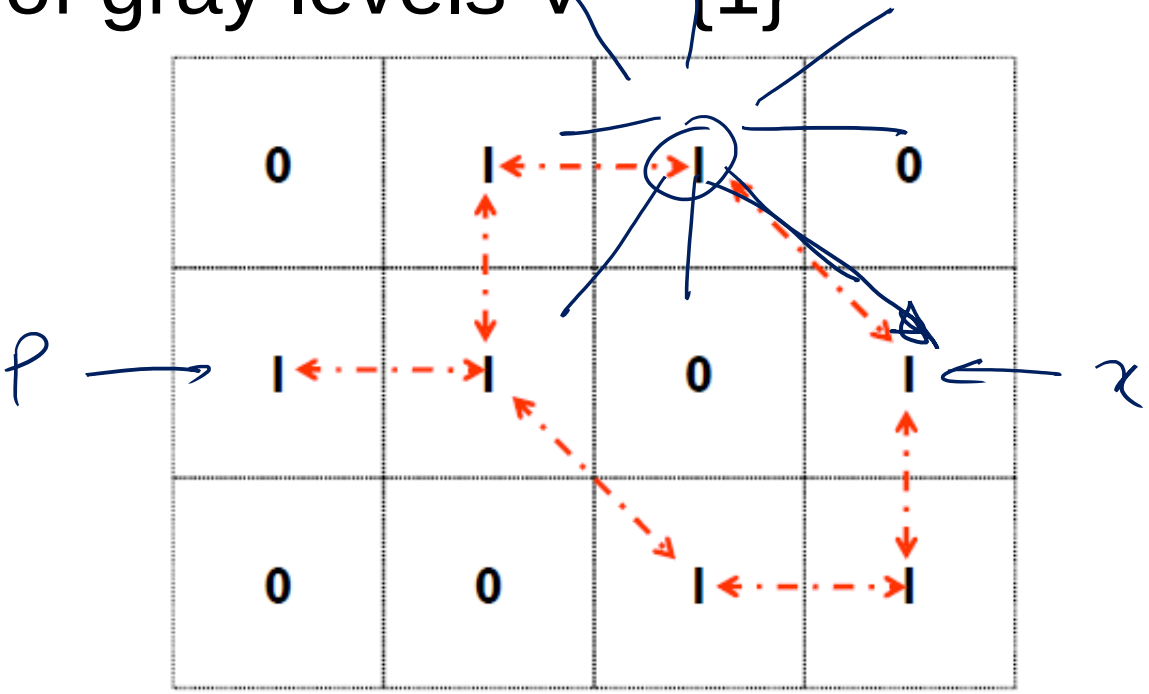
$(p,q) \in V$, *and* q satisfies one of the following:

a. $q \in N_4(p)$ *or*

b. $q \in N_D(p)$ *And* $N_4(p) \cap N_4(q)$ has no pixels whose values are from V

Example: m – Connectivity

- Set of gray levels V = {1}



Note: Mixed connectivity can eliminate the multiple path connections that often occurs in 8-connectivity

Paths

- Path: Let coordinates of pixel *p: (x, y)*, and of pixel *q: (s, t)*
- A *path* from *p* to *q* is a sequence of distinct pixels with coordinates: $(x_0, y_0), (x_1, y_1), \ldots\ldots, (x_n, y_n)$

  where $(x_0, y_0) = (x, y)$ *&* $(x_n, y_n) = (s, t)$*, and* (x_i, y_i) *is adjacent to* (x_{i-1}, y_{i-1}) $1 \leq i \leq n$

Test Yourself

Consider the image segment shown.

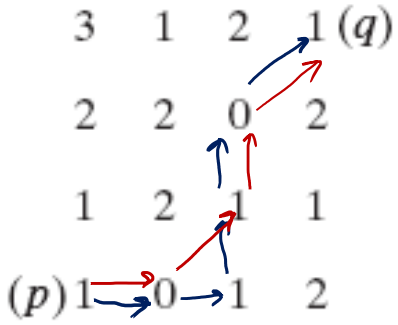
(a) Let $V = \{0, 1\}$ and compute the lengths of the shortest 4-, 8-, and m-path between p and q. If a particular path does not exist between these two points, explain why.

(b) Repeat for $V = \{1, 2\}$.

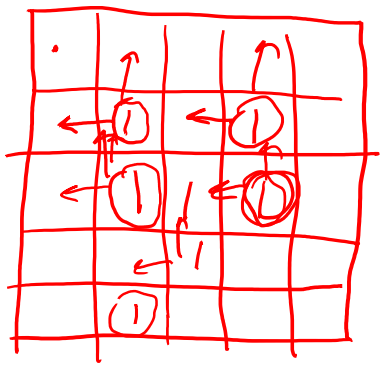
4 → ×

m → ✓ → 5

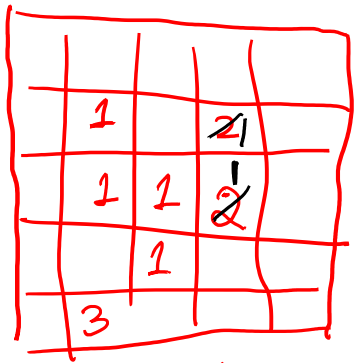
8 → ✓ → 4



$V = \{1\}$



Image



Label Mat A

Eg. List

→ {1}, ~~{2}~~

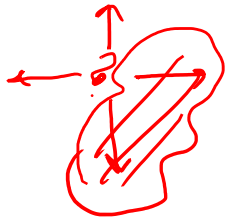
→ ~~{2}~~ {3}

⇓

# of objs

{1} → 6

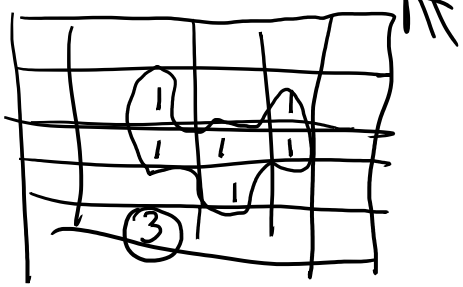
{3} → 1



B ← •

$y[n] = x[n] - x[n-1]$

$y[n] = x[n+1] - x[n]$



- Label = 0

for any pixel → V{1}

See A + B

① if both ≠ V

assign a new Label

② if any one = V

assign same Label

③ both = V

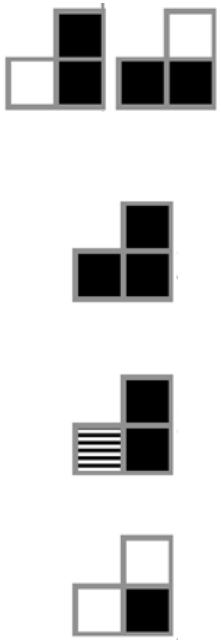
a. both with same Label

↳ assign that label

b. both diff

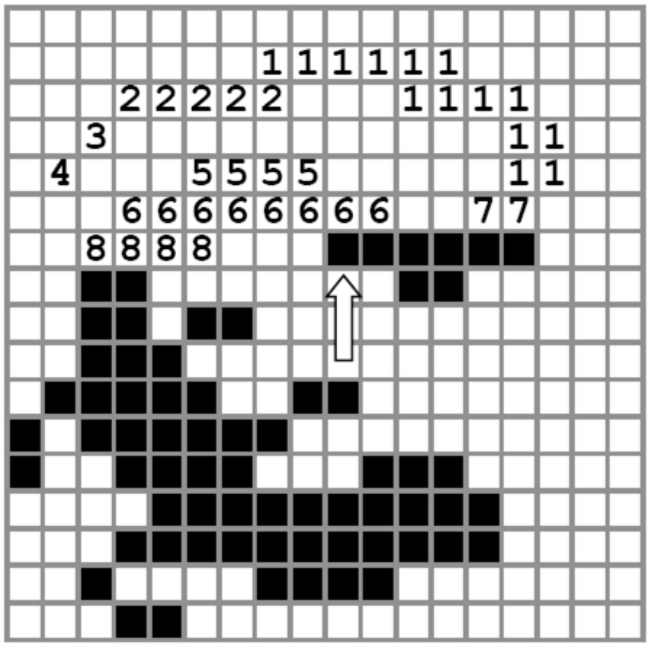
- assign priority label
- update eq table

CC labeling – 4 Connectivity



- Process the image from left to right, top to bottom:
  1.) If the next pixel to process is 1
    i.) If only one of its neighbors (top or left) is 1, copy its label.
    ii.) If both are 1 and have the same label, copy it.
    iii.) If they have different labels
      – Copy the label from the left.
      – Update the equivalence table.
    iv.) Otherwise, assign a new label.

Pass 1

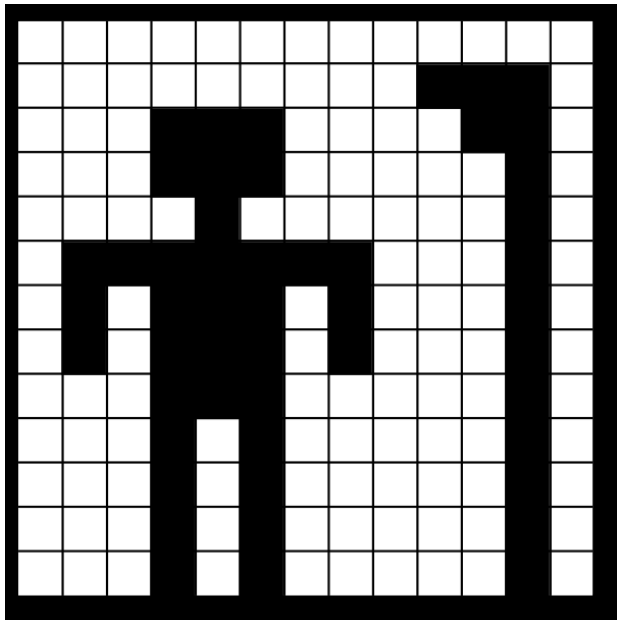


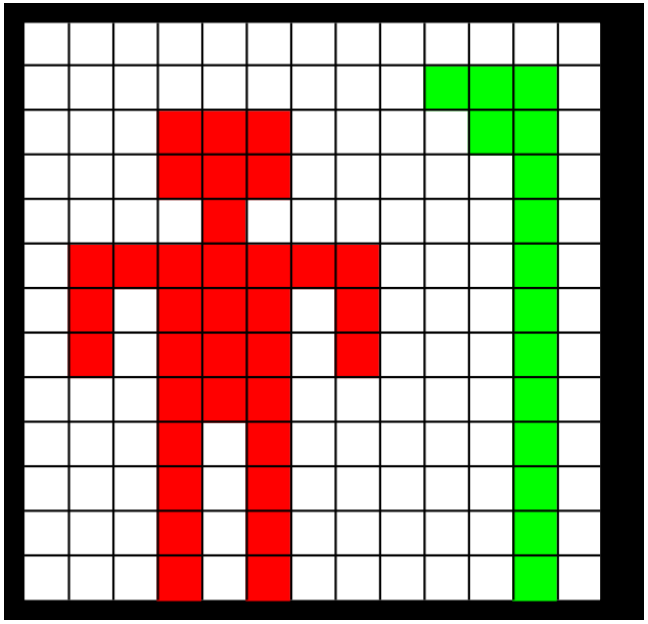
- Re-label with the smallest of equivalent labels

Pass 2

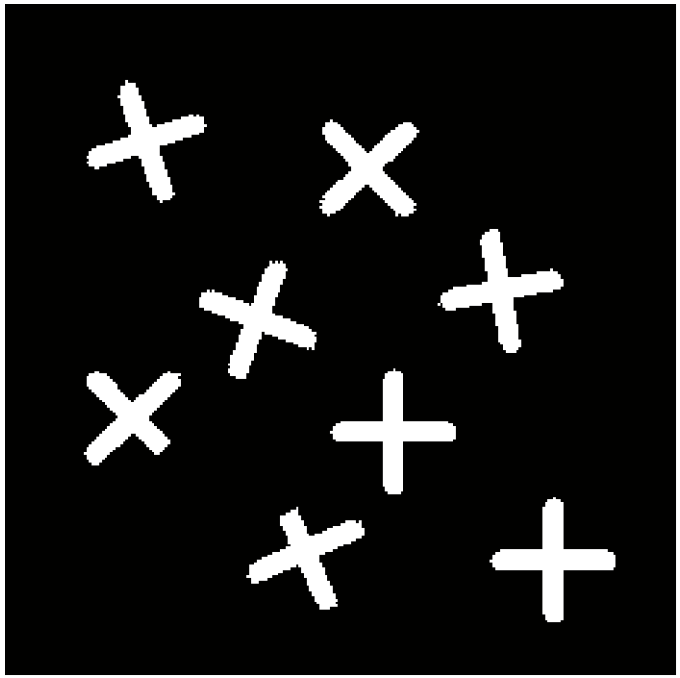
{1, 2, 7}
{3}
{4}
{5, 6, 8}

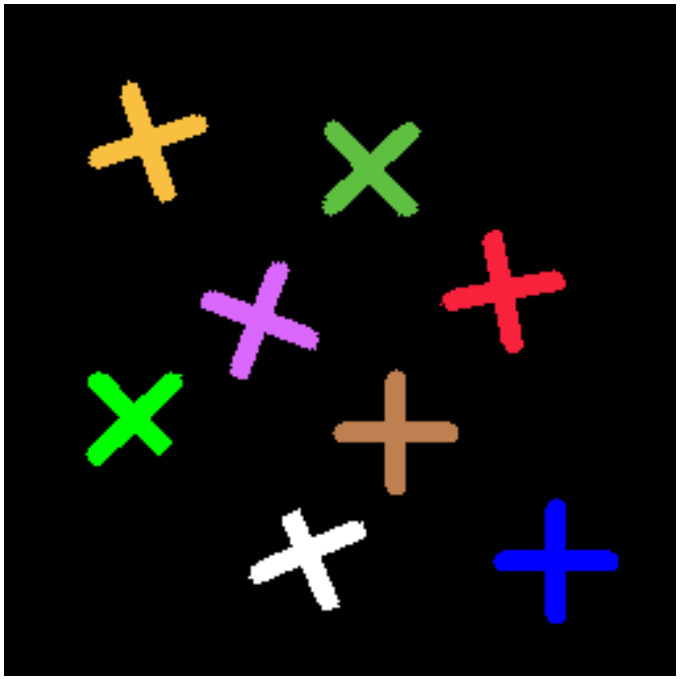
CC labeling – 4 Connectivity





CC labeling – 4 Connectivity

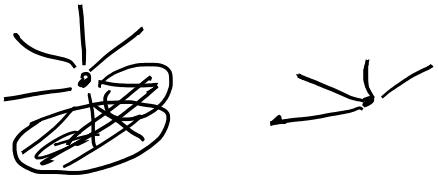


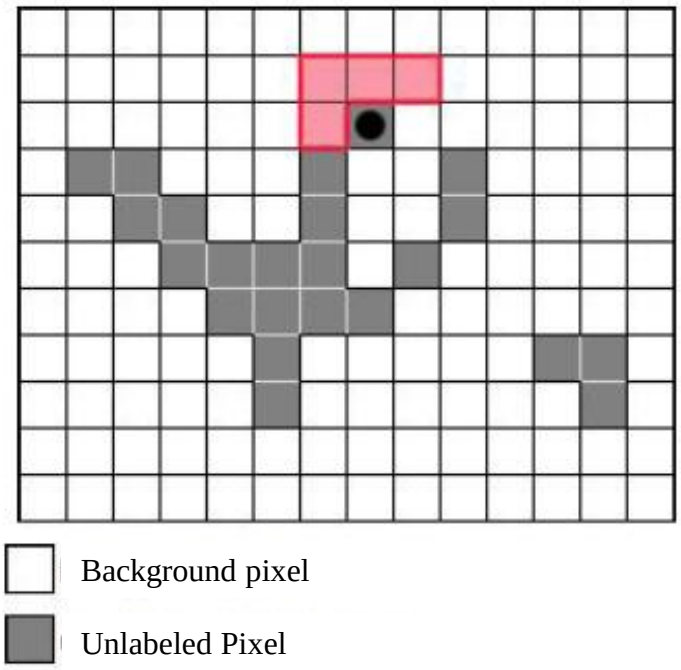


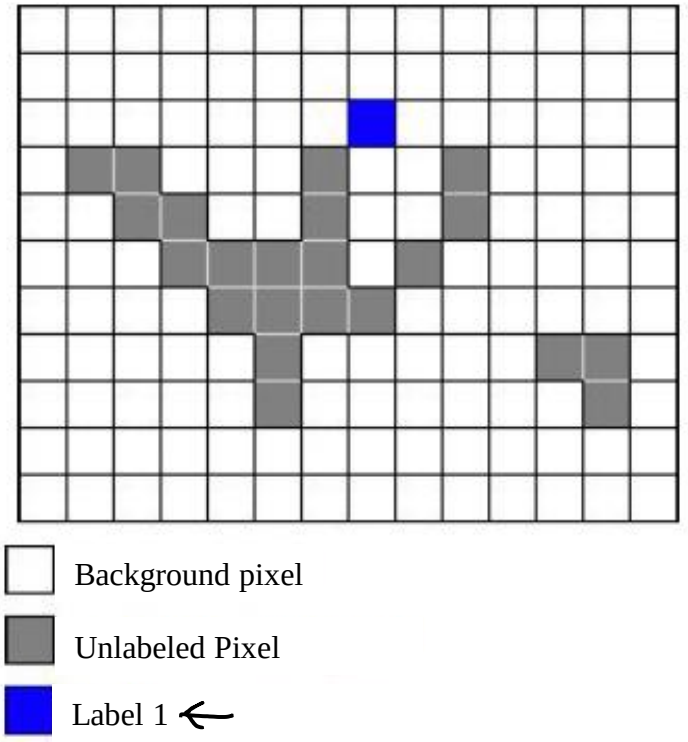
CC labeling – 8 Connectivity

Same algorithm but examine also the upper diagonal neighbors of *p*

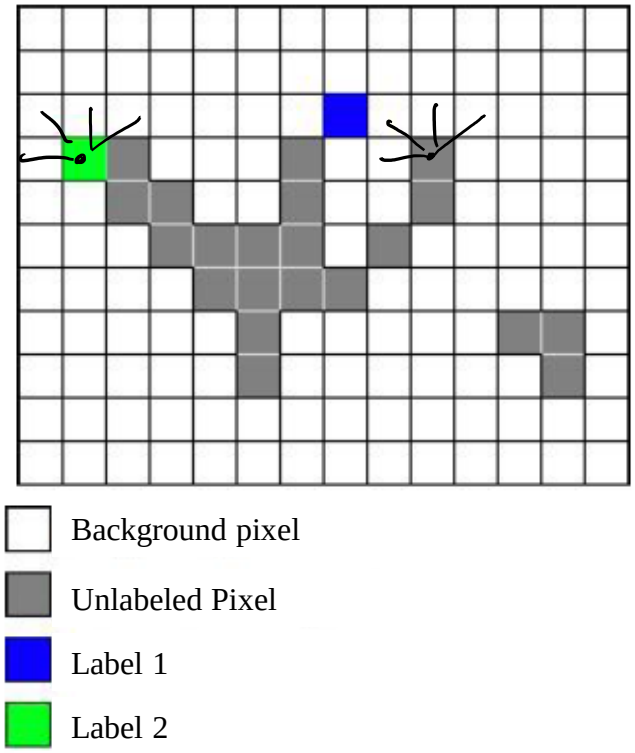
CC labeling – 8 Connectivity

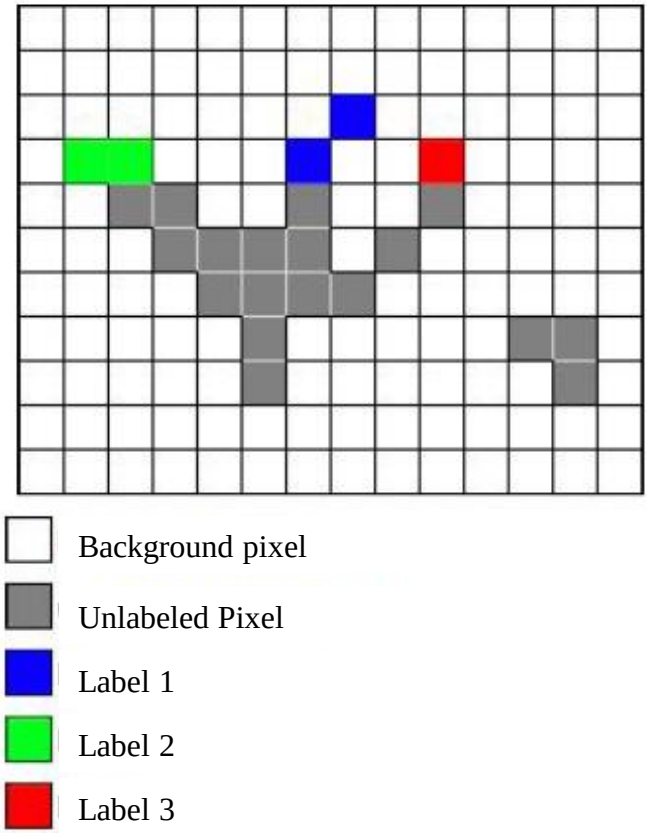




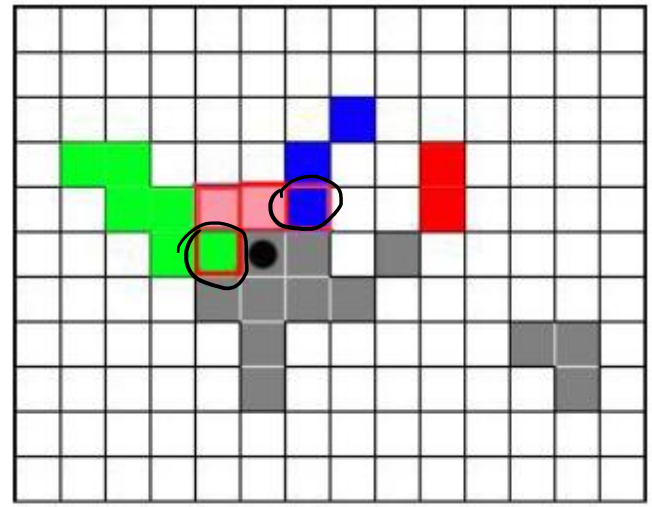


CC labeling – 8 Connectivity





CC labeling – 8 Connectivity



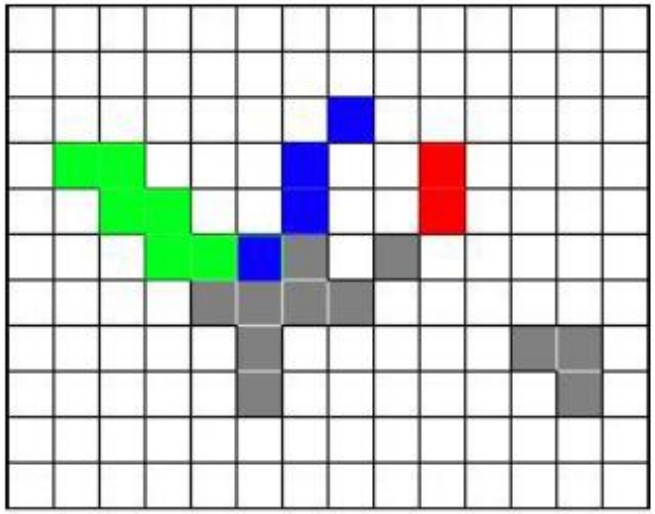
Background pixel

Unlabeled Pixel

Label 1

Label 2

Label 3



Background pixel

Unlabeled Pixel

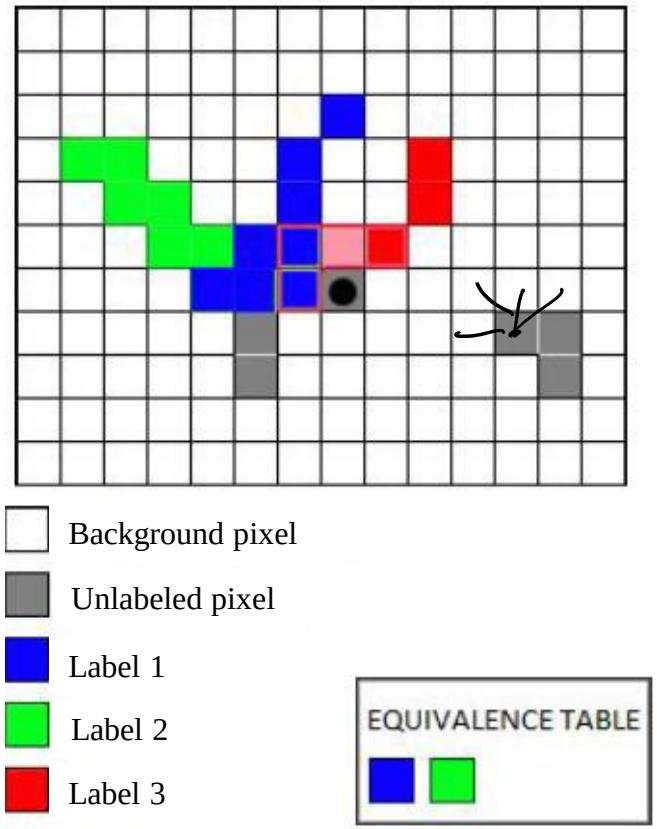
Label 1

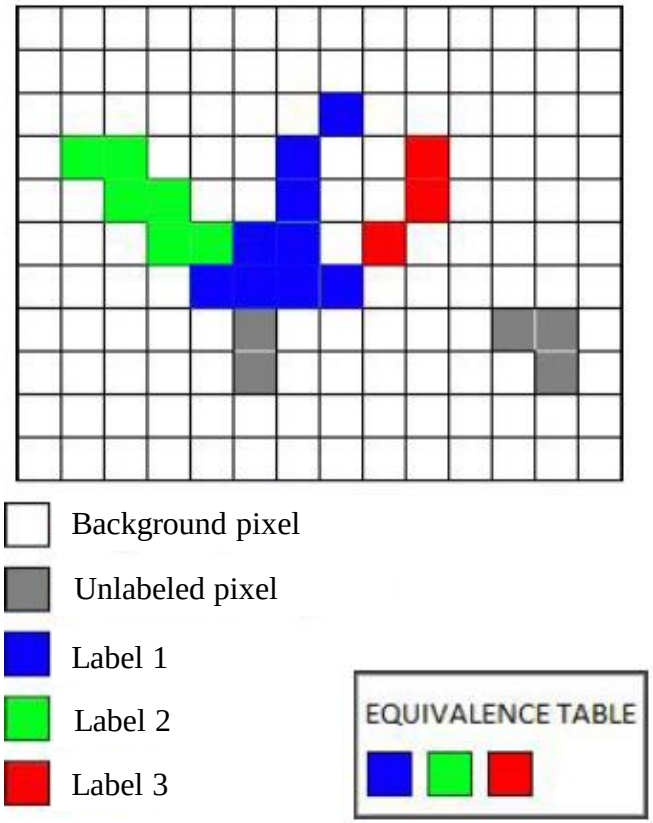
Label 2

Label 3

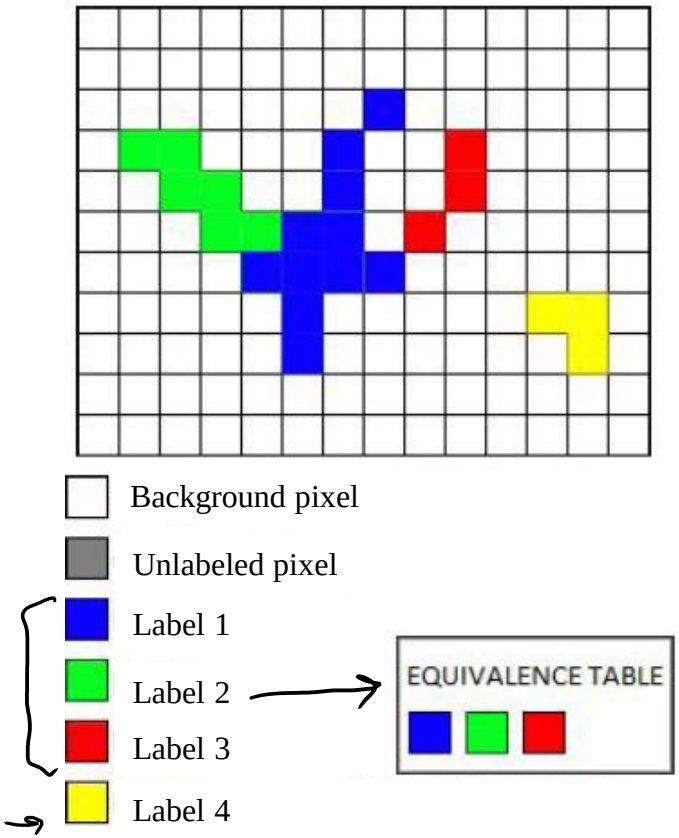
EQUIVALENCE TABLE

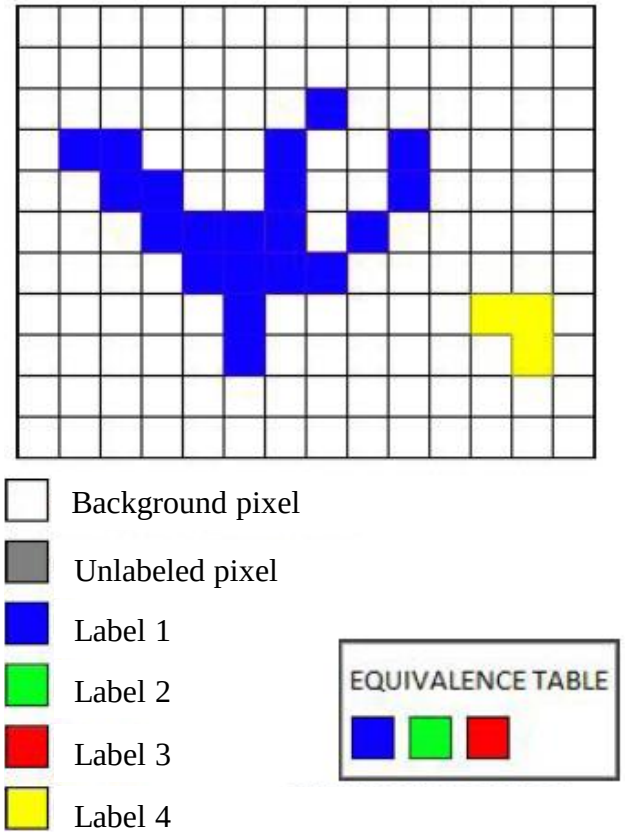
CC labeling – 8 Connectivity

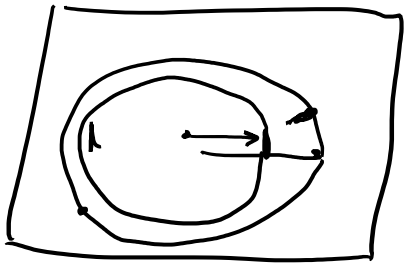




CC labeling – 8 Connectivity







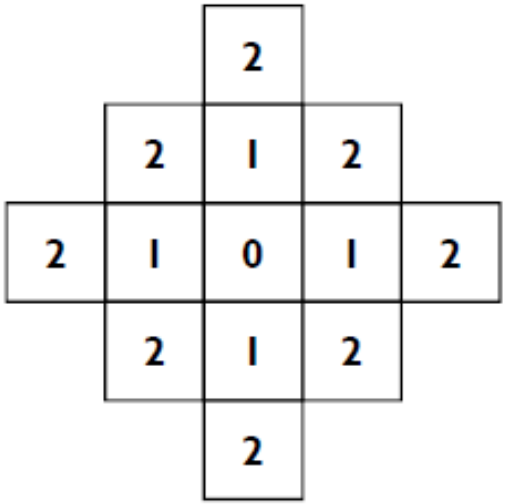
Distance Metrics

- Let pixels *p*, *q* and *z* have coordinates *(x,y)*, *(s,t)* and *(u,v)* respectively.

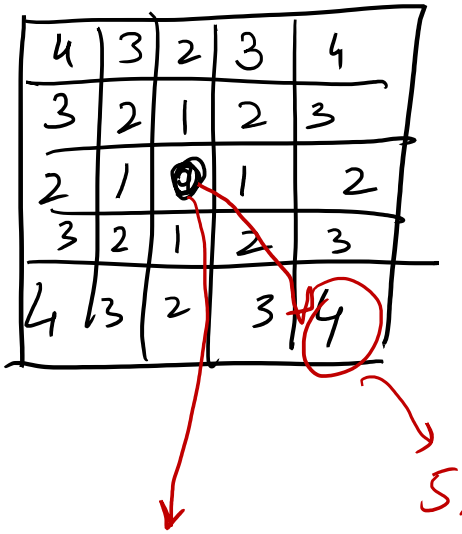
- D is a distance function or metric if
  - *D(p,q) ≥ 0 and*
  - *D(p,q) = 0 iff p = q and*
  - *D(p,q) = D(q,p) and*
  - *D(p,z) ≤ D(p,q) + D(q,z)*

City block distance (D_4 distance)

$$D_4(p,q) = |x-s| + |y-t|$$



- Diamond with center at *(x,y)*
- D_4 = 1 are the 4 neighbors of pixel *p(x,y)*



$$D_4(P,Q) = |x_1 - x_2| + |y_1 - y_2|$$

$$= |3-5| + |3-5|$$

$$= 4$$

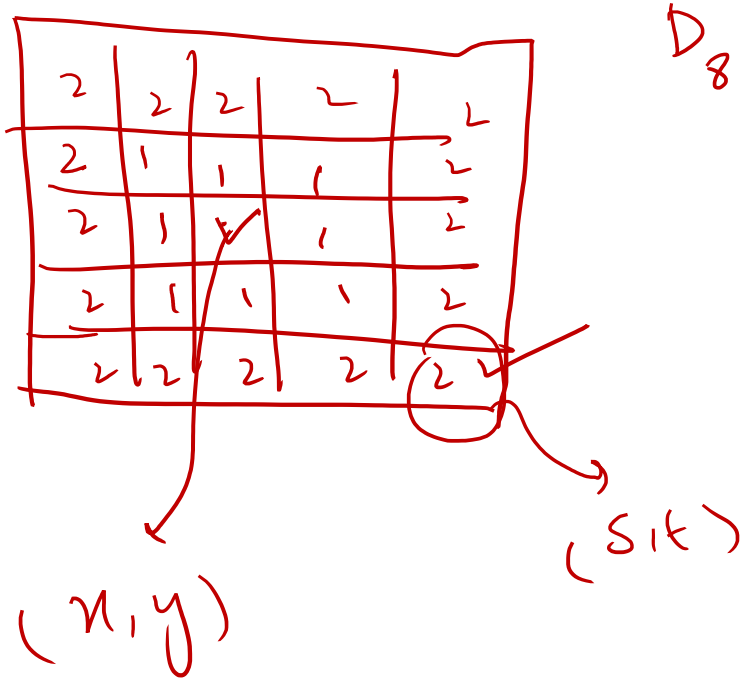
$$D_4(P,Q) = |x-s| + |y-t|$$

Chessboard distance (D_8 distance)

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

- Square centered at *p(x,y)*
- D_8 = 1 are the 8 neighbors of pixel *p(x,y)*



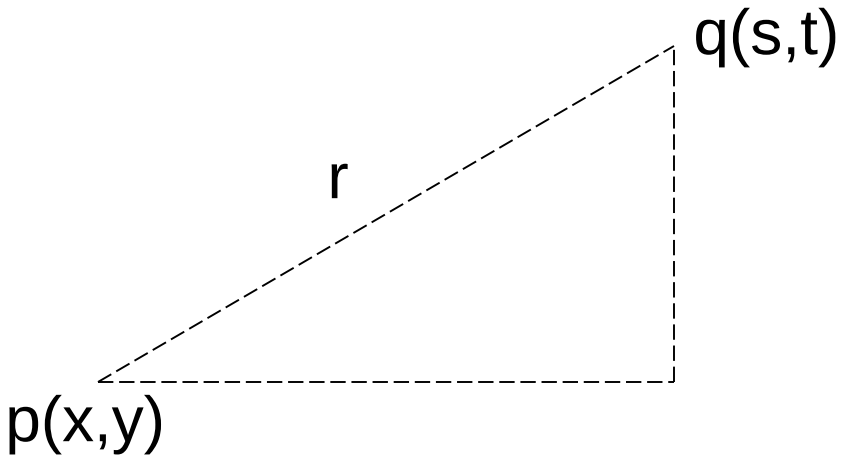
$$D_8(P,Q) = \max\left(|x-s|, |y-t|\right)$$

$$\max\left(|3-5|, |3-5|\right)$$

$$= 2$$

Euclidean Distance

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$



A circle with radius *r* centered at *(x,y)*

Arithmetic Operations

- Carried out between corresponding pixel pairs

$$s(x,y) = f(x,y) + g(x,y)$$

$$d(x,y) = f(x,y) - g(x,y)$$

$$p(x,y) = f(x,y) \times g(x,y)$$

$$d(x,y) = f(x,y) \div g(x,y)$$

Arithmetic Operations

- Conversion to range 0 – 255
- Difference of two 8-bit images: -255 to 255
- Sum of two 8-bit images: 0 to 510
- Solution?

Set all values < 0 to 0

Set all values > 255 to 255

Full range of arithmetic operation not captured

Data Normalization

Arithmetic Operations

- First perform the operation

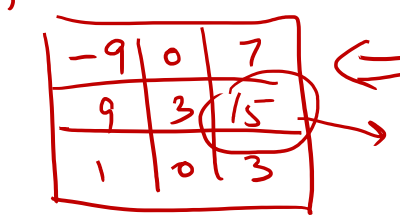
$$f_m = f - \min(f)$$

Creates an image whose minimum value is 0

- Then perform

$$f_s = K\left[f_m / \max(f_m) \right]$$

Creates a scaled image f_s with values in the range [0 K]



$K = 2^l - 1 = 7$

Assignment-1

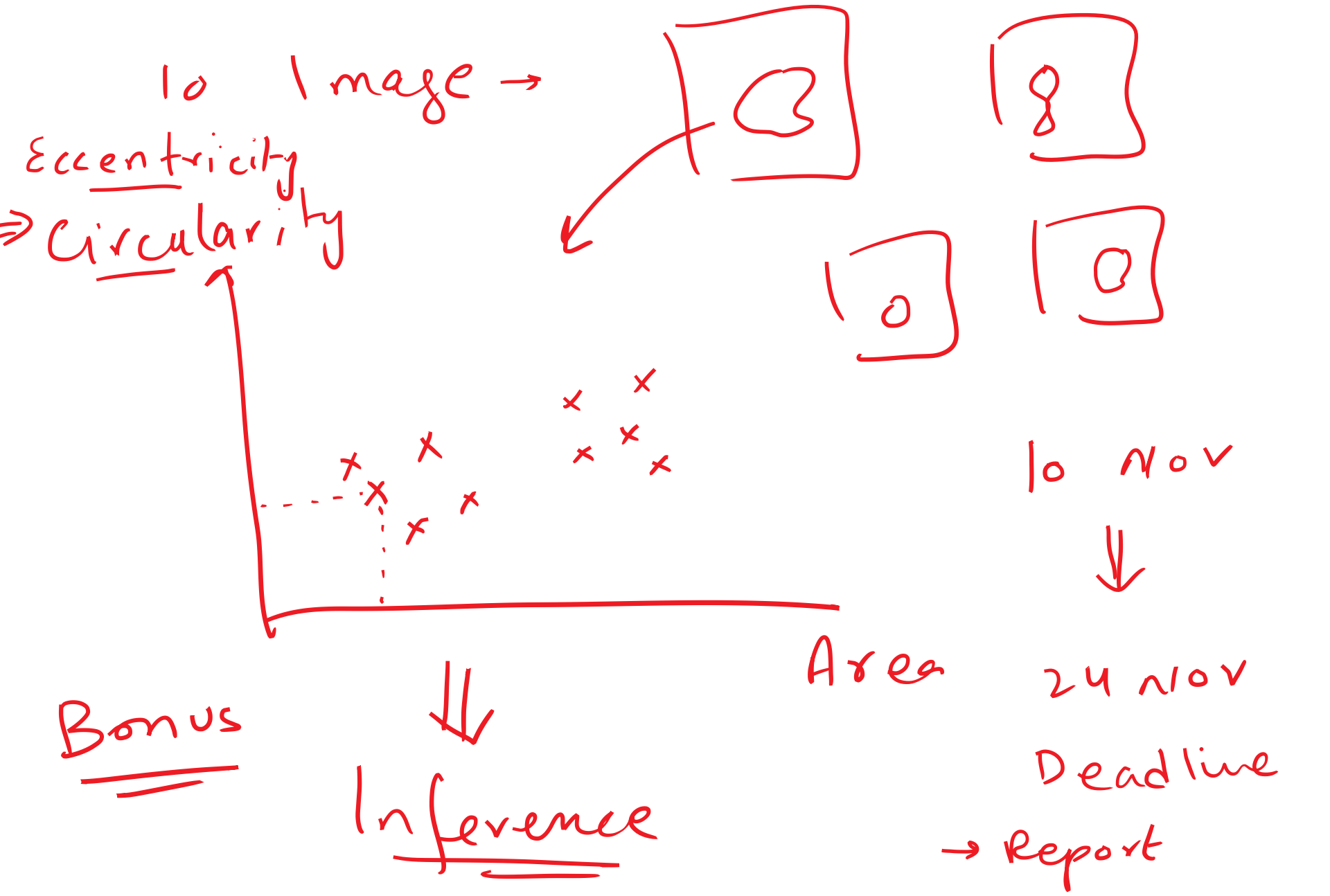
CCA

- Set of binary Images + gray images

Task

1. # of objects → single obj
2. if more objs → remove unwanted objs
3. remaing obj → Mean intensity
4. objs → circularity valve

5. Scatter plot


10 Image →
Eccentricity
⇒ Circularity
Area
10 Nov
24 Nov
Deadline
→ Report
Bonus
Inference

Readings from Book (4th Edn.)

- Chapter – 2

Acknowledgements

- Statistical Pattern Recognition: A Review – A.K Jain et al., PAMI (22) 2000
- Pattern Recognition and Analysis Course – A.K. Jain, MSU
- *Pattern Classification"* by Duda et al., John Wiley & Sons.
- Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- Machine Vision: Automated Visual Inspection and Robot Vision", David Vernon, Prentice Hall, 1991
- www.eu.aibo.com/
- Advances in Human Computer Interaction, Shane Pinder, InTech, Austria, October 2008

Material in these slides has been taken from, the following resources