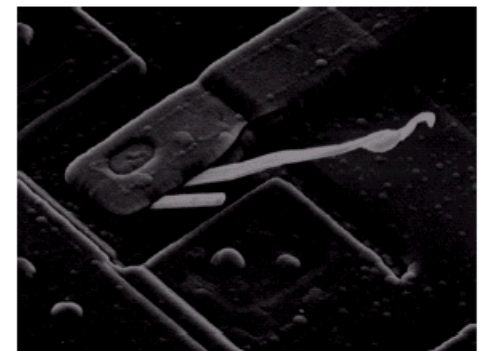
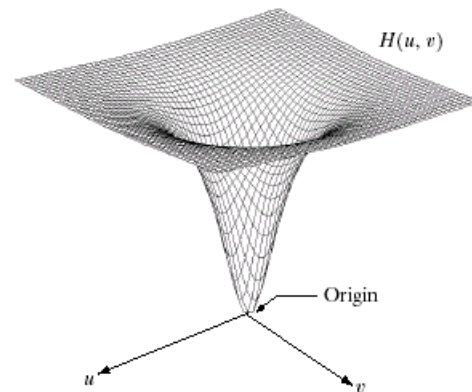
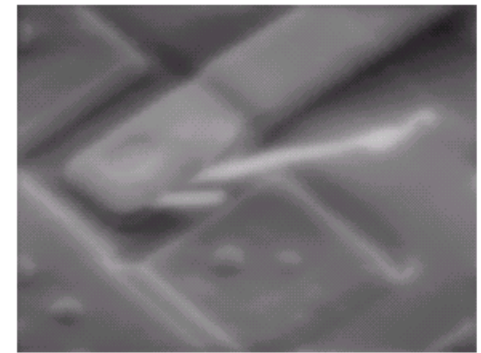
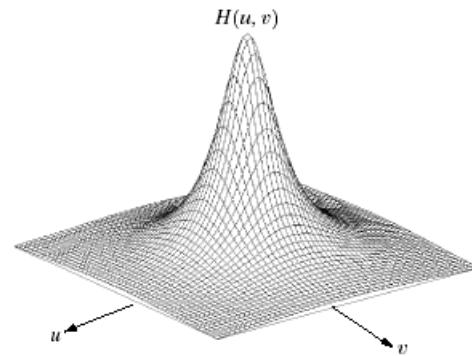


# Digital Image Processing

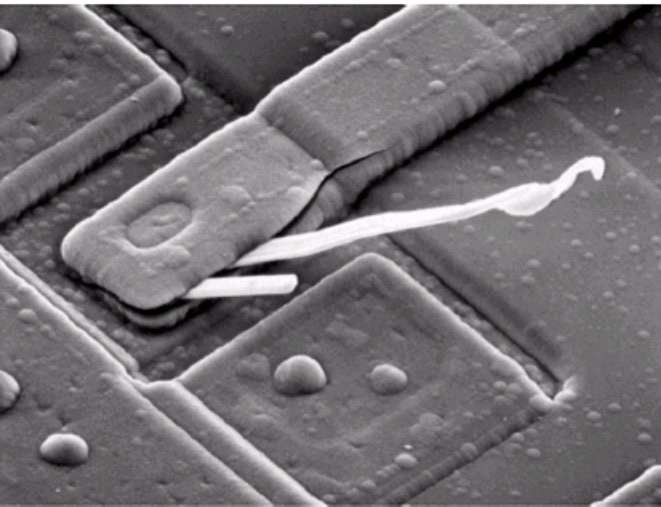
## **Lecture # 8** **Frequency Domain Image Analysis**

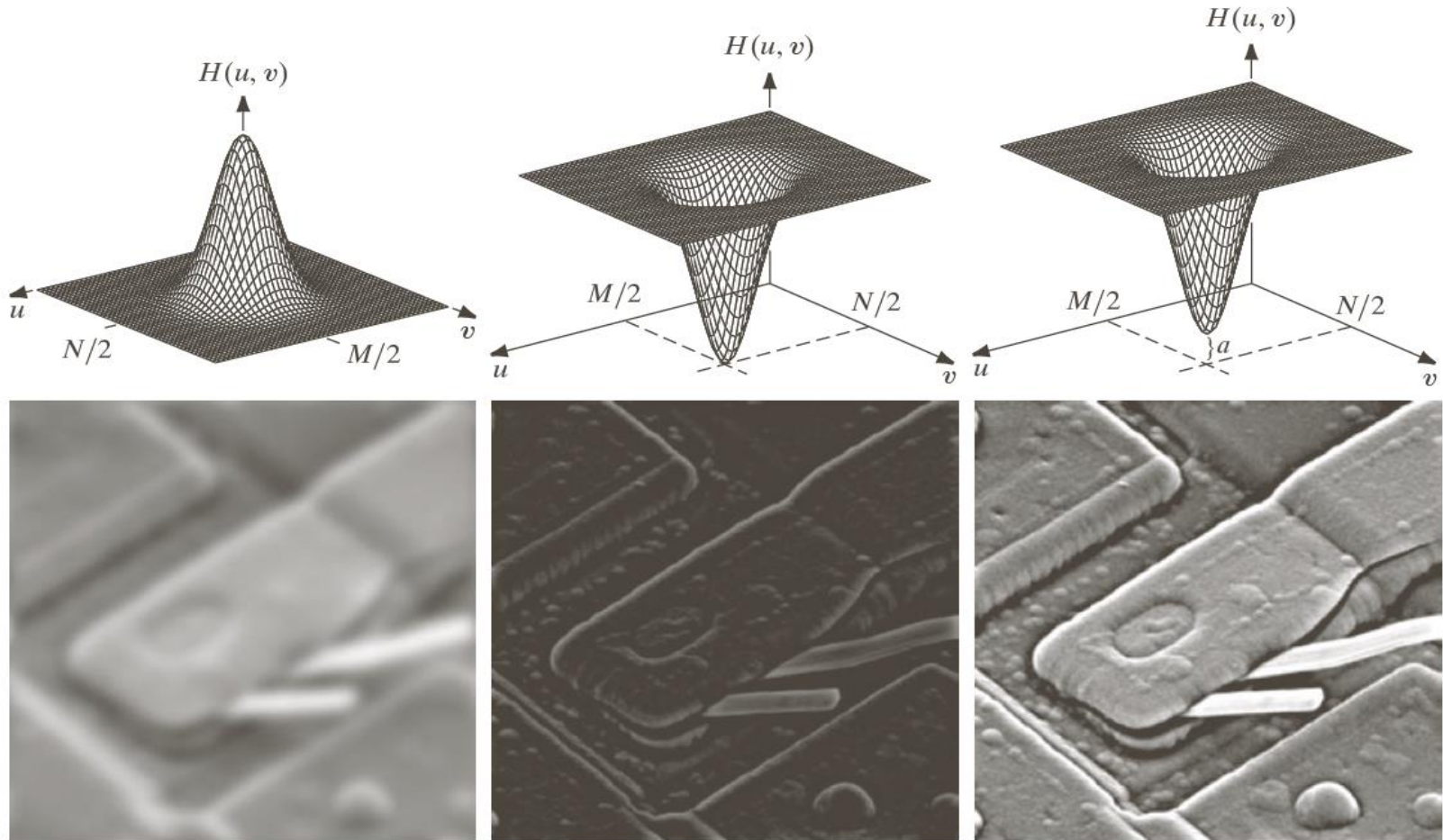
# Some Basic Frequency Domain Filters

## Low Pass Filter



## High Pass Filter



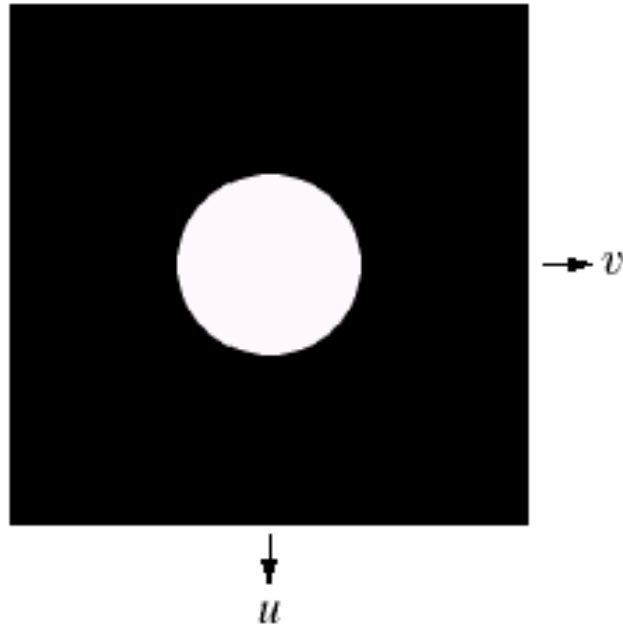


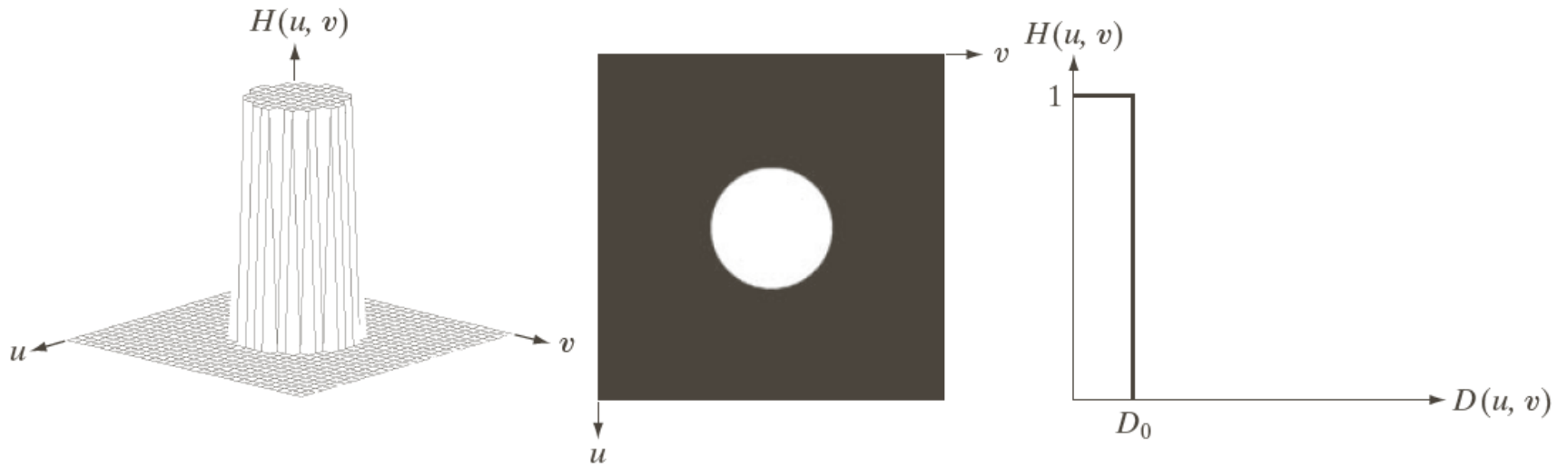
a	b	c
d	e	f

**FIGURE 4.31** Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used  $a = 0.85$  in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

# Ideal Low Pass Filter

Simply cut off all high frequency components that are a specified distance  $D_0$  from the origin

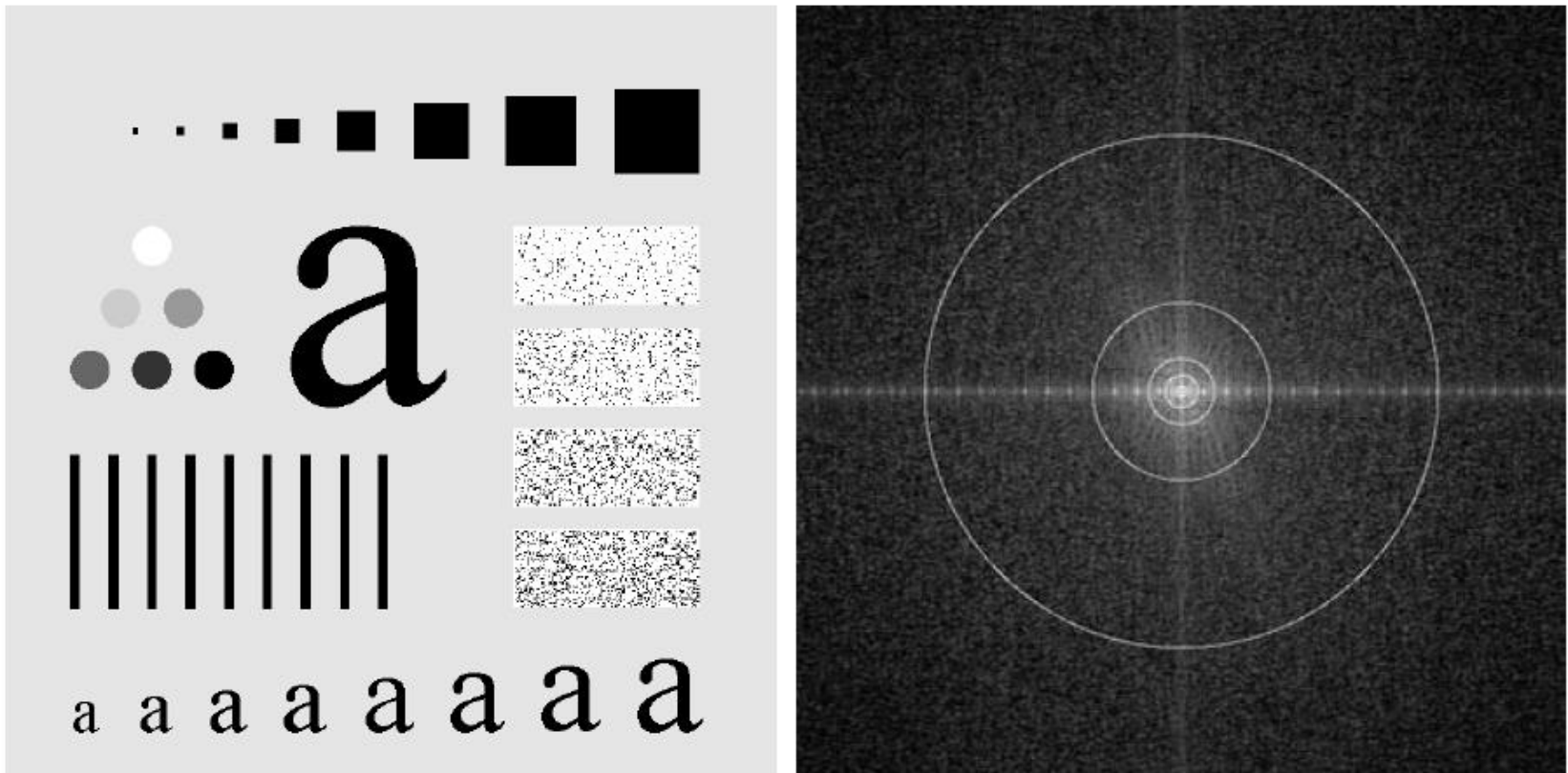




a b c

**FIGURE 4.40** (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

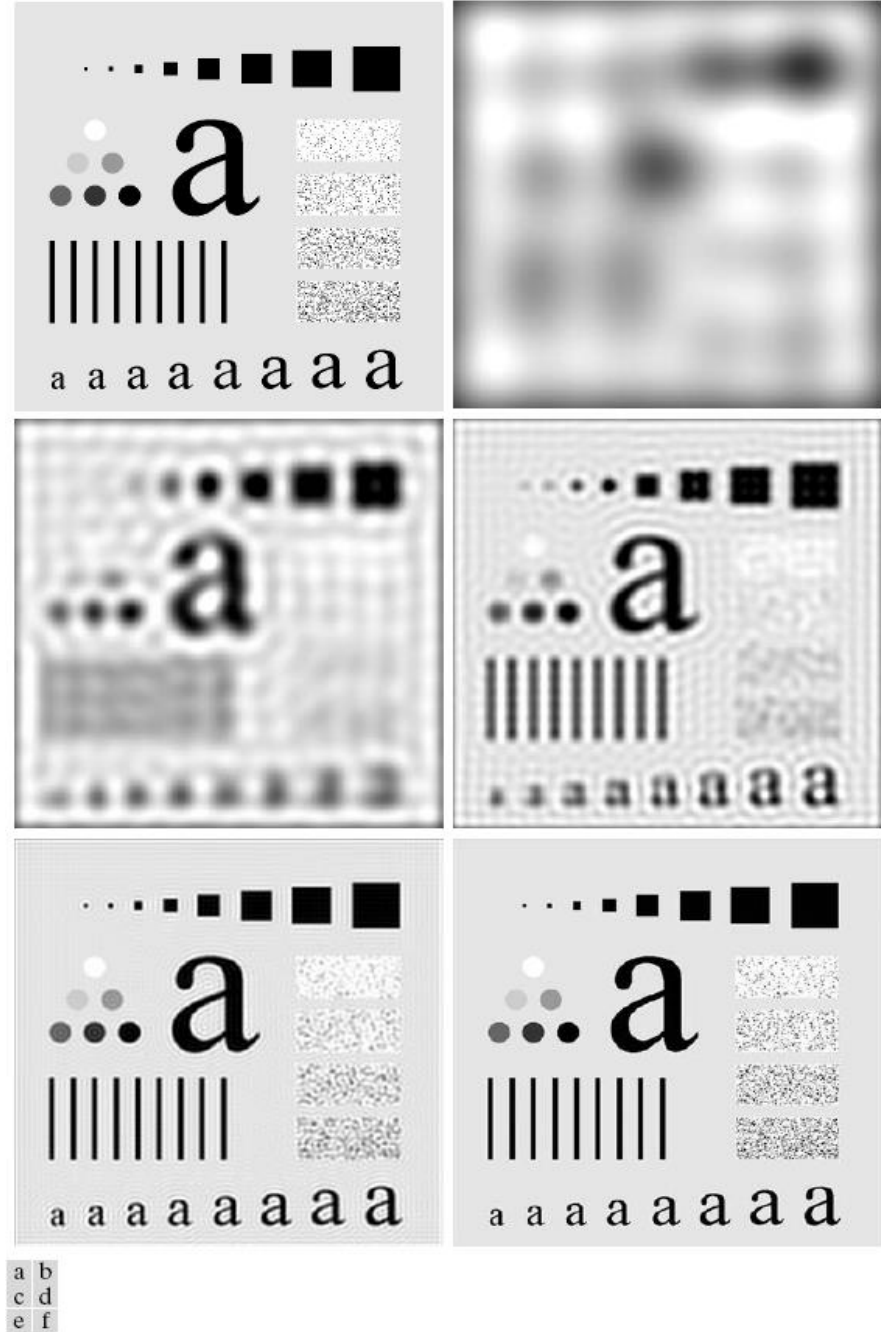
# Ideal Low Pass Filter (cont...)



a b

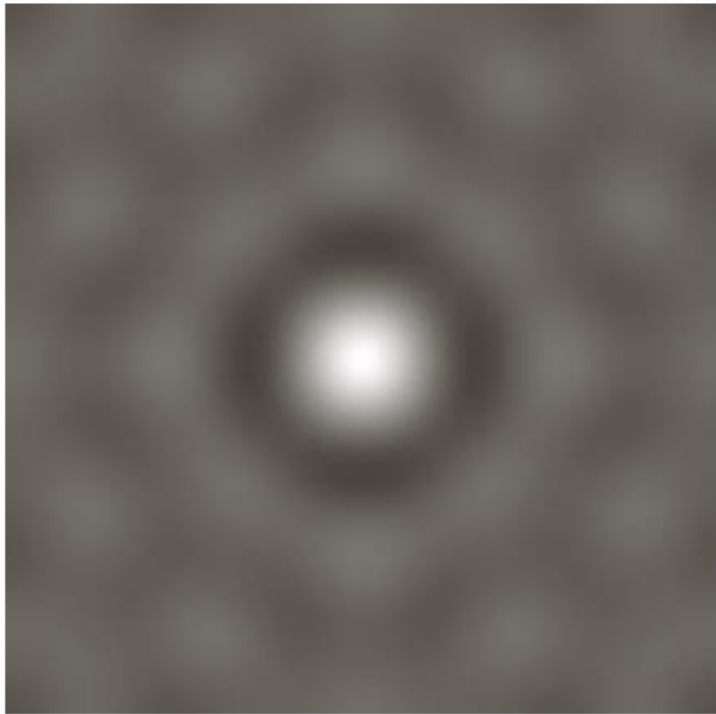
**FIGURE 4.41** (a) Test pattern of size  $688 \times 688$  pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

# Results of ILPF



**FIGURE 4.42** (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

# Spatial representation of ILPF



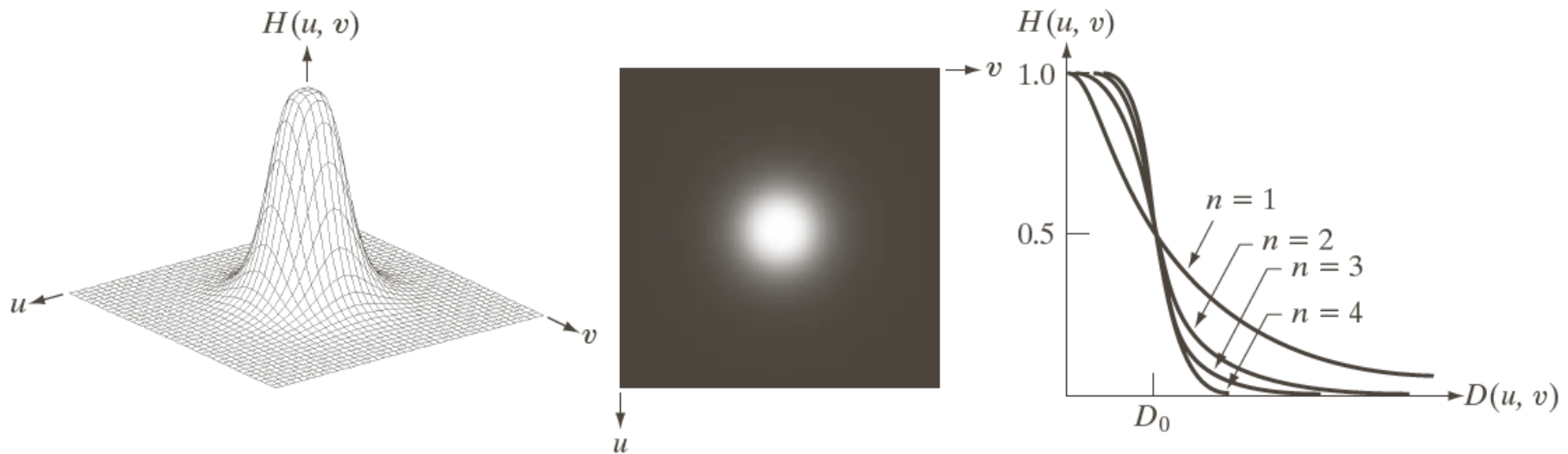
a b

**FIGURE 4.43**

(a) Representation in the spatial domain of an ILPF of radius 5 and size  $1000 \times 1000$ .  
(b) Intensity profile of a horizontal line passing through the center of the image.

---

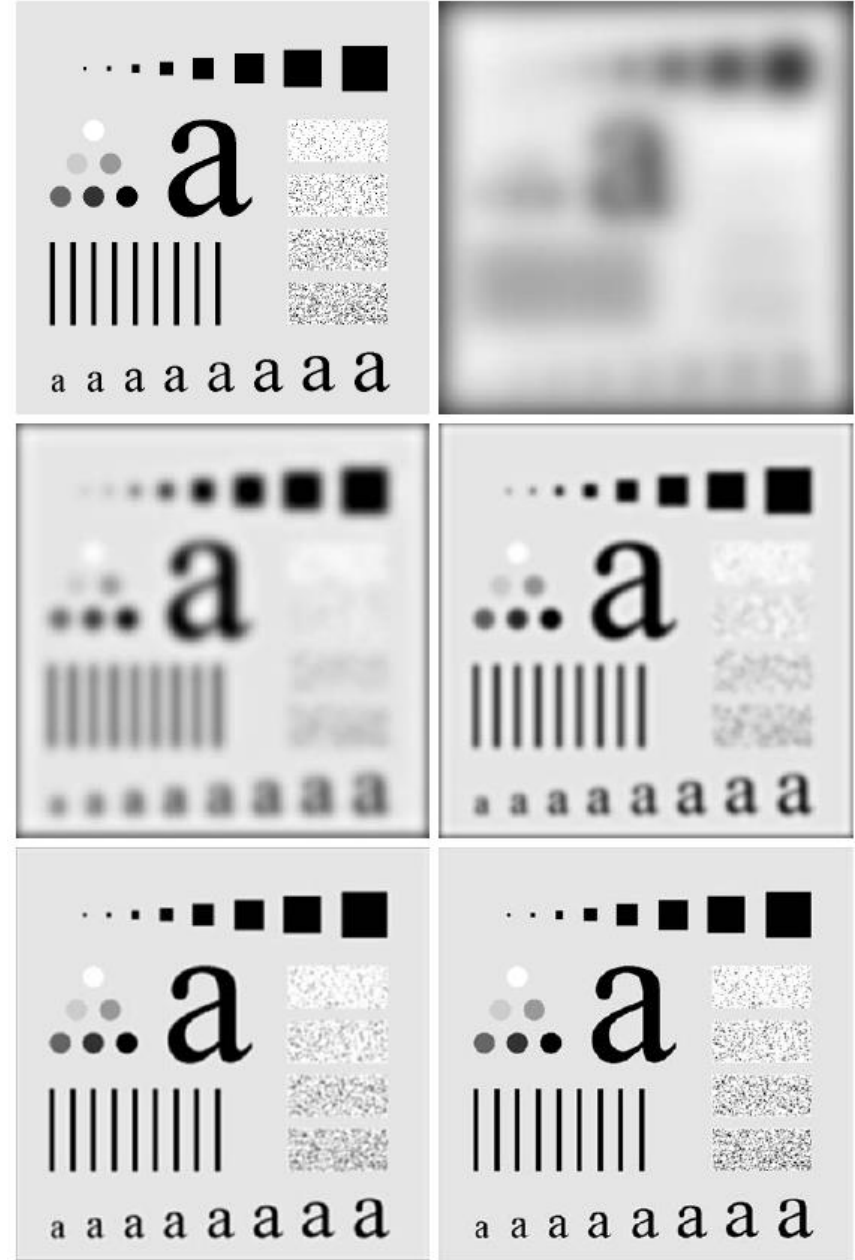
# Butterworth LPF



a b c

**FIGURE 4.44** (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

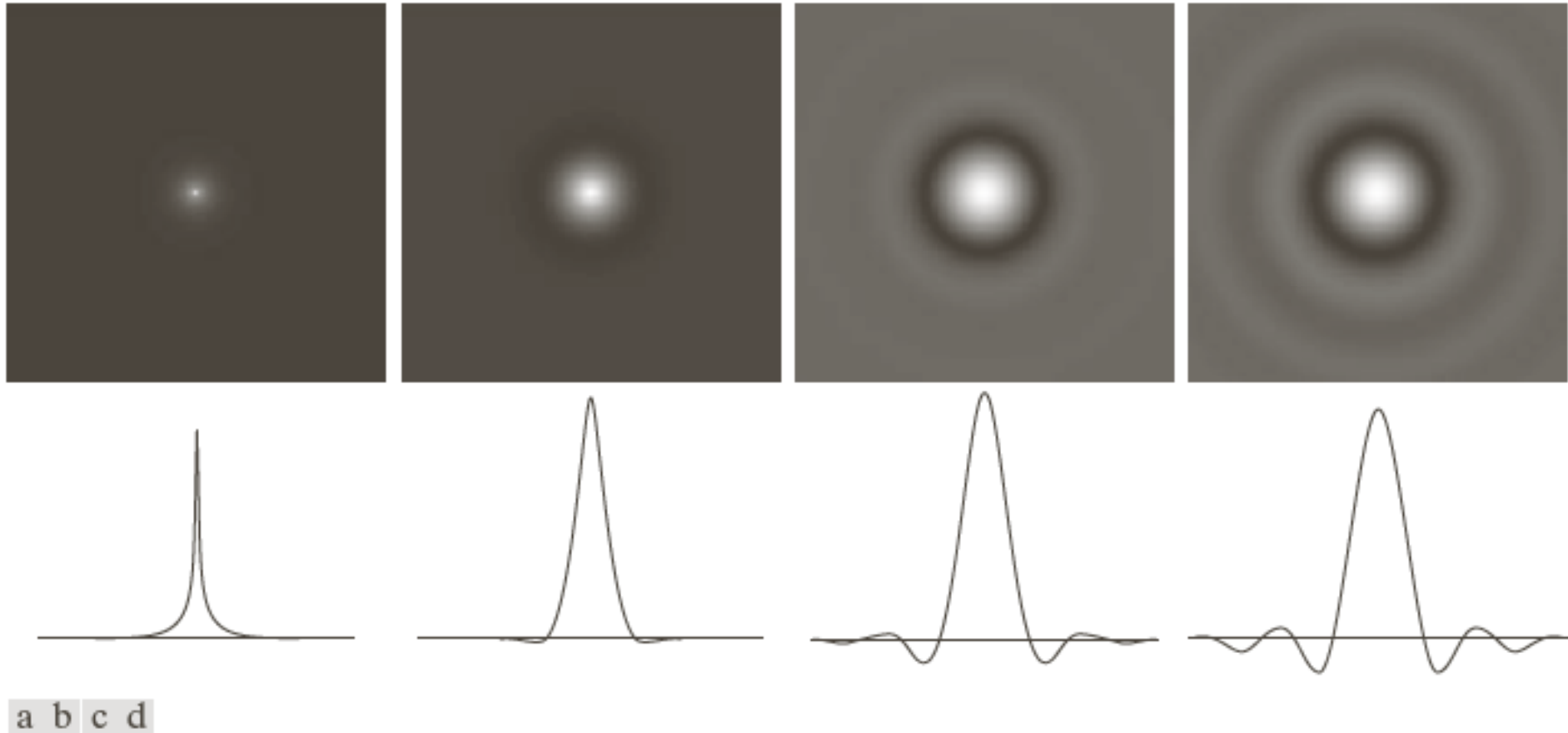
# Results of Butterworth LPF



a b  
c d  
e f

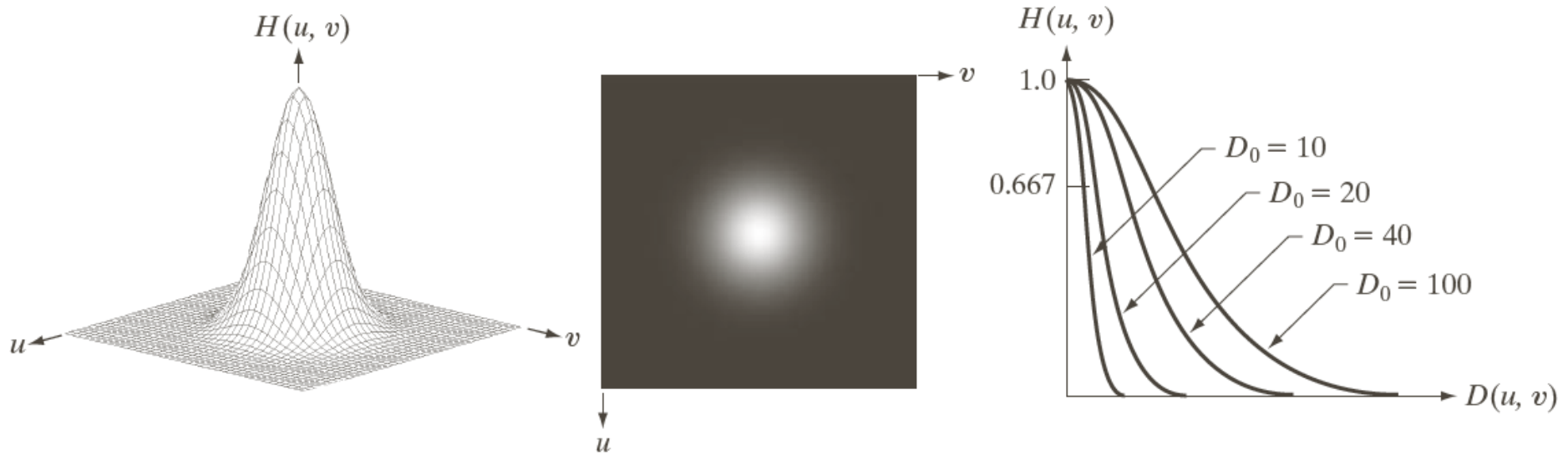
**FIGURE 4.45** (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

# Spatial representation of butterworth LPF



**FIGURE 4.46** (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is  $1000 \times 1000$  and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

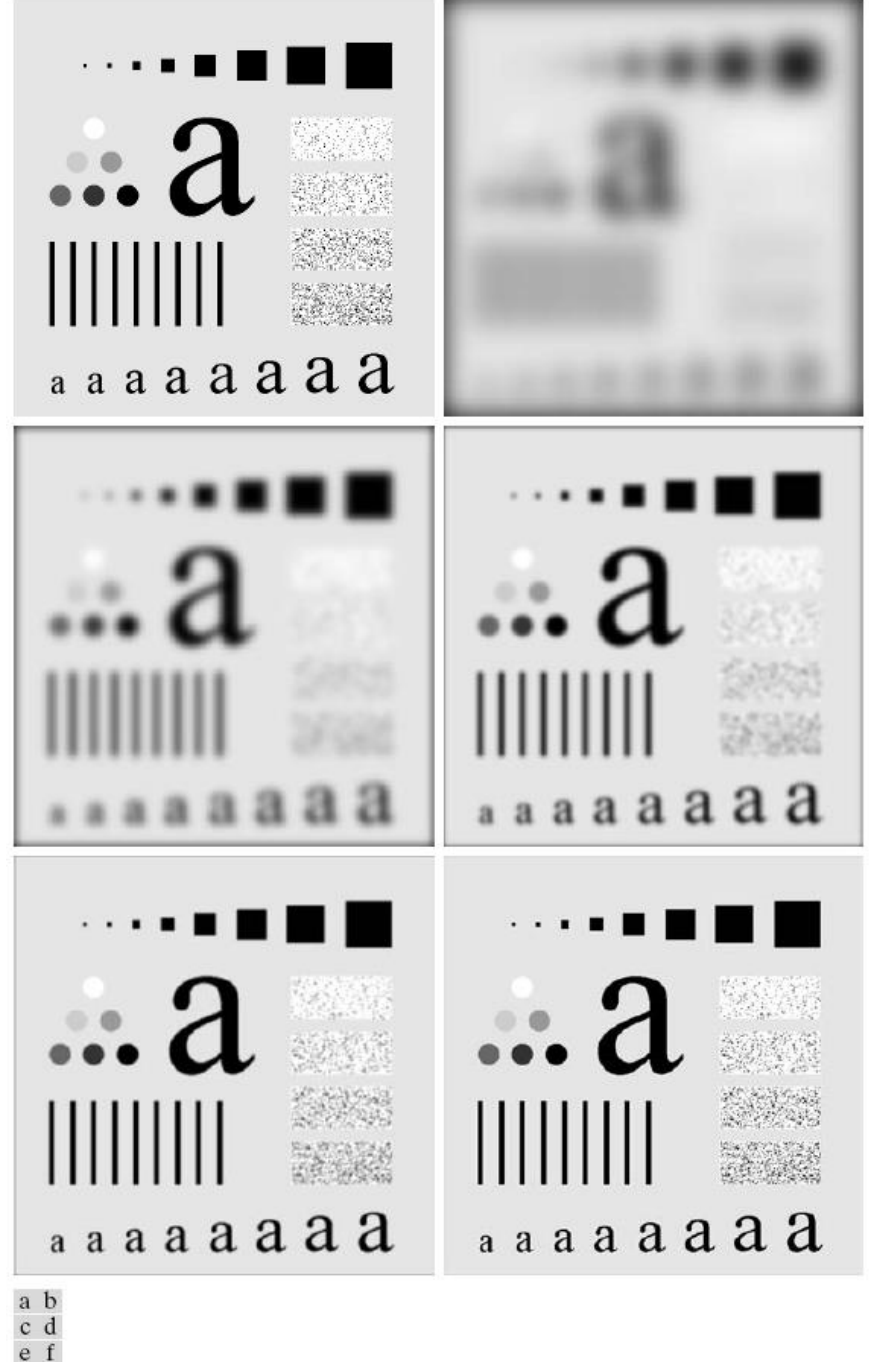
# Gaussian LPF



a b c

**FIGURE 4.47** (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of  $D_0$ .

# Results of Gaussian LPF



**FIGURE 4.48** (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

# Applications of LPFs

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



a b

**FIGURE 4.49**

(a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

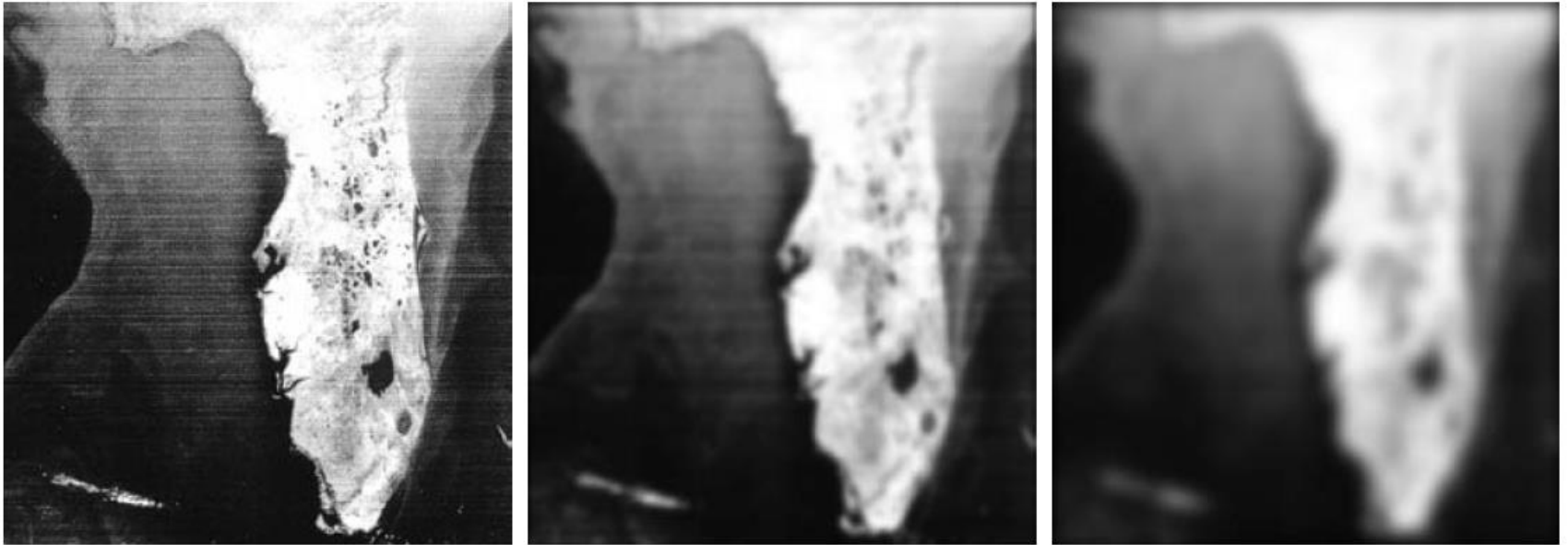
# Applications of LPFs



a b c

**FIGURE 4.50** (a) Original image ( $784 \times 732$  pixels). (b) Result of filtering using a GLPF with  $D_0 = 100$ . (c) Result of filtering using a GLPF with  $D_0 = 80$ . Note the reduction in fine skin lines in the magnified sections in (b) and (c).

# Applications of LPFs



a b c

**FIGURE 4.51** (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with  $D_0 = 50$ . (c) Result of using a GLPF with  $D_0 = 20$ . (Original image courtesy of NOAA.)

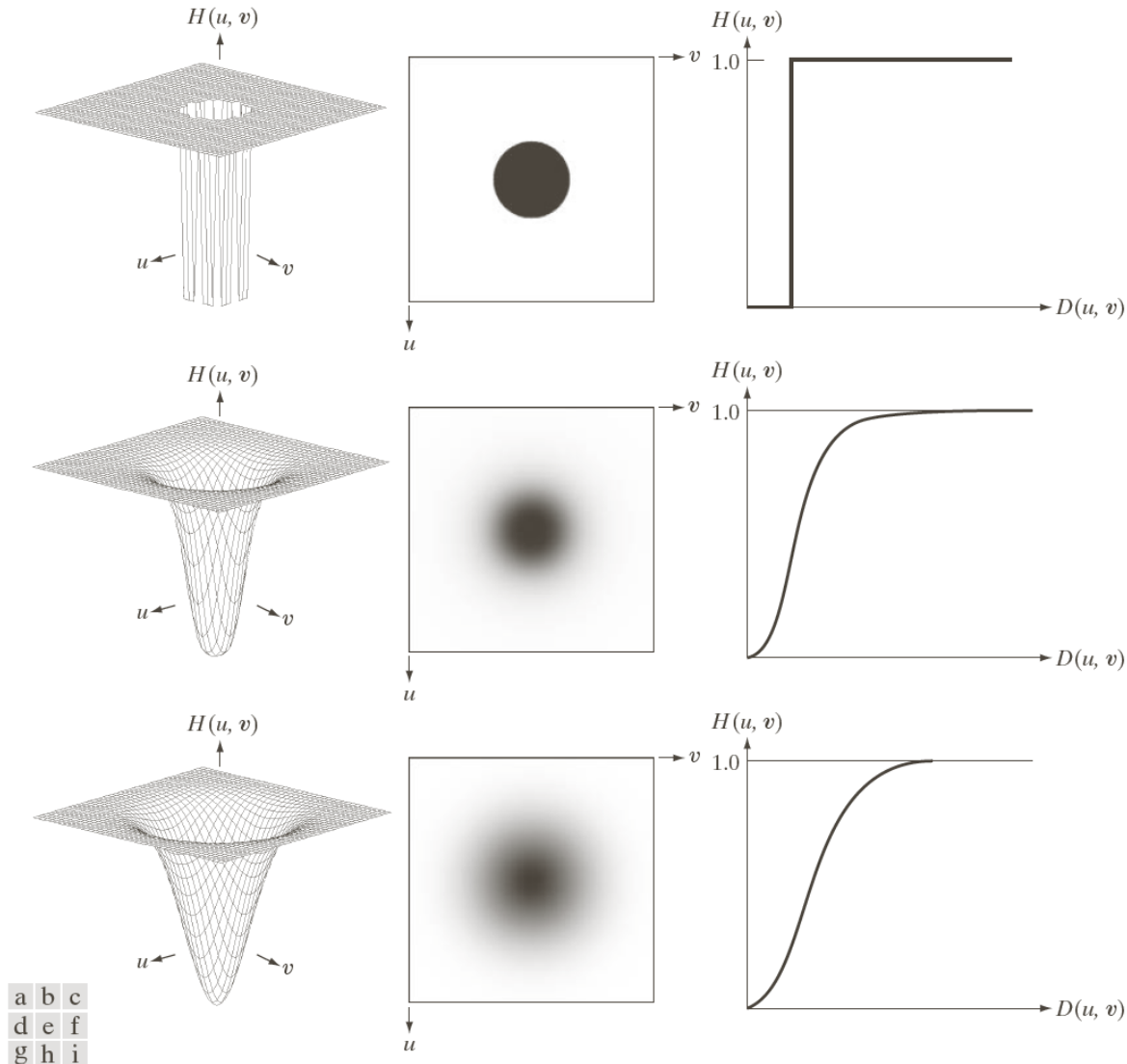
# LPF Summary

**TABLE 4.4**

Lowpass filters.  $D_0$  is the cutoff frequency and  $n$  is the order of the Butterworth filter.

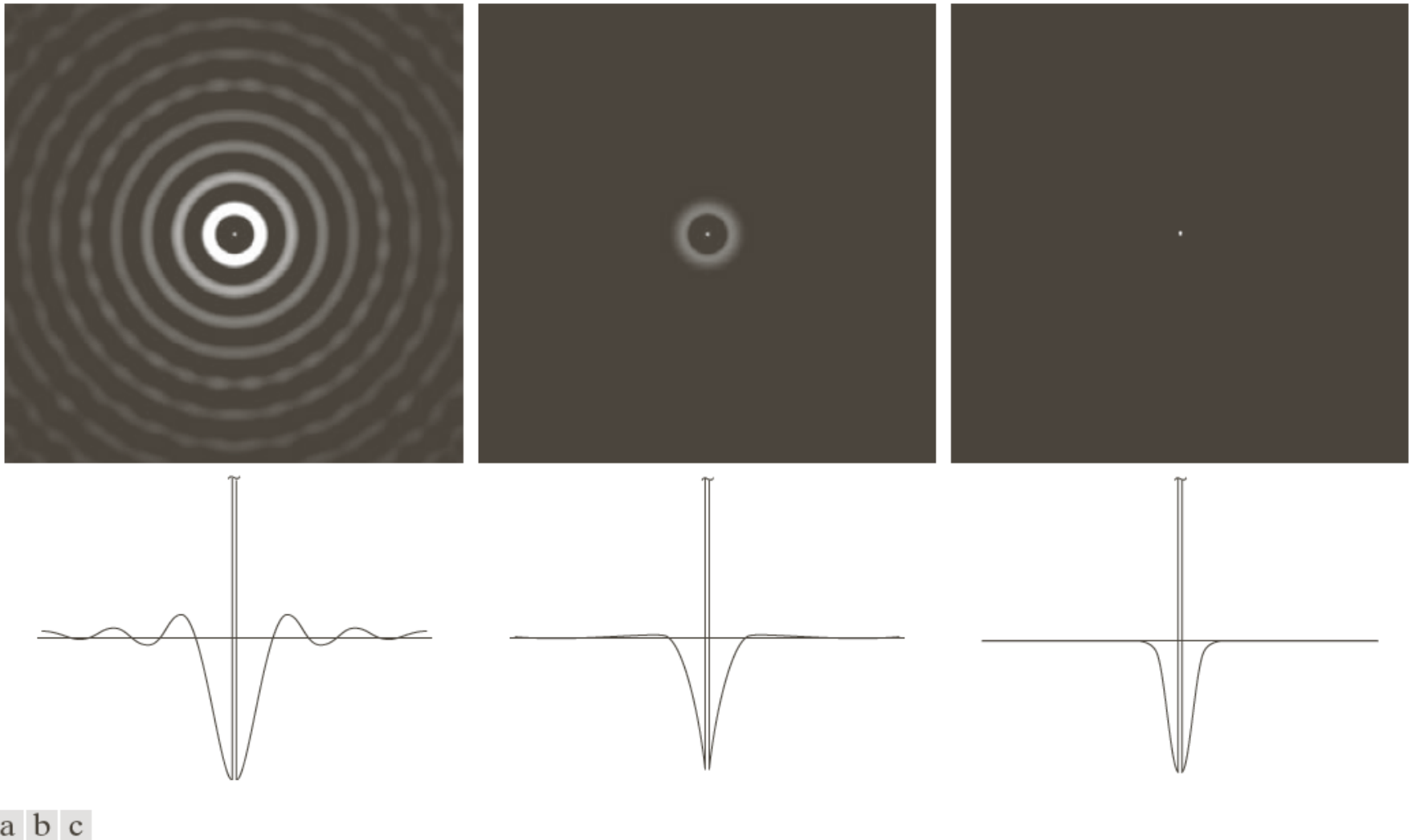
Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$	$H(u, v) = \frac{1}{1 + [D(u, v)/D_0]^{2n}}$	$H(u, v) = e^{-D^2(u,v)/2D_0^2}$

# Highpass Filter (HPFs)



**FIGURE 4.52** Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

# Spatial representation of IHPF



**FIGURE 4.53** Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

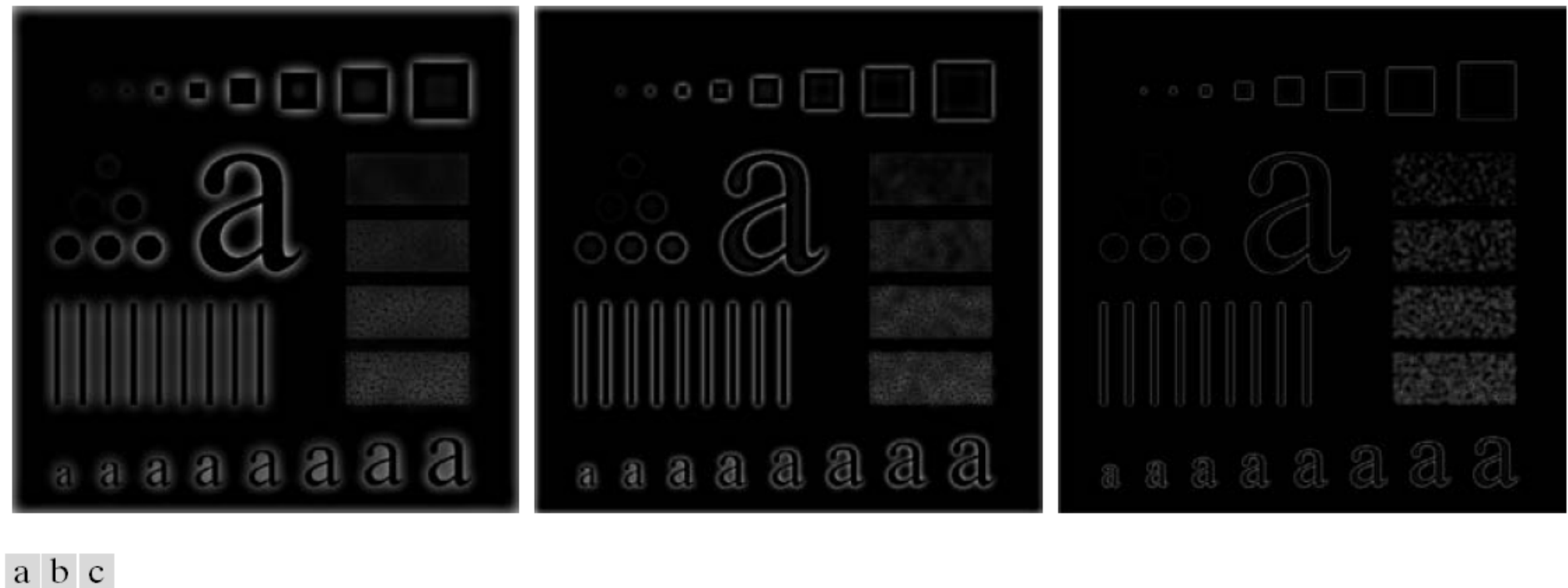
# Results of IHPF



a b c

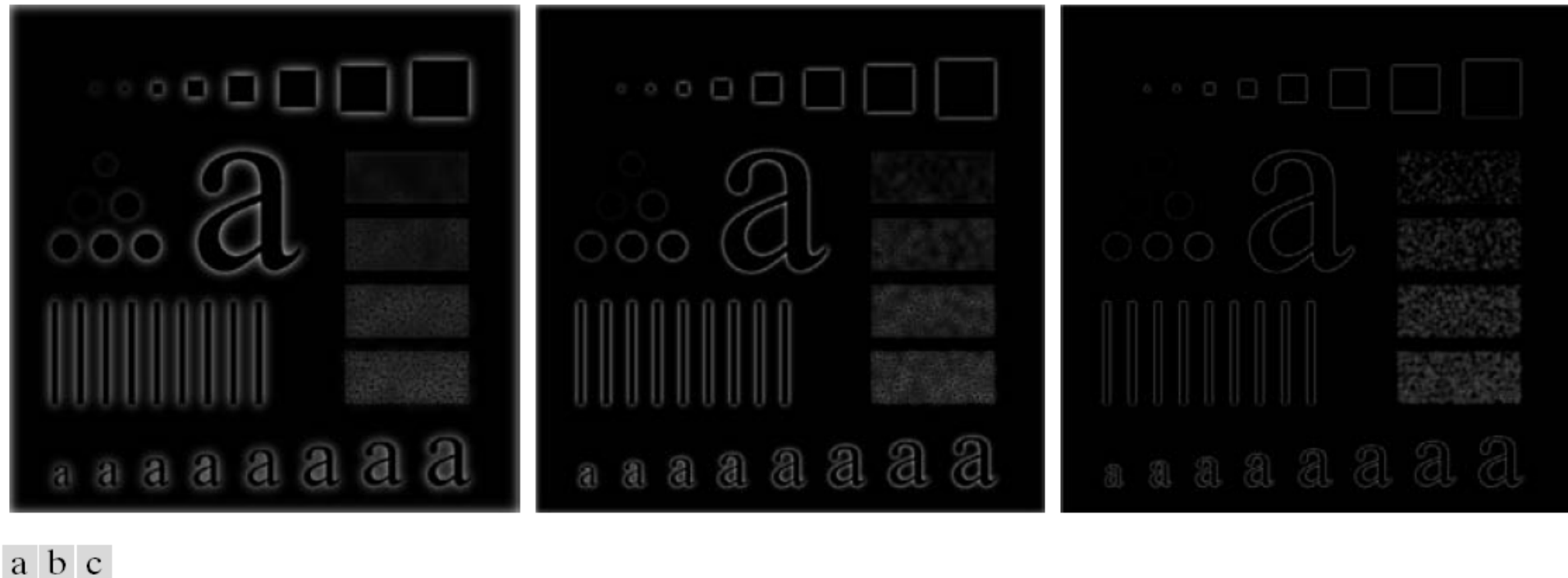
**FIGURE 4.54** Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with  $D_0 = 30, 60,$  and  $160$ .

# Results of BHPF



**FIGURE 4.55** Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with  $D_0 = 30, 60,$  and  $160$ , corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

# Results of GHPF



**FIGURE 4.56** Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with  $D_0 = 30, 60,$  and  $160,$  corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

# Applications of HPFs



a b c

**FIGURE 4.57** (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

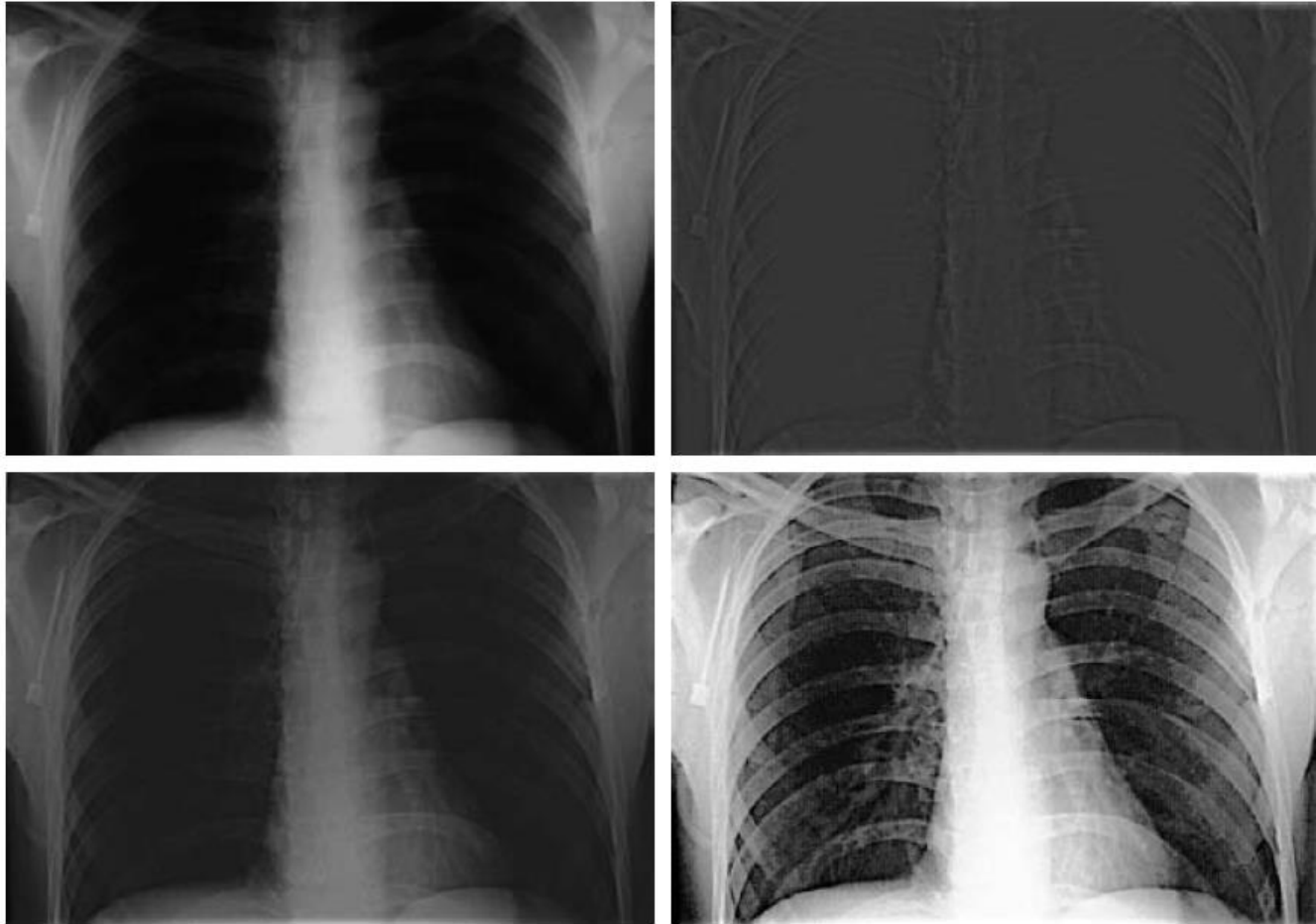
# Applications of HPFs

a b

**FIGURE 4.58**  
(a) Original, blurry image.  
(b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).



# Applications of HPFs



a b  
c d

**FIGURE 4.59** (a) A chest X-ray image. (b) Result of highpass filtering with a Gaussian filter. (c) Result of high-frequency-emphasis filtering using the same filter. (d) Result of performing histogram equalization on (c). (Original image courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School.)

# Summary of HPFs

**TABLE 4.5**

Highpass filters.  $D_0$  is the cutoff frequency and  $n$  is the order of the Butterworth filter.

<b>Ideal</b>	<b>Butterworth</b>	<b>Gaussian</b>
$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$	$H(u, v) = \frac{1}{1 + [D_0/D(u, v)]^{2n}}$	$H(u, v) = 1 - e^{-D^2(u,v)/2D_0^2}$

# Bandreject Filters

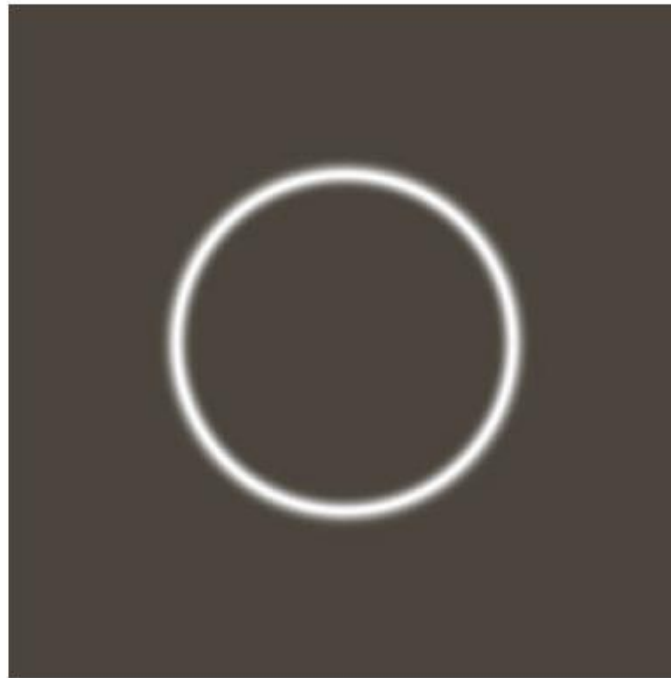
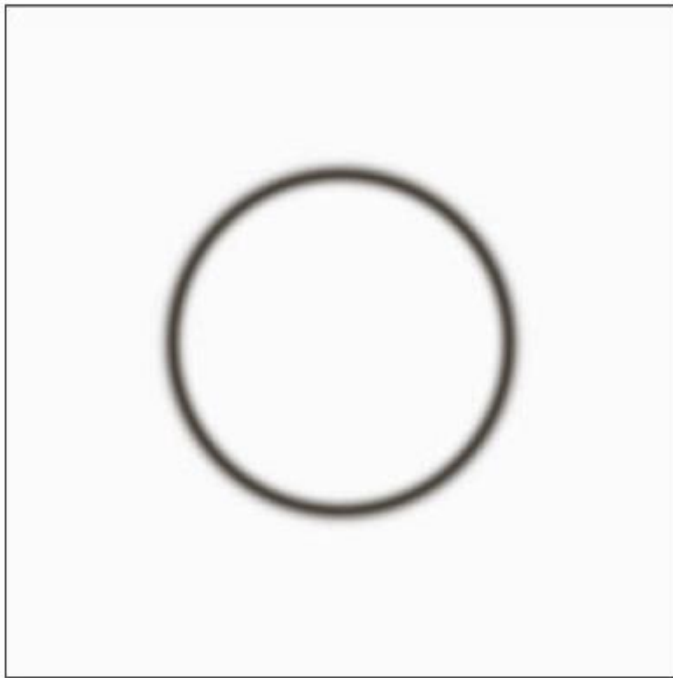


a b c

**FIGURE 5.15** From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

---

# Bandreject and bandpass Filters



a b

## FIGURE 4.63

(a) Bandreject Gaussian filter.

(b) Corresponding bandpass filter.

The thin black border in (a) was added for clarity; it is not part of the data.

---

# Summary of Bandreject Filters

**TABLE 4.6**

Bandreject filters.  $W$  is the width of the band,  $D$  is the distance  $D(u, v)$  from the center of the filter,  $D_0$  is the cutoff frequency, and  $n$  is the order of the Butterworth filter. We show  $D$  instead of  $D(u, v)$  to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u, v) = \frac{1}{1 + \left[ \frac{DW}{D^2 - D_0^2} \right]^{2n}}$	$H(u, v) = 1 - e^{-\left[ \frac{D^2 - D_0^2}{DW} \right]^2}$

# Readings from Book (3<sup>rd</sup> Edn.)

- Frequency Filters (Chapter-4)



# Acknowledgements

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- ◆ Digital Image processing Lectures: Coursera