Digital Image Processing

Lecture # 3
Image Enhancement

Image Enhancement

Image Enhancement

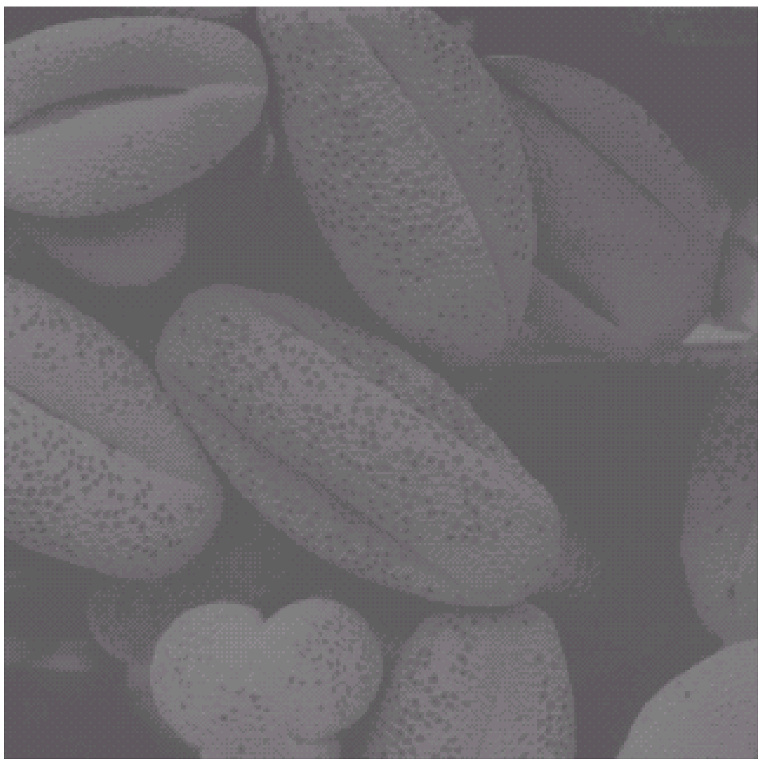


Image Enhancement

Process an image so that the result is more suitable than the original image for a specific application

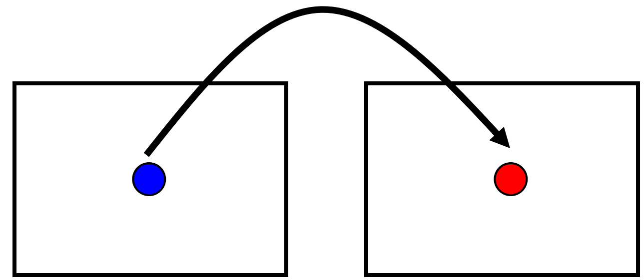
- Image Enhancement Methods
  - Spatial Domain: Direct manipulation of pixels in an image
  - Frequency Domain: Process the image by modifying the Fourier transform of an image

This Chapter – Spatial Domain

Types of image enhancement operations

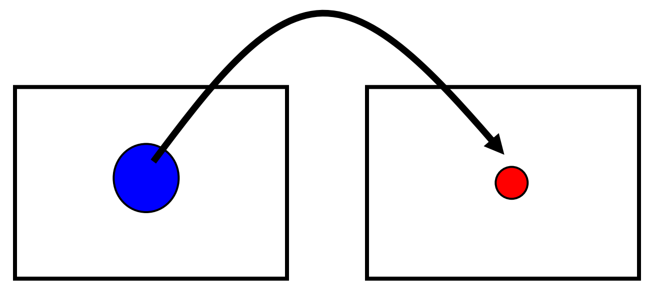
- Point/Pixel operations

  Output value at specific coordinates *(x,y)* is dependent only on the input value at *(x,y)*



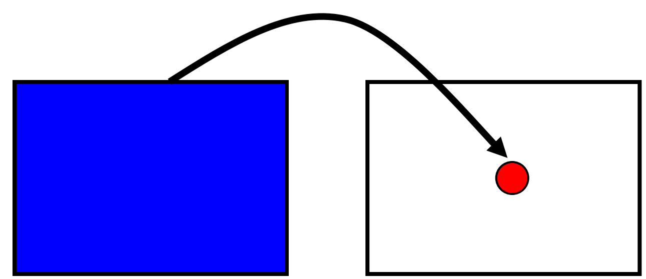
- Local operations

  The output value at *(x,y)* is dependent on the input values in the neighborhood of *(x,y)*



- Global operations

  The output value at *(x,y)* is dependent on all the values in the input image



Basic Concepts

- Most spatial domain enhancement operations can be generalized as:

$$g(x, y) = T\left[f(x, y) \right]$$

f (x, y) = the input image

g (x, y) = the processed/output image

T = some operator defined over some neighbourhood of *(x, y)*

Point Processing

- In a digital image, point = pixel
- Point processing transforms a pixel’s value as function of its value alone;
- It does not depend on the values of the pixel’s neighbors.

Point Processing

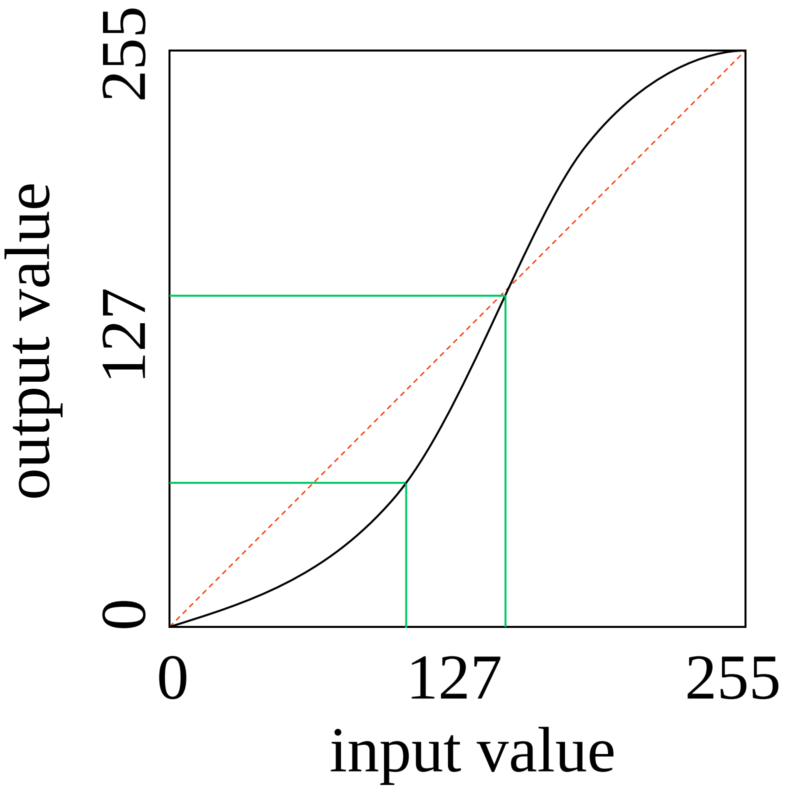
- Neighborhood of size 1x1:
- *g* depends only on *f* at *(x,y)*
- *T*: Gray-level/intensity transformation/ mapping function

$$s = T(r)$$

- r = gray level of *f* at *(x,y)*
- s = gray level of *g* at *(x,y)*

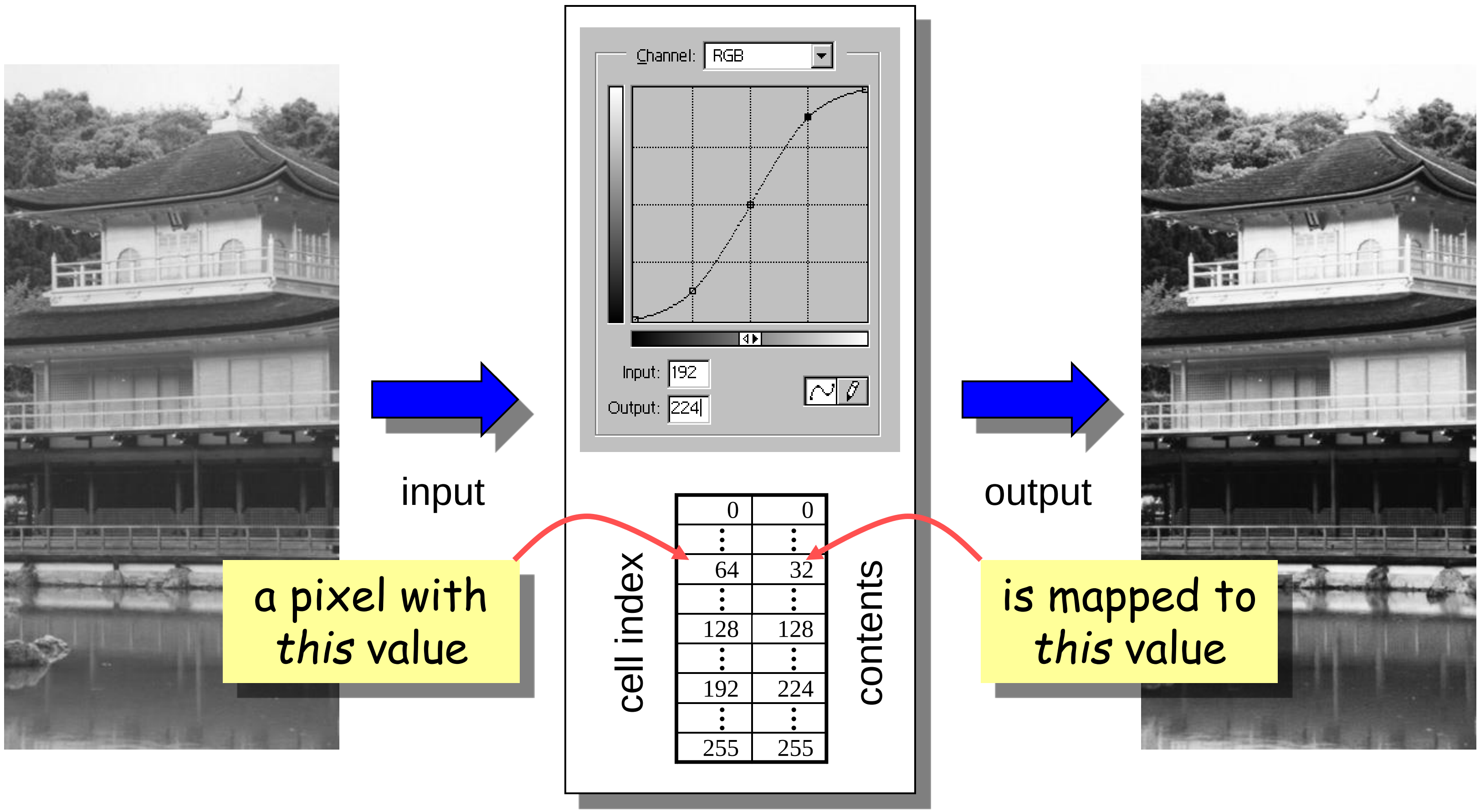
Point Processing using Look-up Tables

A look-up table (LUT) implements a functional mapping.



E.g.:	index	value
	...	...
	101	64
	102	68
	103	69
	104	70
	105	70
	106	71
	...	...
	input	output

Point Processing using Look-up Tables



POINT PROCESSING

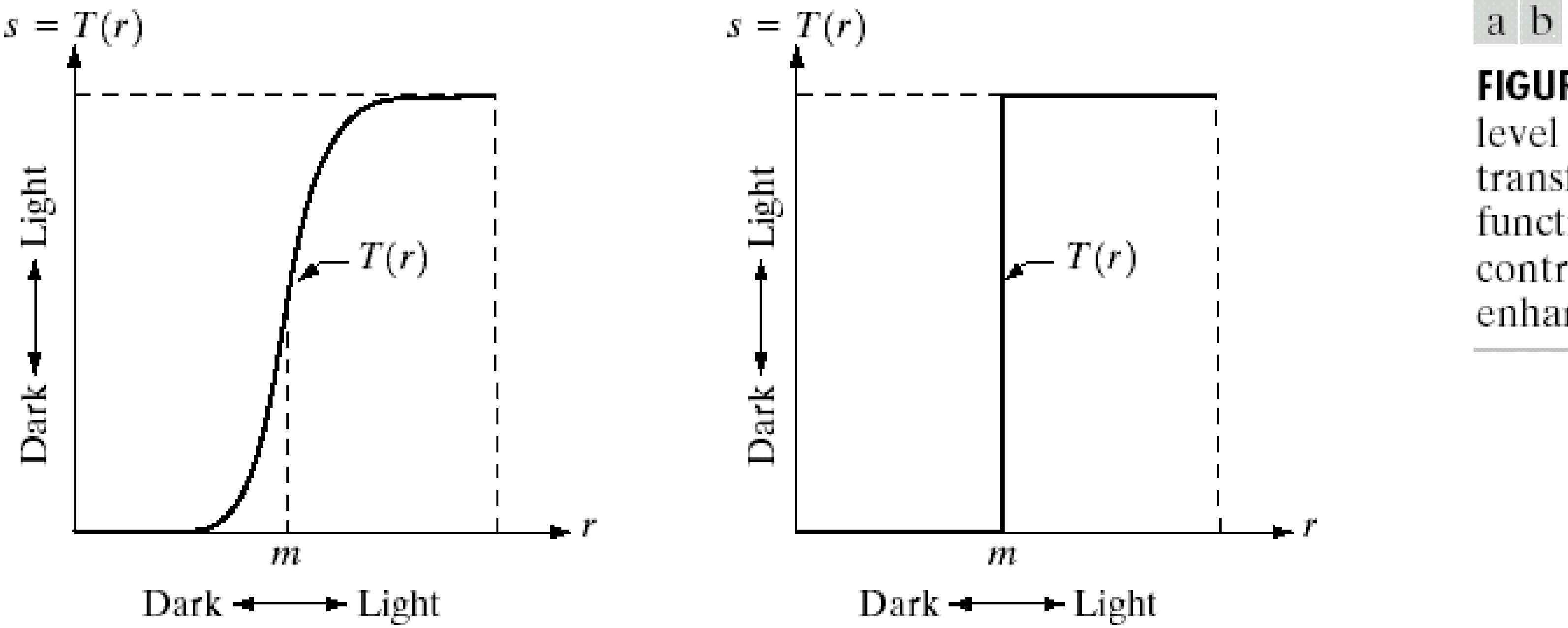


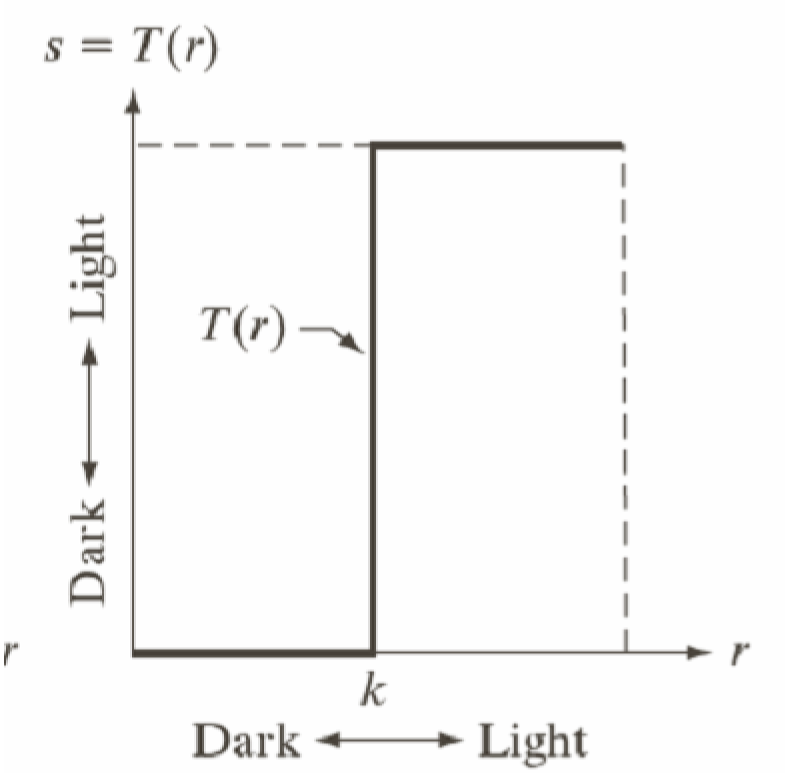
FIGURE 3.2 Gray-level transformation functions for contrast enhancement.

Contrast Stretching

Thresholding

Point Processing Example: Thresholding

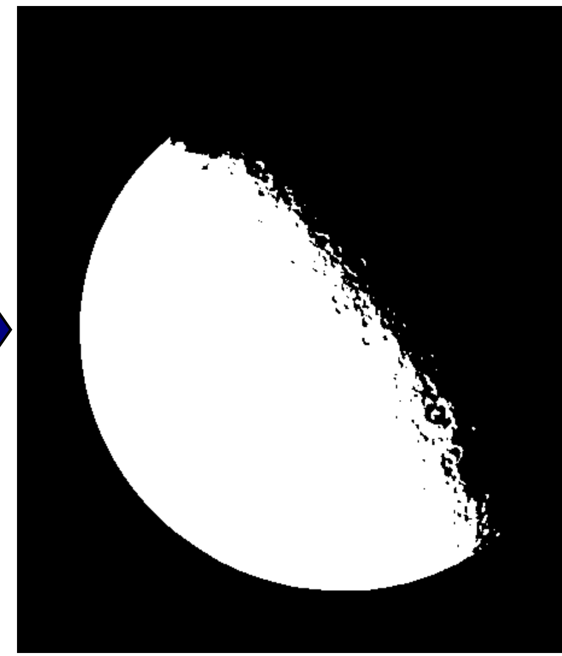
$$s = \begin{cases} 1.0 & r > threshold \\ 0.0 & r <= threshold \end{cases}$$



Point Processing Example: Thresholding

- Segmentation of an object of interest from a background

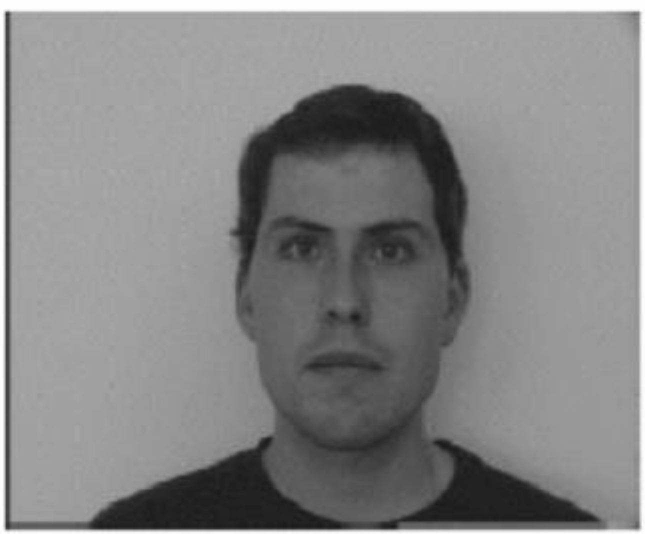
$$s = \begin{cases} 1.0 & r > \text{threshold} \\ 0.0 & r <= \text{threshold} \end{cases}$$



Point Processing Example: Intensity Scaling

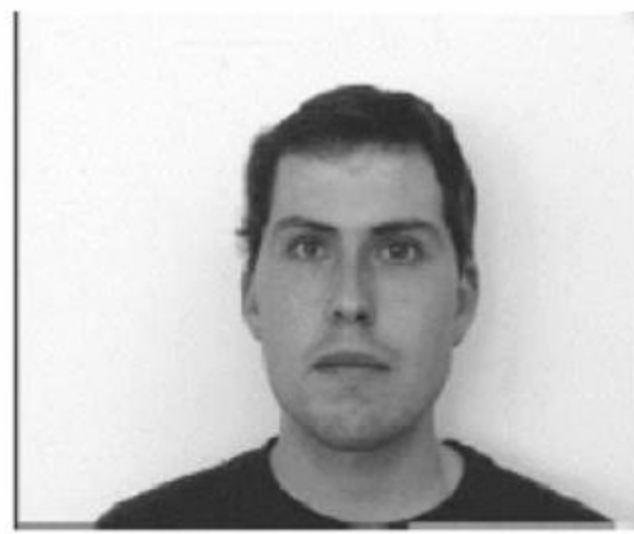
$$s = T(r) = a.r$$

Original image



$f(x,y)$

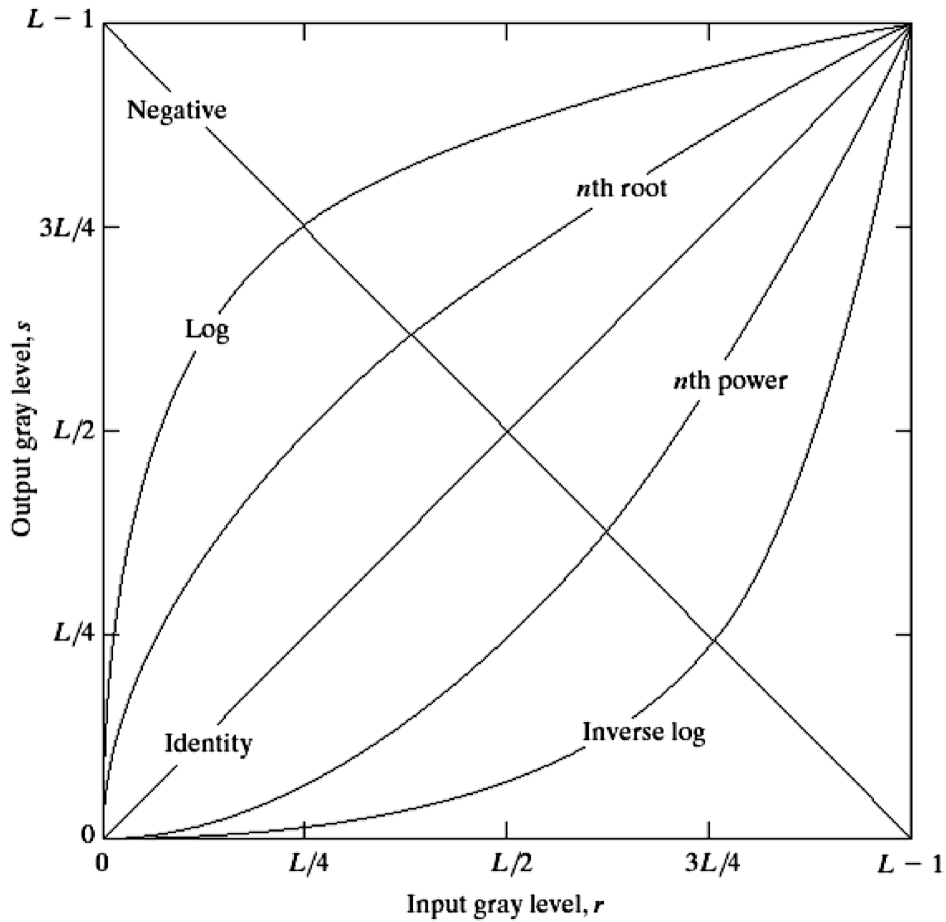
Scaled image



$a \cdot f(x,y)$

Point Processing Transformations

- There are many different kinds of grey level transformations
- Three of the most common are shown here
  - Linear
    - Negative/Identity
  - Logarithmic
    - Log/Inverse log
  - Power law
    - n^{th} power/n^{th} root



Point Processing Example: Negative Images

- Reverses the gray level order
- For L gray levels, the transformation has the form:

$$s = (L - 1) - r$$

- Negative images are useful for enhancing white or grey detail embedded in dark regions of an image

Point Processing Example: Negative Images

Input image (X-ray image)

Output image (negative)

Logarithmic Transformations

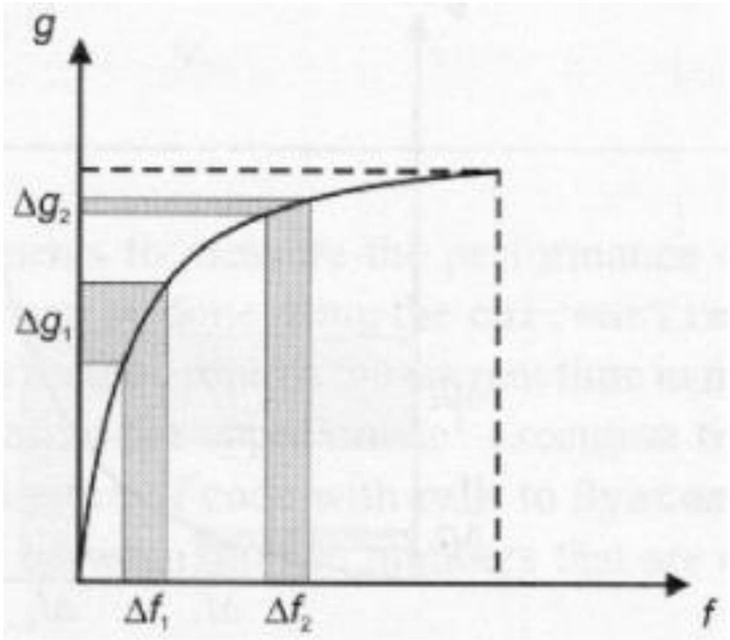
- The general form of the log transformation is

$$s = c \times \log(1 + r)$$

- The log transformation maps a narrow range of low input grey level values into a wider range of output values
- The inverse log transformation performs the opposite transformation

Logarithmic Transformations

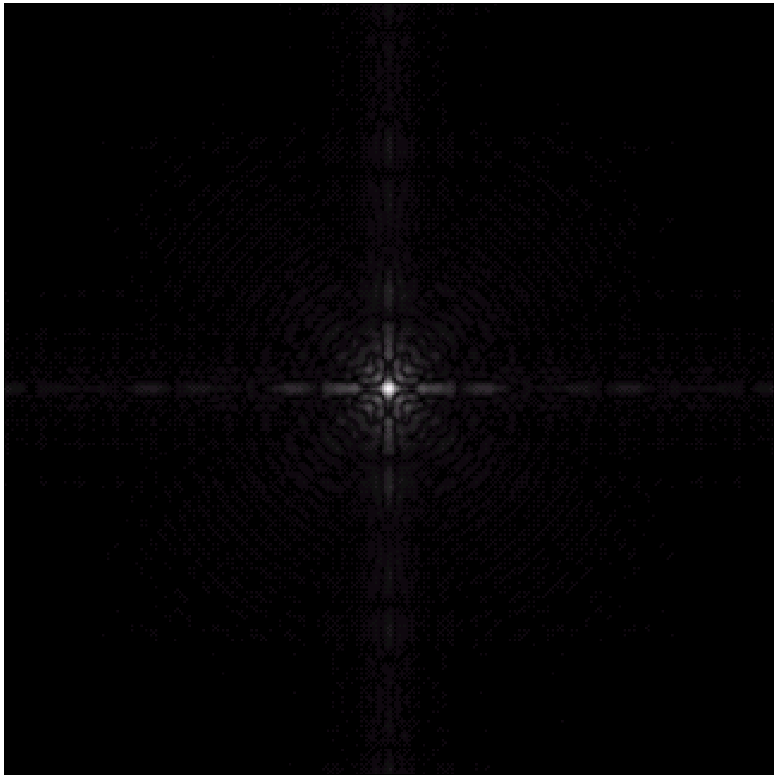
- **Properties**
  - For lower amplitudes of input image the range of gray levels is expanded
  - For higher amplitudes of input image the range of gray levels is compressed



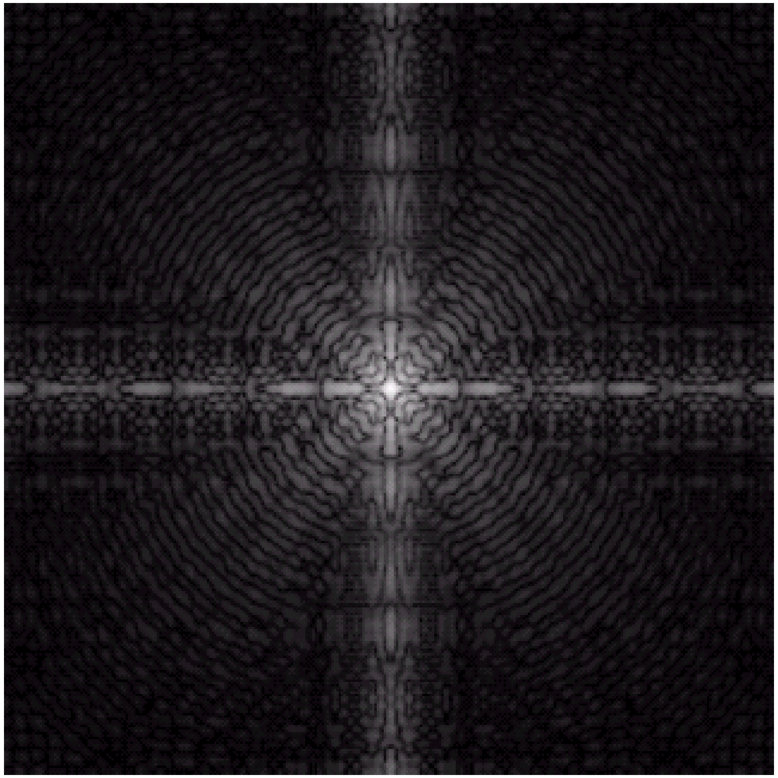
Logarithmic Transformations

- **Application**
  - This transformation is suitable for the case when the dynamic range of a processed image far exceeds the capability of the display device (e.g. display of the Fourier spectrum of an image)
  - Also called "dynamic-range compression / expansion"

Logarithmic Transformations



Fourier spectrum: image values ranging from 0 to 1.5×10^6



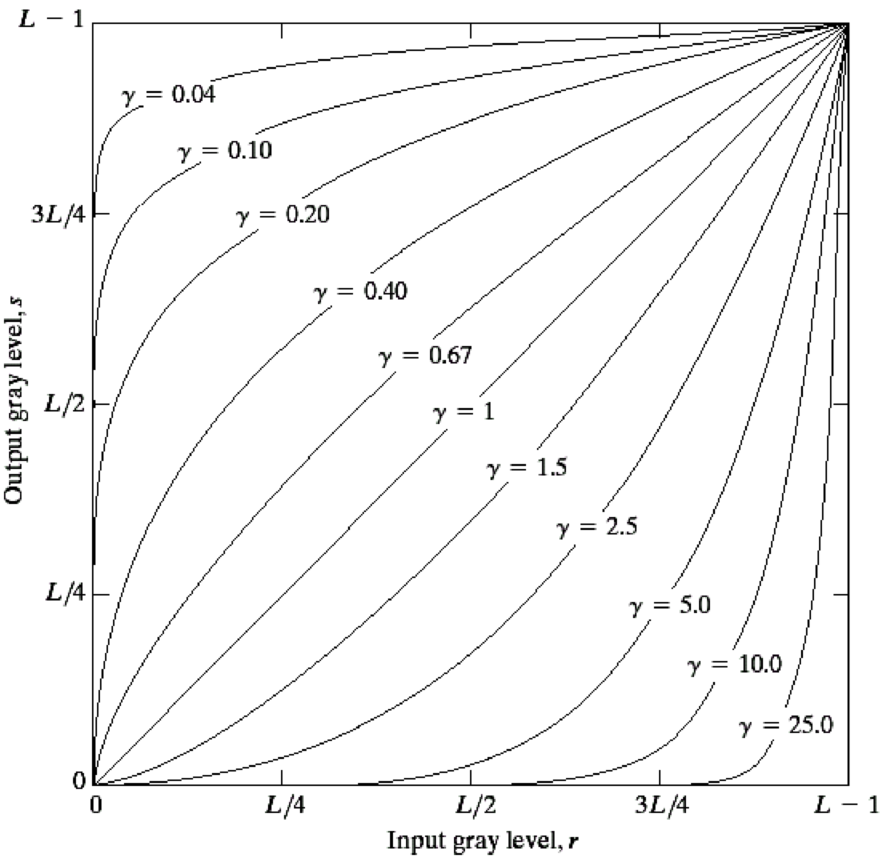
The result of log transformation with c = 1

Power Law Transformations

- Power law transformations have the following form

$$s = c \times r^{\gamma}$$

- Map a narrow range of dark input values into a wider range of output values or vice versa
- Varying γ gives a whole family of curves



Power Law Transformations

- For $\gamma < 1$: Expands values of dark pixels, compress values of brighter pixels
- For $\gamma > 1$: Compresses values of dark pixels, expand values of brighter pixels
- If $\gamma=1$ & $c=1$: Identity transformation ($s = r$)

- A variety of devices (image capture, printing, display) respond according to a power law and need to be corrected

- **Gamma (γ) correction**

  The process used to correct the power-law response phenomena

Power Law Transformations: Gamma Correction

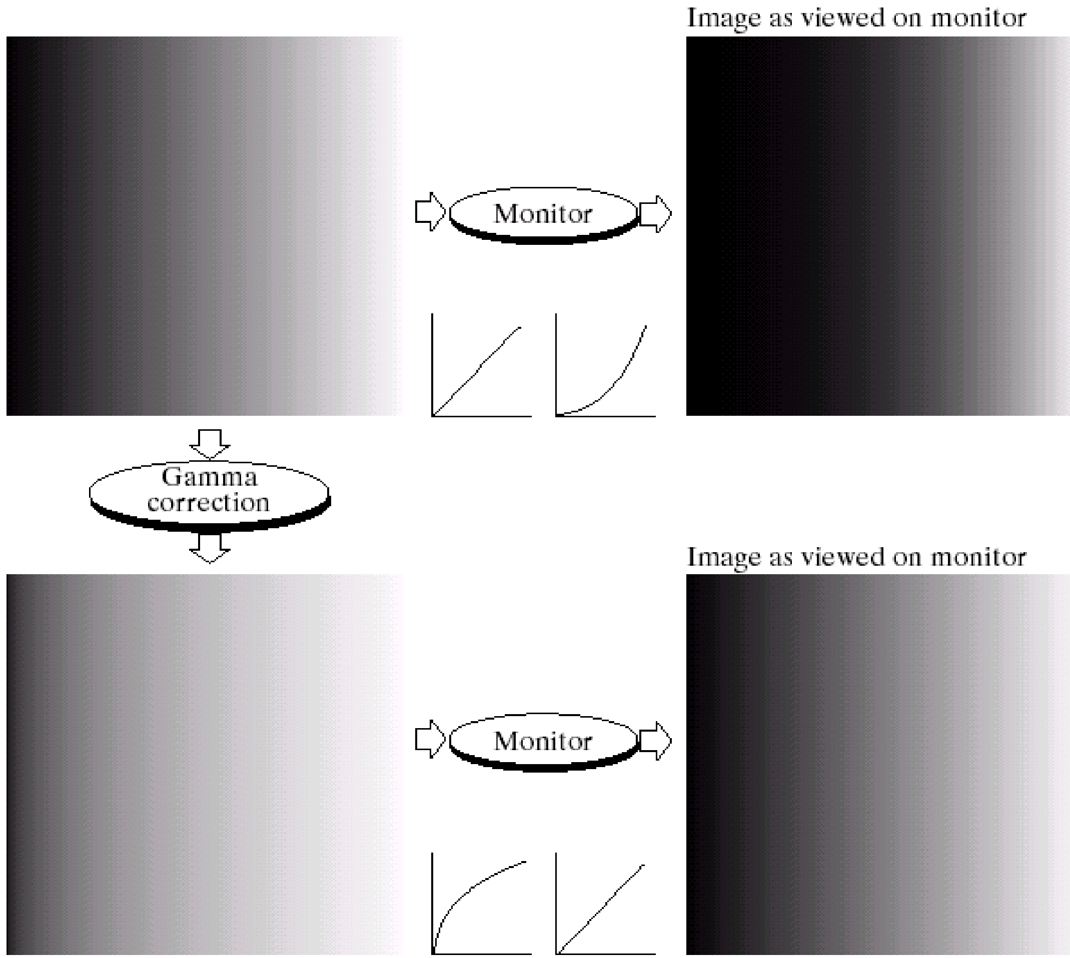


FIGURE 3.7
(a) Linear-wedge gray-scale image.
(b) Response of monitor to linear wedge.
(c) Gamma-corrected wedge.
(d) Output of monitor.

Power Law Transformations
Contrast Enhancement

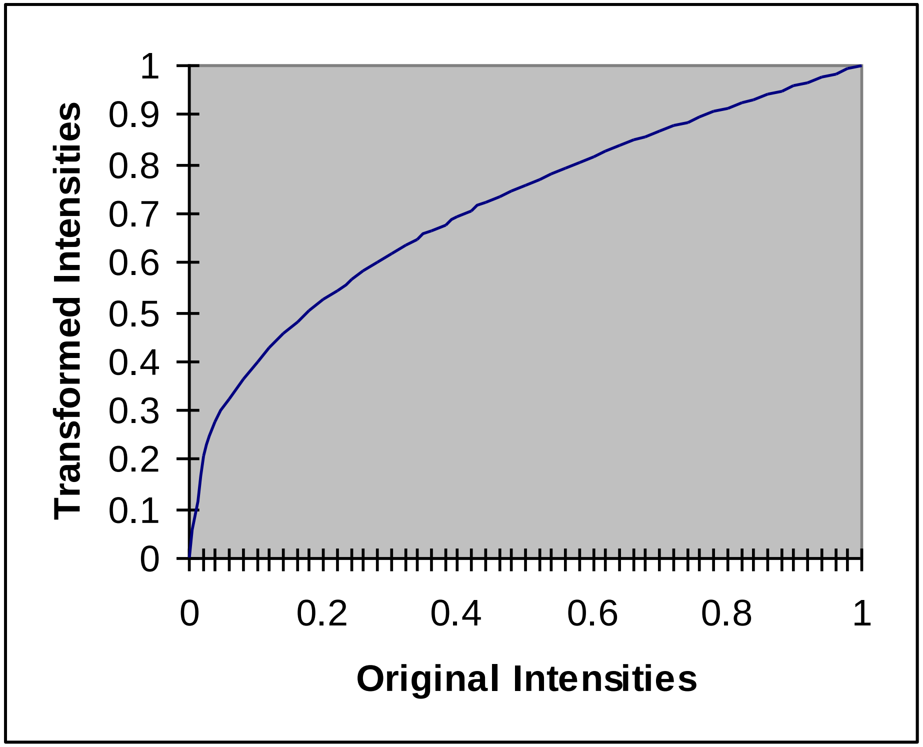
The images to the right show a magnetic resonance (MR) image of a fractured human spine

Power Law Transformations
Contrast Enhancement

γ = 0.6

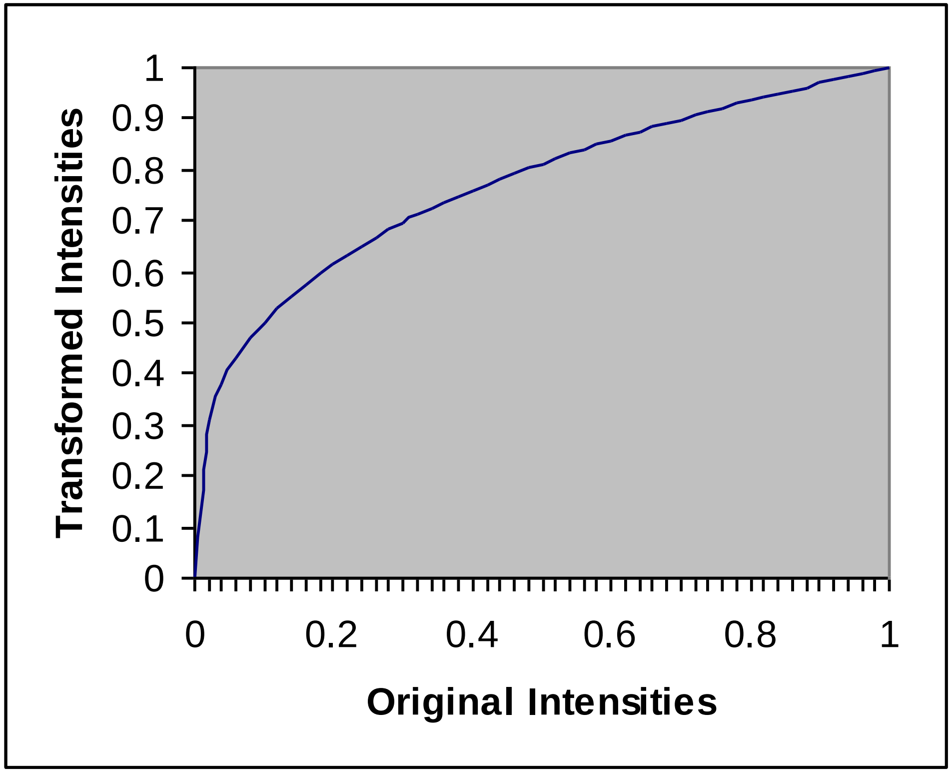
Power Law Transformations
Contrast Enhancement

$\gamma = 0.4$



Power Law Transformations
Contrast Enhancement

$\gamma = 0.3$



Power Law Transformations
Contrast Enhancement

MR image of fractured human spine

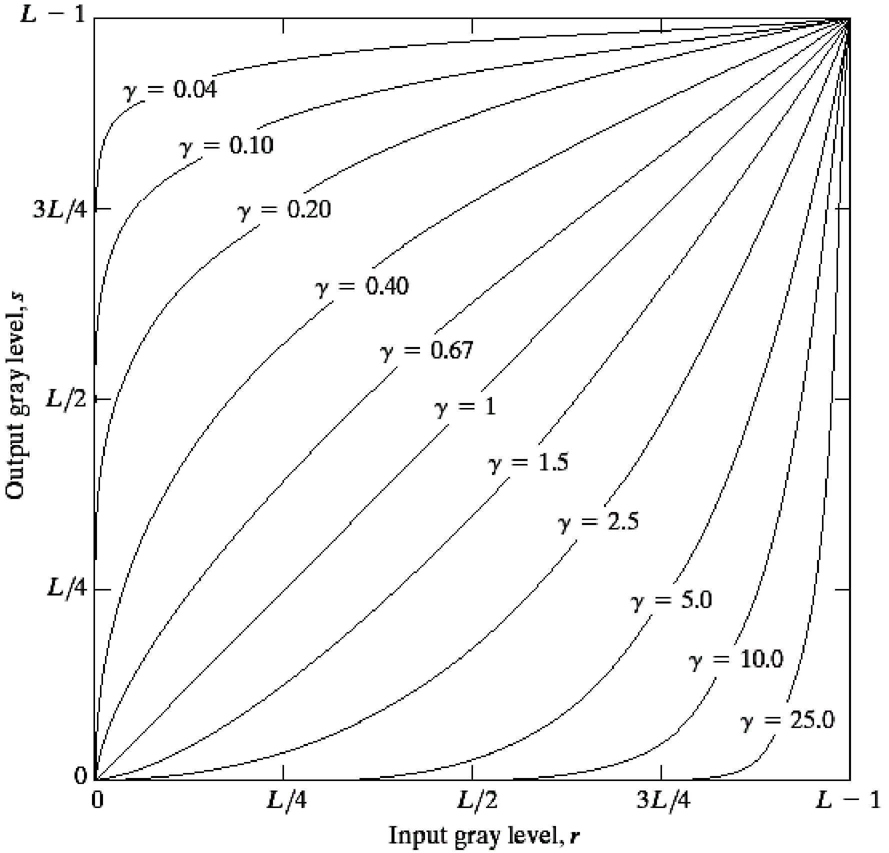
Result after Power law transformation $c = 1, \gamma = 0.6$

Result after Power law transformation $c = 1, \gamma = 0.4$

Result after Power law transformation $c = 1, \gamma = 0.3$

Power Law Transformations
Contrast Enhancement

When the γ is reduced too much, the image begins to reduce contrast to the point where the image started to have very slight “wash-out” look.



Power Law Transformations
Contrast Enhancement

Image has a washed-out appearance – needs γ > 1

Image Enhancement

Aerial Image

Result of Power law transformation c = 1, γ = 3.0 (suitable)

Result of Power law transformation c = 1, γ = 4.0 (suitable)

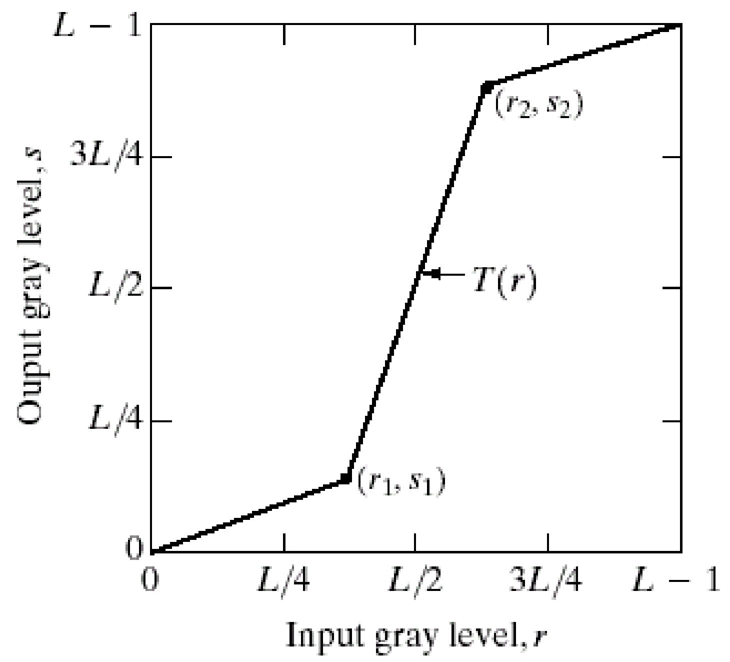
Result of Power law transformation c = 1, γ = 5.0 (high contrast, some regions are too dark)

Piecewise Linear Transformation Functions

- Contrast stretching
- Intensity level slicing

Contrast Stretching

- Objective
  - Increase the dynamic range of the gray levels for low contrast images
- Rather than using a well defined mathematical function we can use arbitrary user-defined transforms

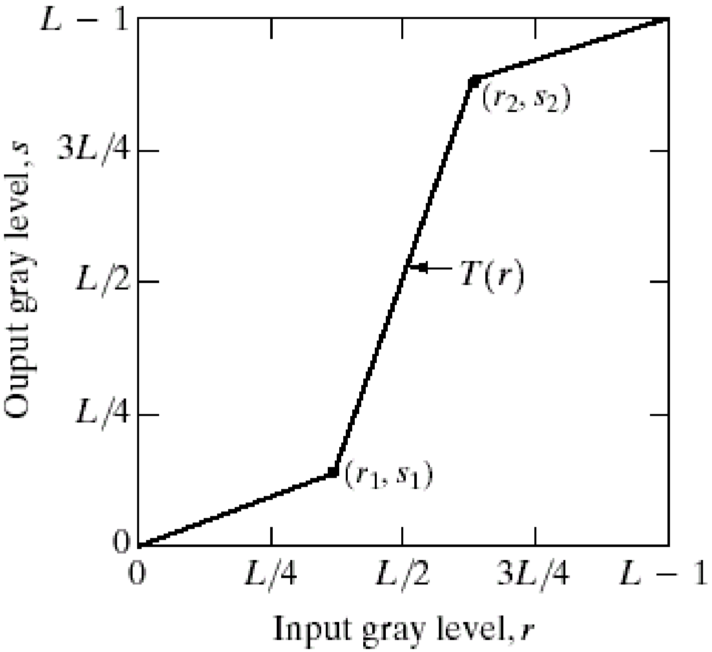


- If $r_1 = s_1$ & $r_2 = s_2$, no change in gray levels
- If $r_1 = r_2$, $s_1 = 0$ & $s_2 = L\text{-}1$, then it is a threshold function. The resulting image is binary

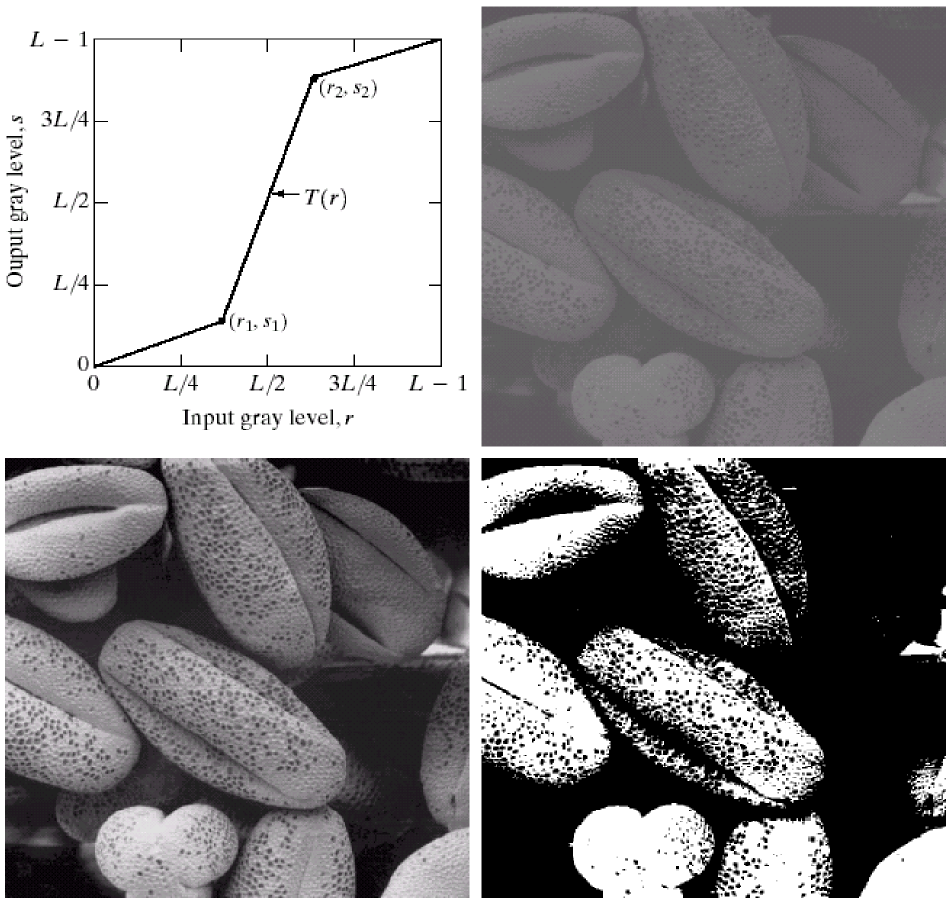
Contrast Stretching

$r_1 = r_{min}$ & $s_1 = 0$

$r_2 = r_{max}$ & $s_2 = L\text{-}1$



Contrast Stretching

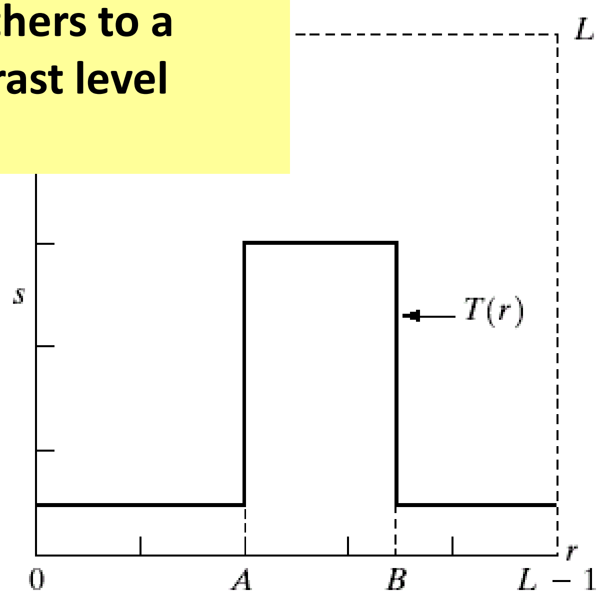


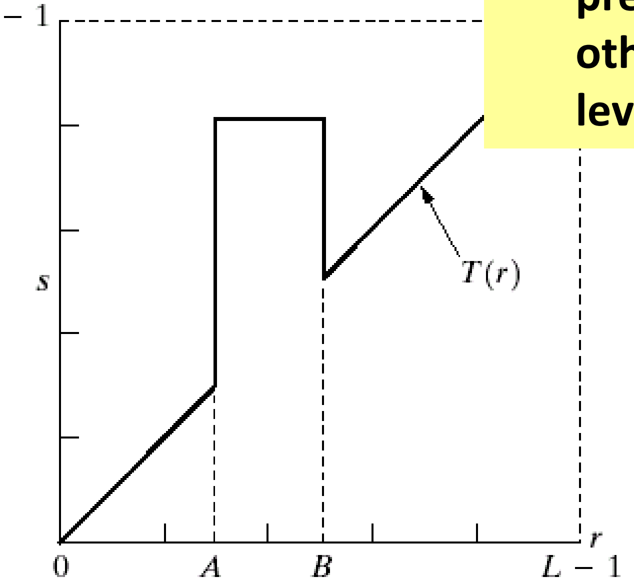
a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

Grey Level Slicing

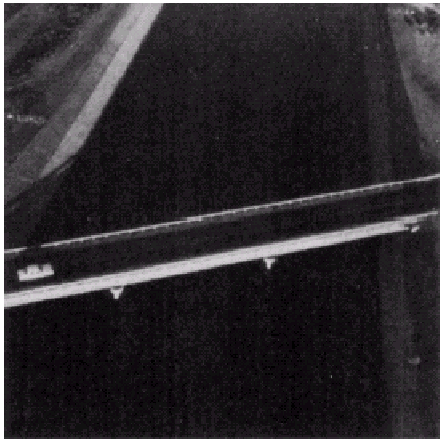
Highlights range [A,B] of gray levels and reduces all others to a contrast level

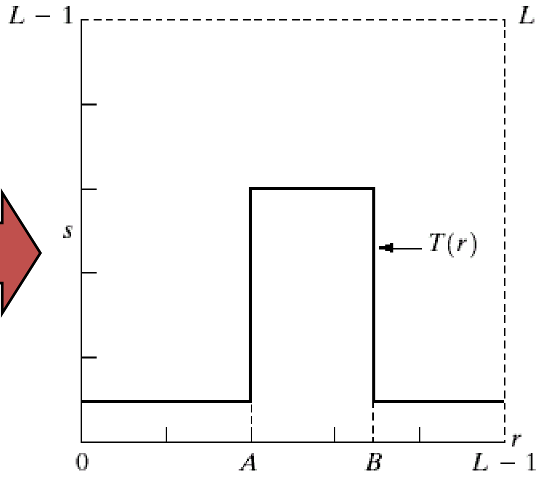


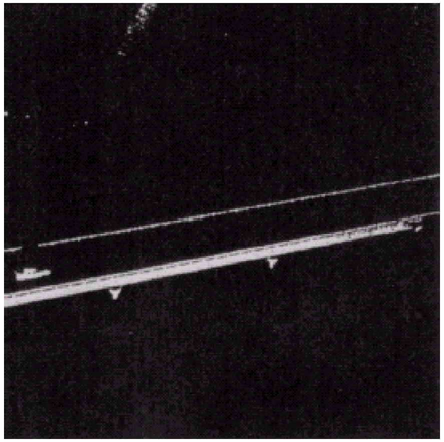


Highlights range [A,B] but preserves all other gray levels

Grey Level Slicing







Histogram of a Grayscale Image

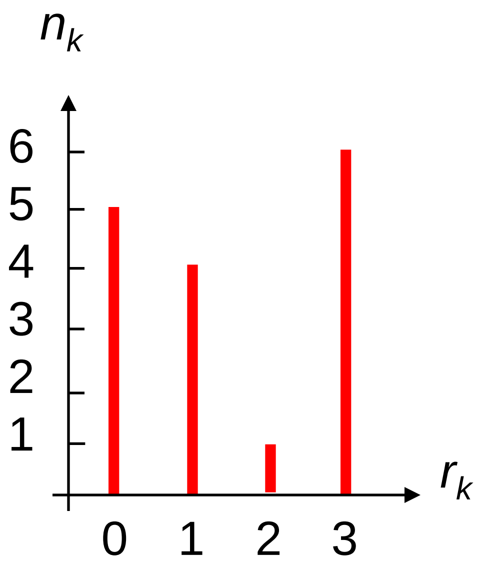
- Let I be a 1-band (grayscale) image.
- $I(r,c)$ is an 8-bit integer between 0 and 255.
- Histogram, h_I, of I:
  - a 256-element array, h_I
  - $h_I(g)$ = number of pixels in I that have value g.

    for $g = 0, 1, 2, 3, \ldots, 255$

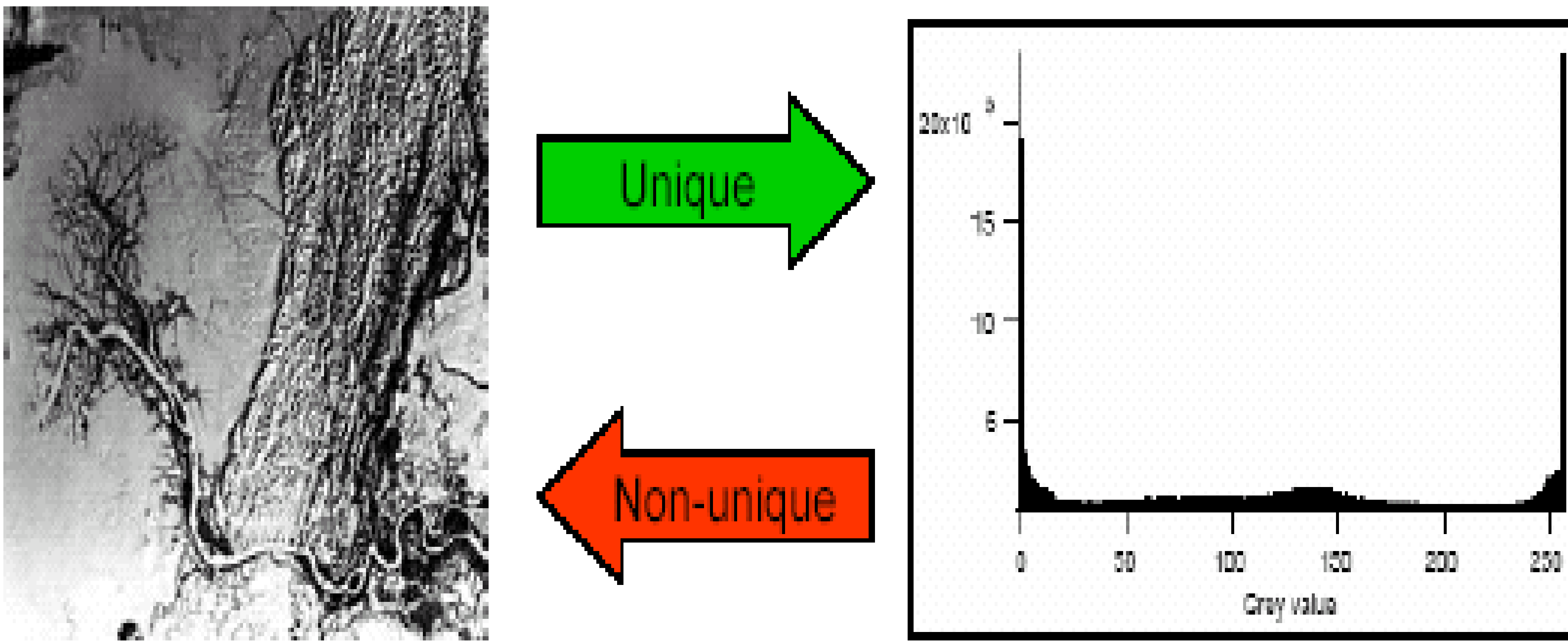
HISTOGRAM

- A discrete function $h(r_k)=n_k$
  - r_k is the k^{th} gray level
  - n_k is the number of pixels having gray level r_k in the image
- Ex:

0	1	2	3
1	3	3	0
0	1	3	0
3	0	3	1



UNIQUENESS



Histogram of a Grayscale Image

- Histogram of a digital image with gray levels in the range *[0,L-1]* is a discrete function

$$h(r_k) = n_k$$

Where

- $r_k = k^{th}$ gray level
- n_k = number of pixels in the image having gray level r_k
- $h(r_k)$ = histogram of an image having r_k gray levels

Normalized Histogram

- Dividing each of histogram at gray level r_k by the total number of pixels in the image, n

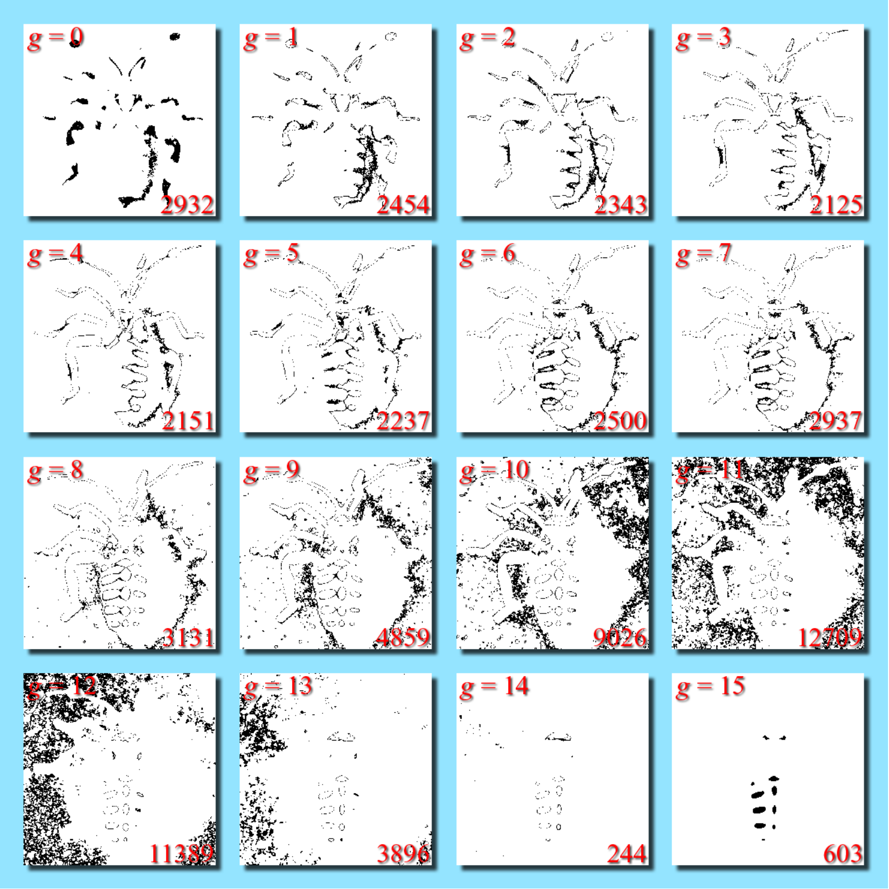
$$p(r_k) = n_k / n \quad for\ k = 0, 1, \ldots, L-1$$

- $p(r_k)$ gives an estimate of the probability of occurrence of gray level r_k
- The sum of all components of a normalized histogram is equal to 1

Histogram of a Grayscale Image

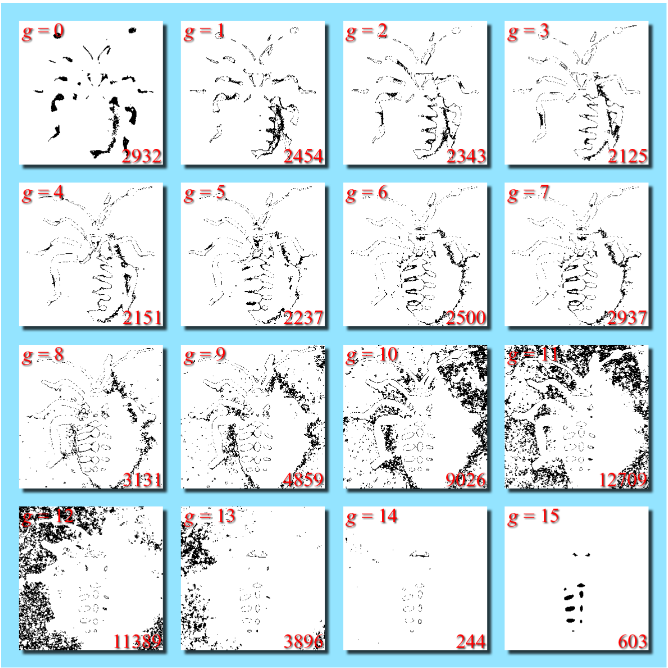
16-level (4-bit) image

lower RHC: number of pixels with intensity *g*



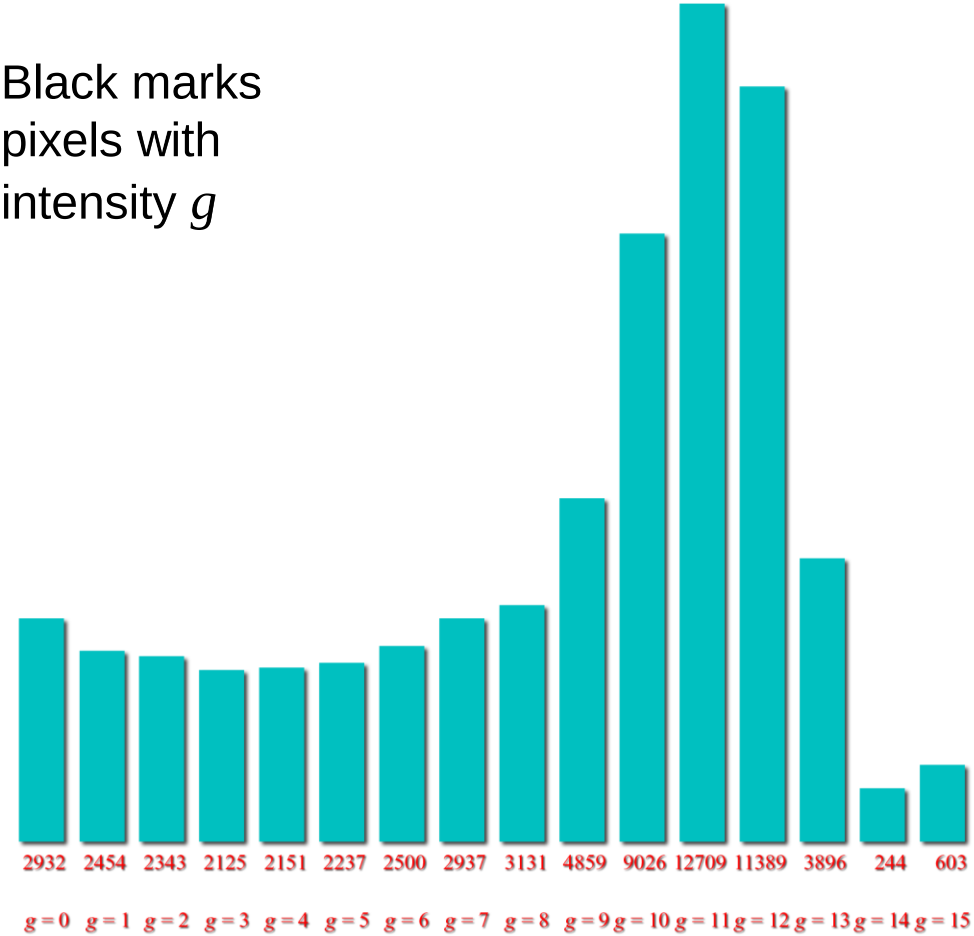
black marks pixels with intensity *g*

Histogram of a Grayscale Image

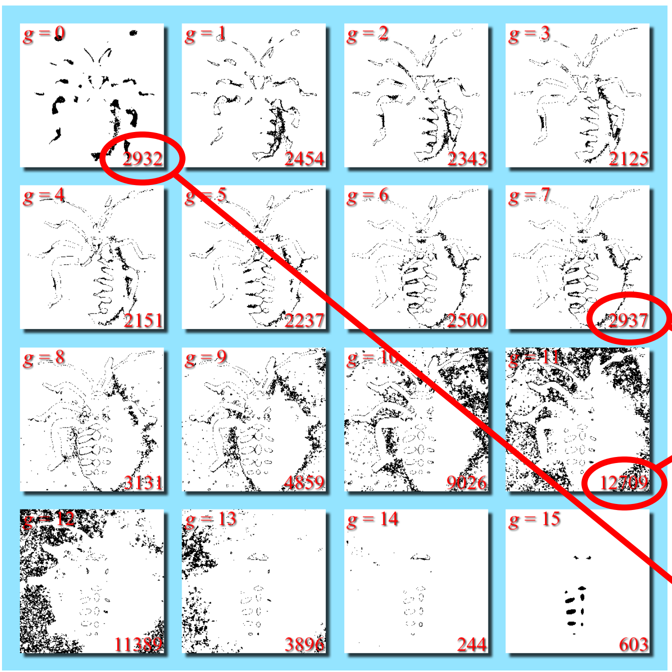


Black marks pixels with intensity g

Plot of histogram:
number of pixels with intensity g

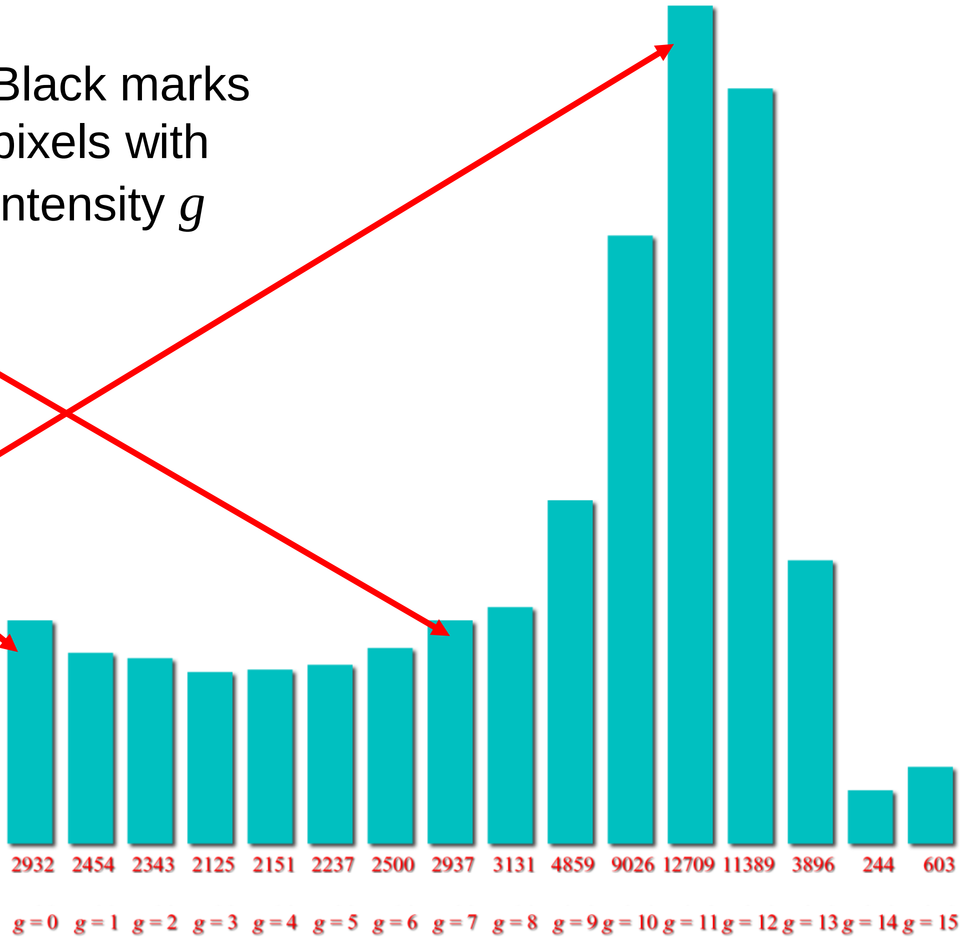


Histogram of a Grayscale Image



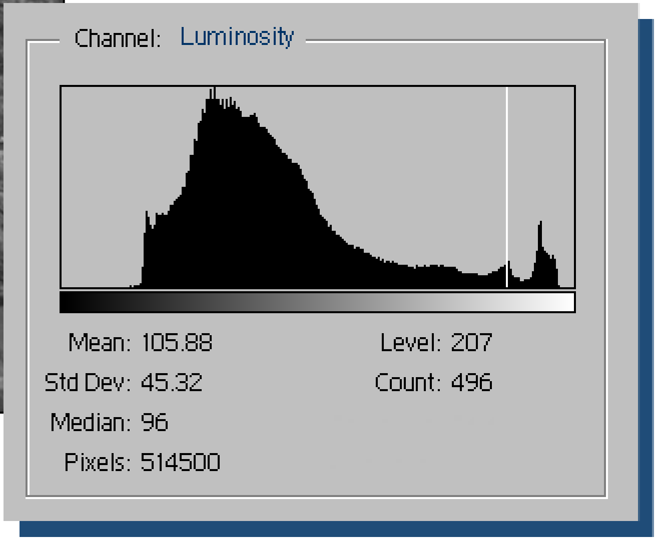
Black marks pixels with intensity g

Plot of histogram:
number of pixels with intensity g



Histogram of a Grayscale Image

$h_I(g) =$ the number of pixels in I with graylevel g.

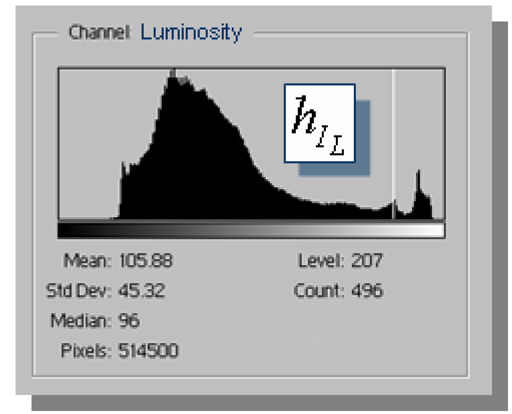


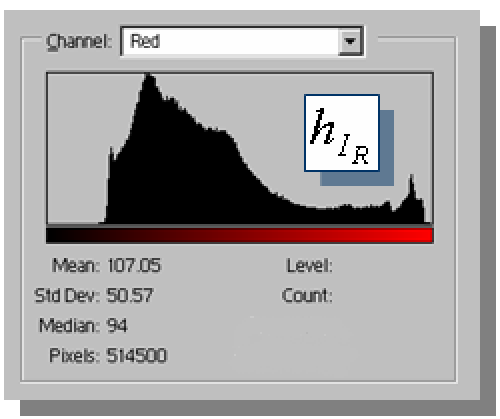
Histogram of a Color Image

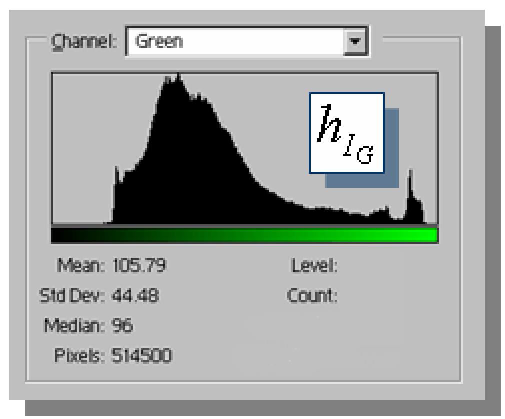
- If I is a 3-band image
- then I(r,c,b) is an integer between 0 and 255.
- I has 3 histograms:
  - $h_R(g)$ = # of pixels in I(:,:,1) with intensity value g
  - $h_G(g)$ = # of pixels in I(:,:,2) with intensity value g
  - $h_B(g)$ = # of pixels in I(:,:,3) with intensity value g

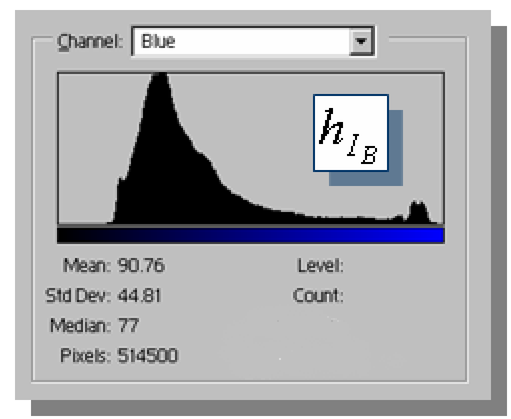
Histogram of a Color Image

There is one histogram per color band R, G, & B. Luminosity histogram is from 1 band = (R+G+B)/3

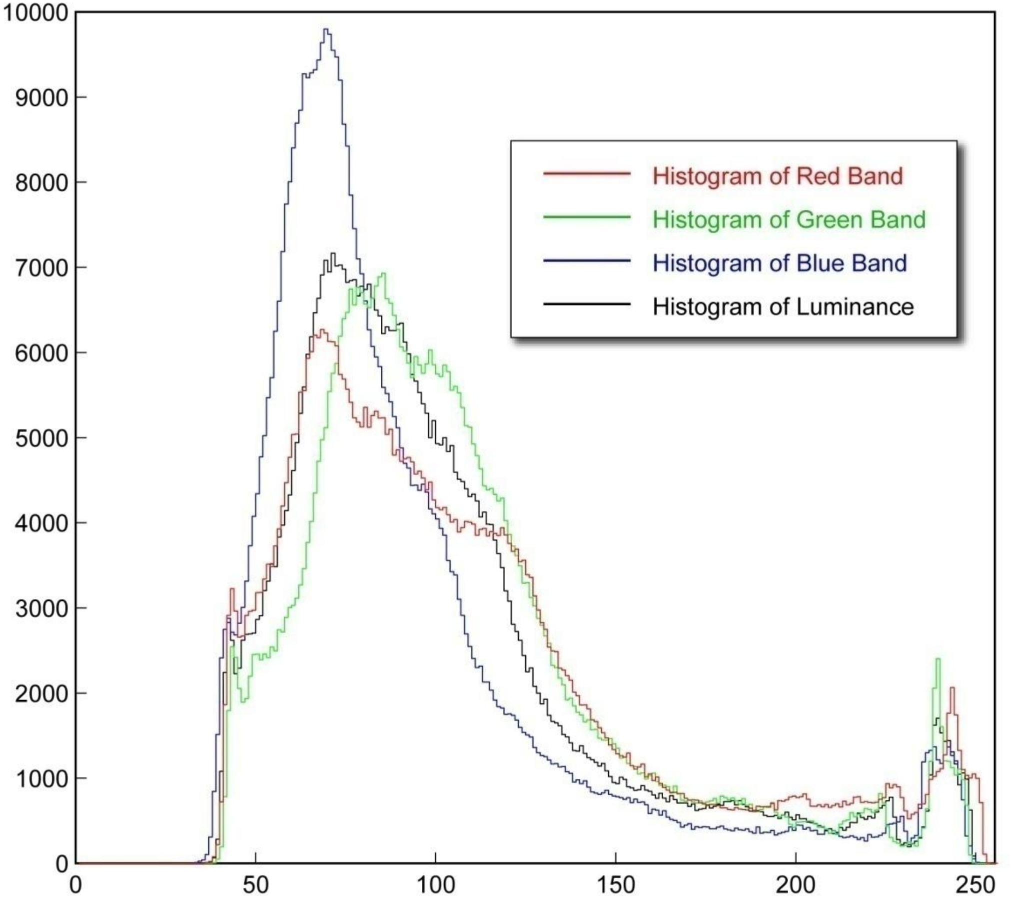




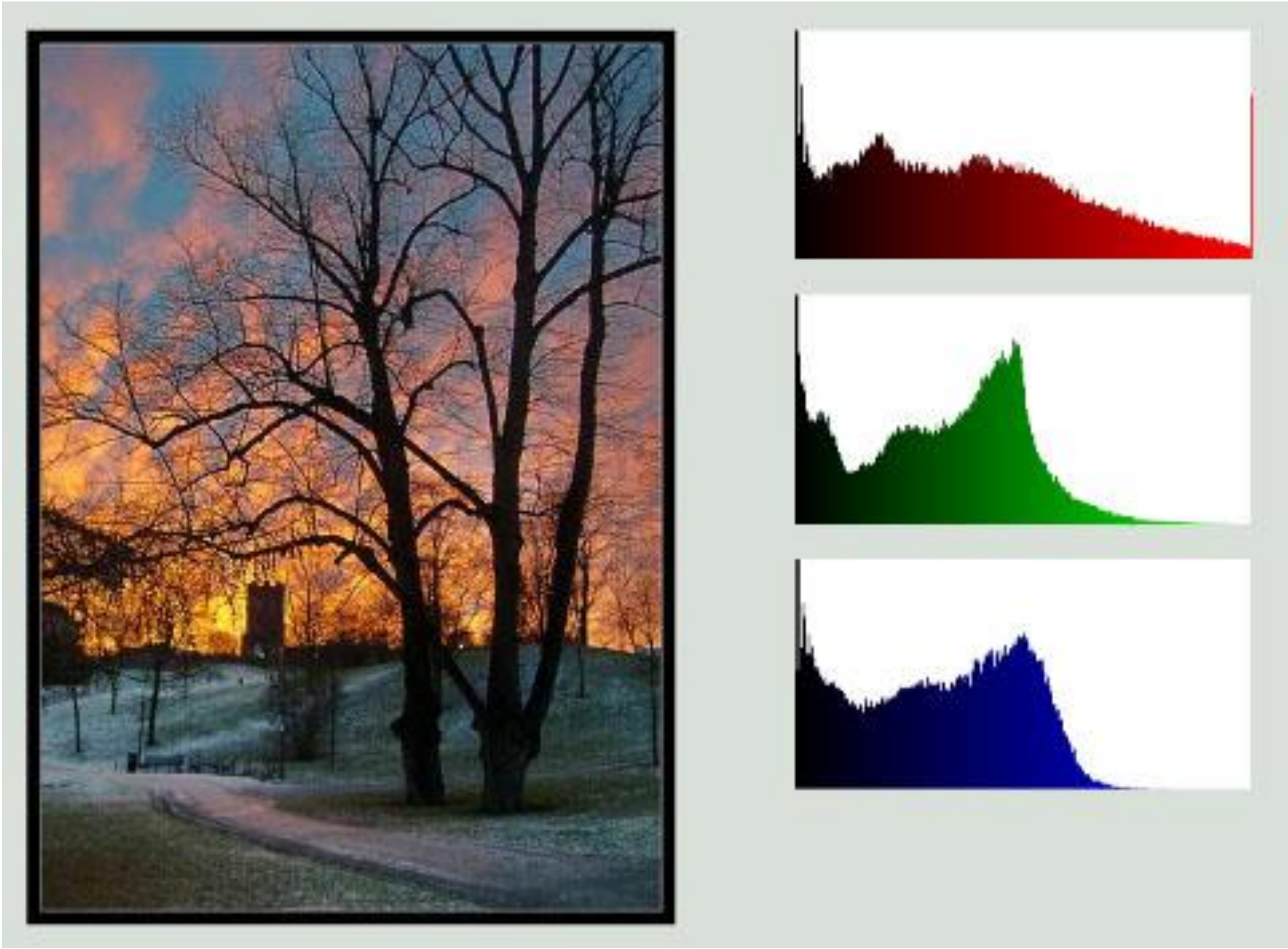




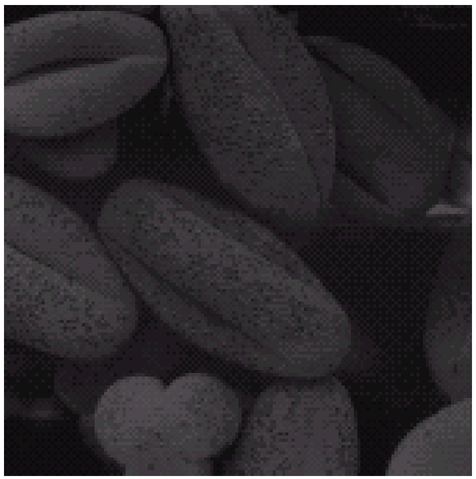
Histogram of a Color Image



Histogram of a Color Image



Histogram: Example

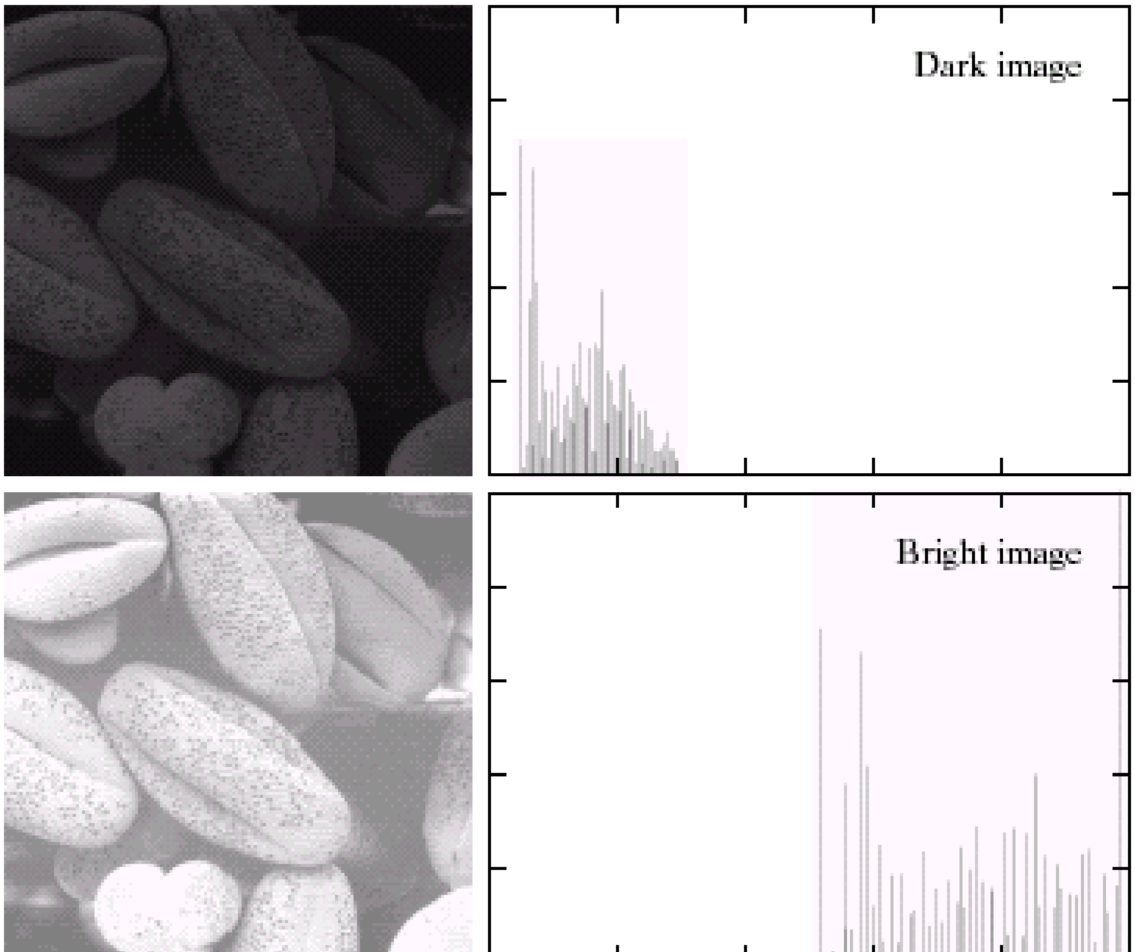


Dark Image

How would the histograms of these images look like?

Bright Image

Histogram: Example

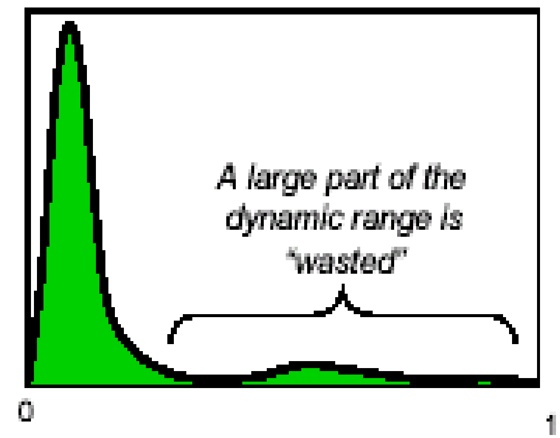


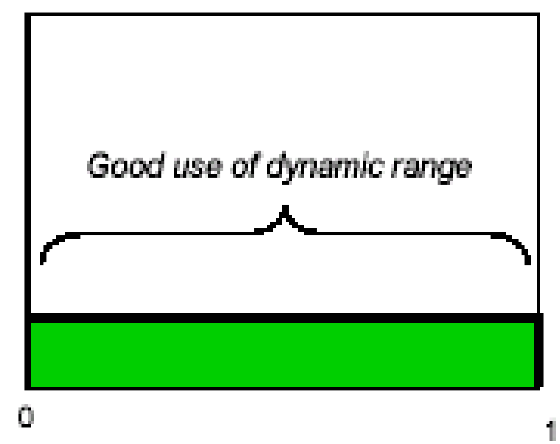
Dark image
Components of histogram are concentrated on the low side of the gray scale

Bright image
Components of histogram are concentrated on the high side of the gray scale

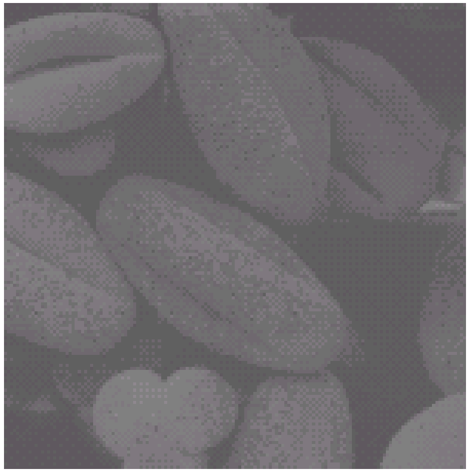
HISTOGRAM INSIGHT INTO CONTRAST

- A high contrast image makes good use of the full dynamic range available.
- Hence in some applications it may be desirable to make more optimal use of the full dynamic range.
- In some circumstances this results in a clearer image.





Histogram: Example

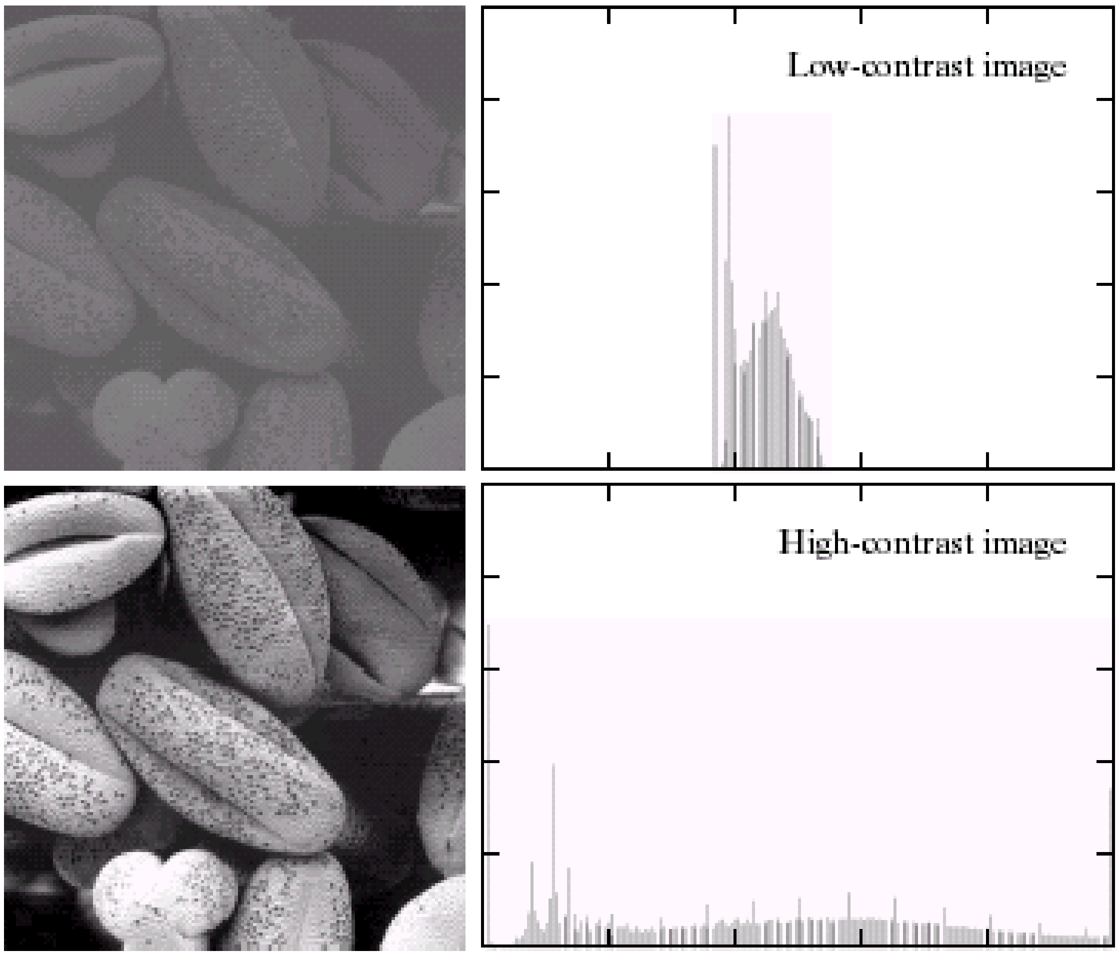


Low Contrast Image

How would the histograms of these images look like?

High Contrast Image

Histogram: Example



Low contrast image

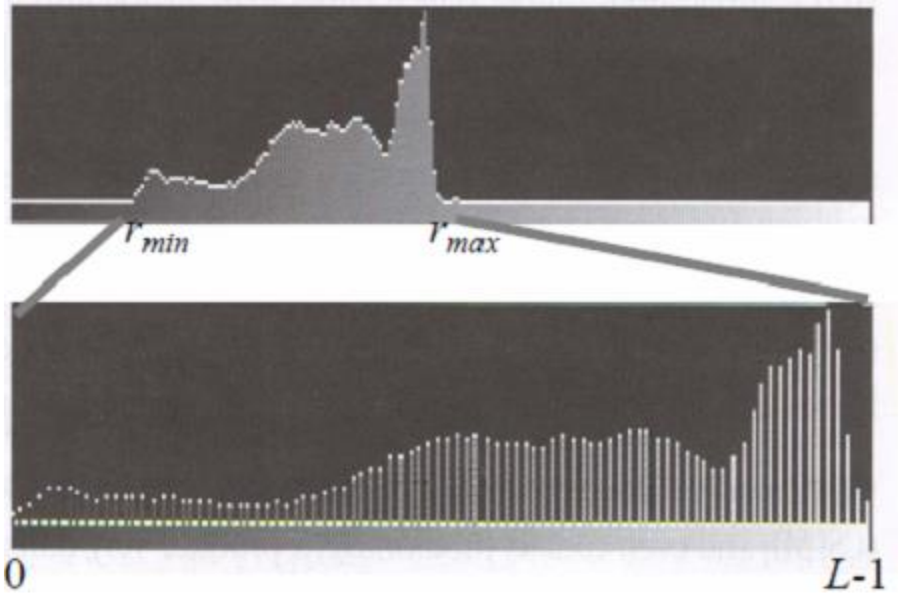
Histogram is narrow and centered toward the middle of the gray scale

High contrast image

Histogram covers broad range of the gray scale and the distribution of pixels is not too far from uniform with very few vertical lines being much higher than the others

Contrast Stretching

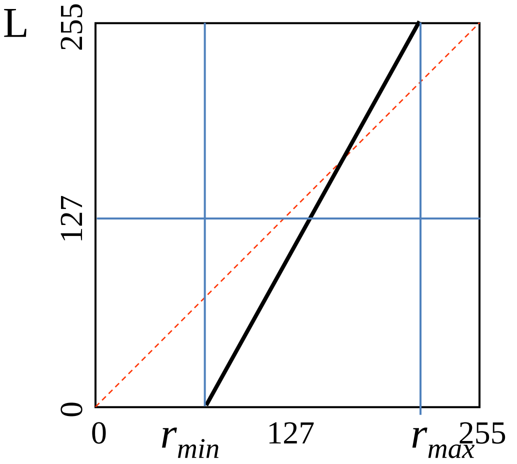
Improve the contrast in an image by `stretching' the range of intensity values it contains to span a desired range of values, *e.g.* the the full range of pixel values



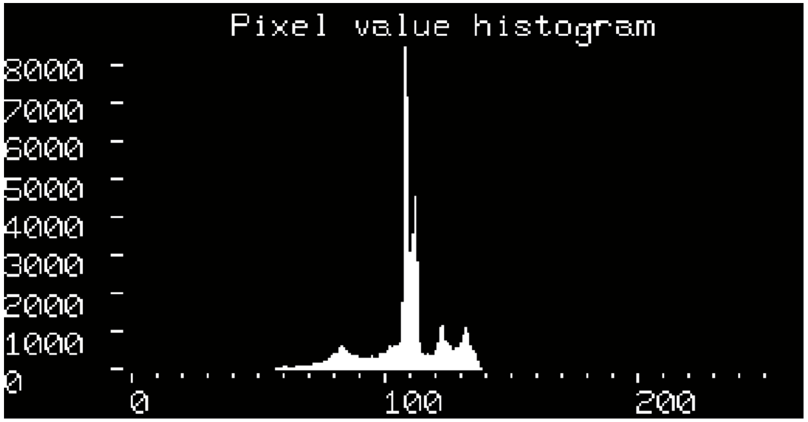
Contrast Stretching

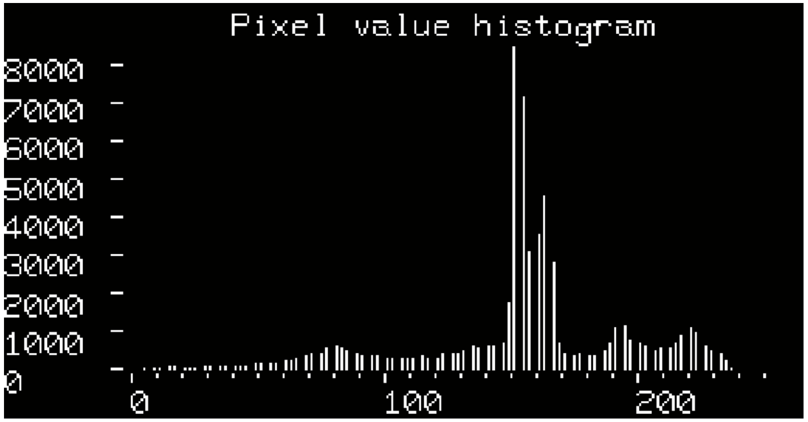
If r_{max} and r_{min} are the maximum and minimum gray level of the input image and L is the total gray levels of output image, the transformation function for contrast stretch will be

$$s = T(r) = (r - r_{\min})\left[\frac{L-1}{r_{\max} - r_{\min}}\right]$$



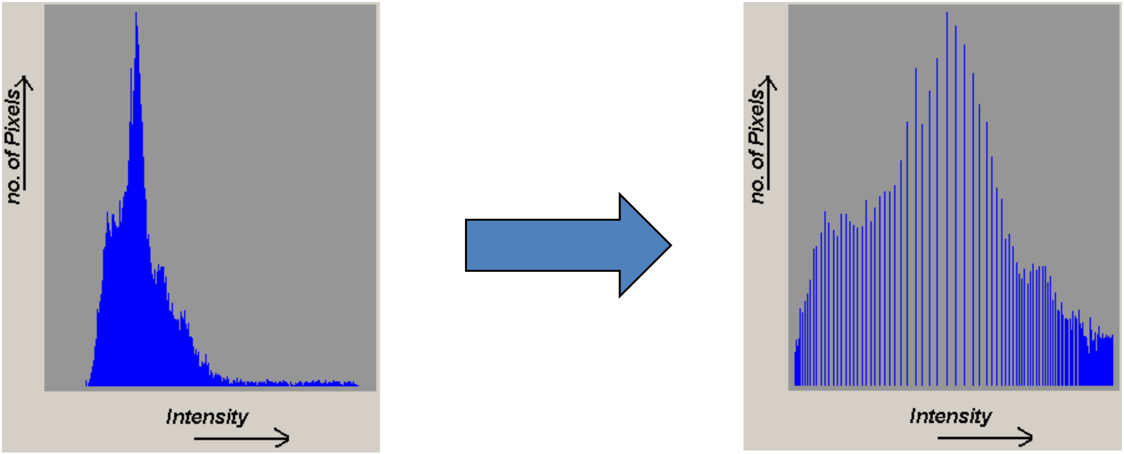
Contrast Stretching



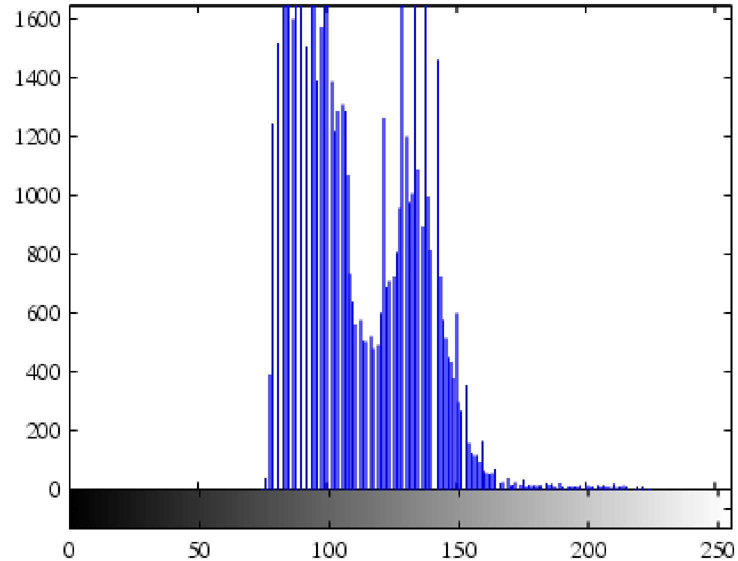


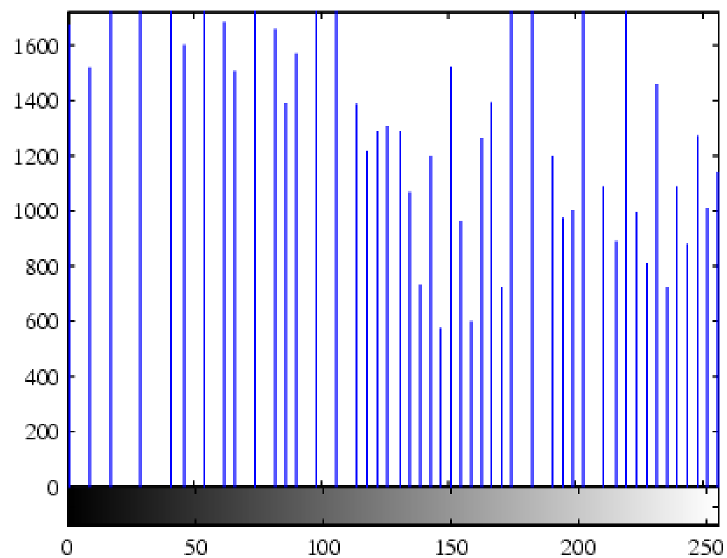
Histogram Equalization

Histogram equalization re-assigns the intensity values of pixels in the input image such that the output image contains a uniform distribution of intensities

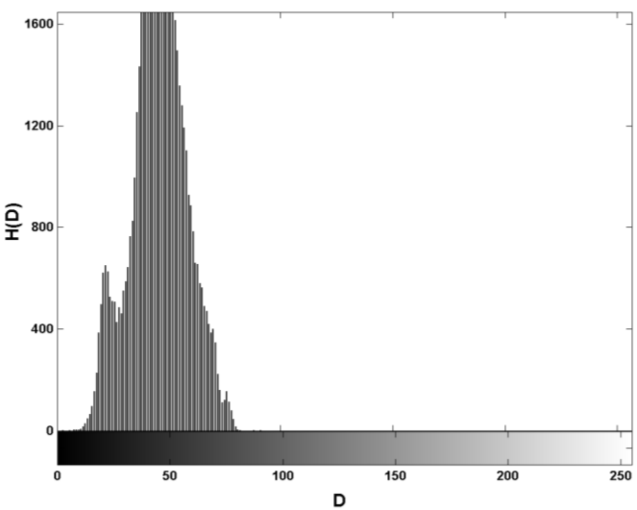


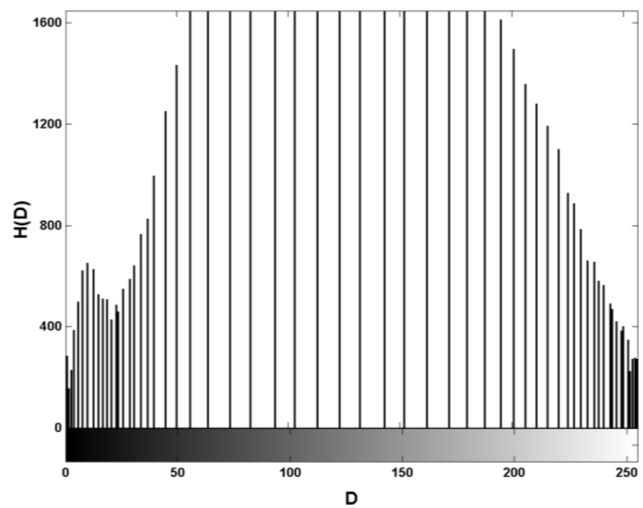
HISTOGRAM EQUALIZATION





AERIAL PHOTOGRAPH OF THE PENTAGON





Resulting image uses more of dynamic range.
Resulting histogram almost, but not completely, flat.

The Probability Distribution Function of an Image

Let $A = \sum_{g=0}^{255} h_I(g)$

Note that since $h_I(g)$ is the number of pixels in I with value g,

A is the number of pixels in I. That is if I is R rows by C columns then $A = R \times C$.

Then,

$$p_I(g) = \frac{1}{A} h_I(g)$$

This is the probability that an arbitrary pixel from *I* has value *g*.

The Probability Distribution Function of an Image

- *p*(*g*) is the fraction of pixels in an image that have intensity value *g.*
- *p*(*g*) is the probability that a pixel randomly selected from the given image has intensity value *g.*
- Whereas the sum of the histogram *h*(*g*) over all *g* from *0* to 255 is equal to the number of pixels in the image, the sum of *p*(*g*) over all *g* is 1*.*
- *p* is the normalized histogram of the image

The Cumulative Distribution Function of an Image

Let **q** = $I(r,c)$ be the value of a randomly selected pixel from I. Let g be a specific gray level. The probability that $q \le g$ is given by

$$P_I(g) = \sum_{\gamma=0}^{g} p_I(\gamma) = \frac{1}{A}\sum_{\gamma=0}^{g} h_I(\gamma) = \frac{\sum_{\gamma=0}^{g} h_I(\gamma)}{\sum_{\gamma=0}^{255} h_I(\gamma)},$$

where $h_I(\gamma)$ is the histogram of image I.

This is the probability that any given pixel from *I* has value less than or equal to g.

The Cumulative Distribution Function of an Image

Let **q** = $I(r,c)$ be the value of a randomly selected pixel from I. Let g be a specific gray level. The probability that $q \le g$ is given by

$$P_I(g) = \sum_{\gamma=0}^{g} p_I(\gamma) = \frac{1}{A}\sum_{\gamma=0}^{g} h_I(\gamma) = \frac{\sum_{\gamma=0}^{g} h_I(\gamma)}{\sum_{\gamma=0}^{255} h_I(\gamma)},$$

where $h_I(\gamma)$ is the histogram of image I.

Also called CDF for "Cumulative Distribution Function".

This is the probability that any given pixel from *I* has value less than or equal to g.

The Cumulative Distribution Function of an Image

- $P(g)$ is the fraction of pixels in an image that have intensity values less than or equal to *g.*
- $P(g)$ is the probability that a pixel randomly selected from the given band has an intensity value less than or equal to *g.*
- $P(g)$ is the cumulative (or running) sum of $p(g)$ from 0 through *g* inclusive.
- $P(0) = p(0)$ and $P(255) = 1$;

Histogram Equalization

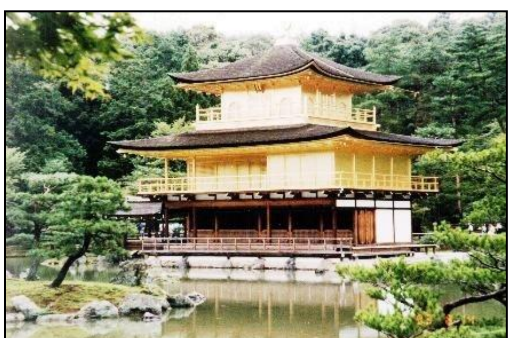
Task: remap image I so that its histogram is as close to constant as possible

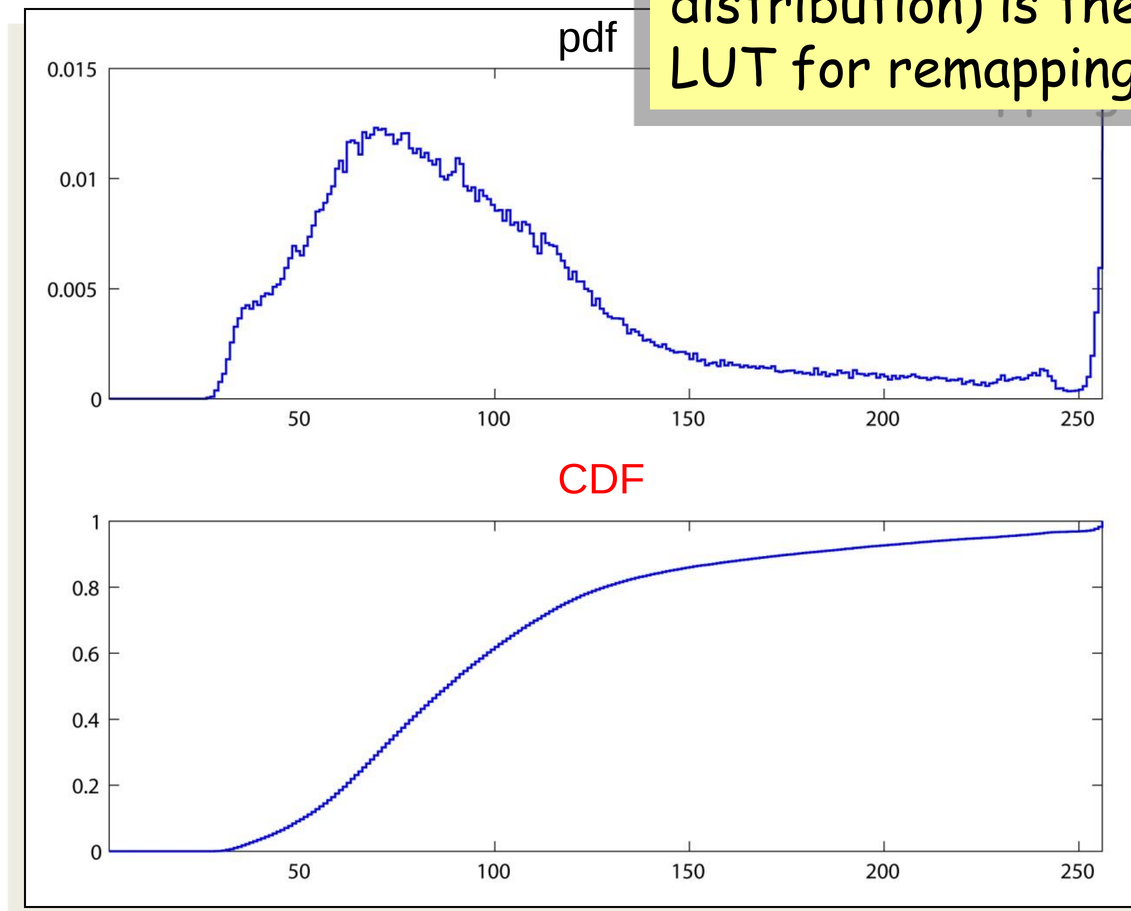
Let $P_I(\gamma)$

be the cumulative (probability) distribution function of I.

The CDF itself is used as the LUT.

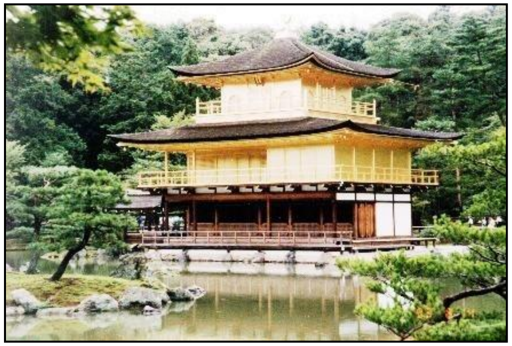
Histogram Equalization

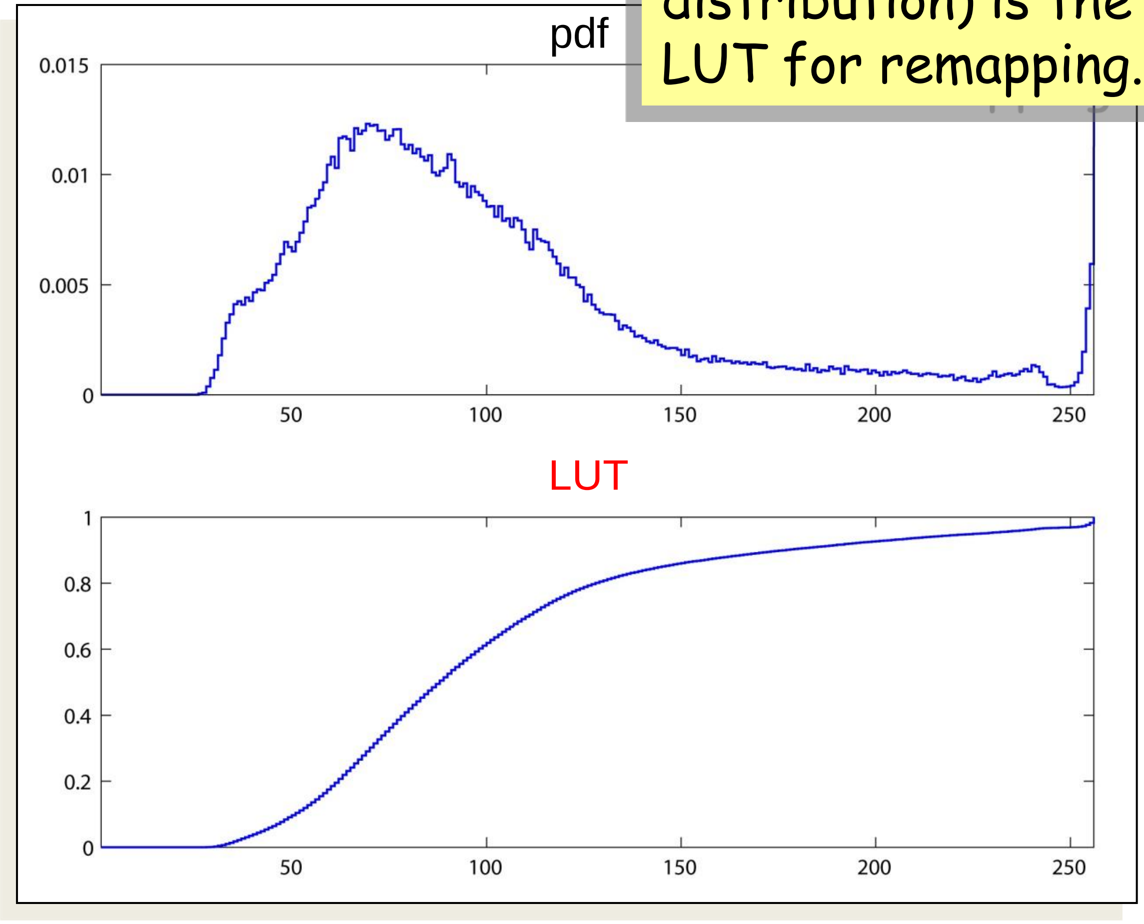




The CDF (cumulative distribution) is the LUT for remapping.

Histogram Equalization

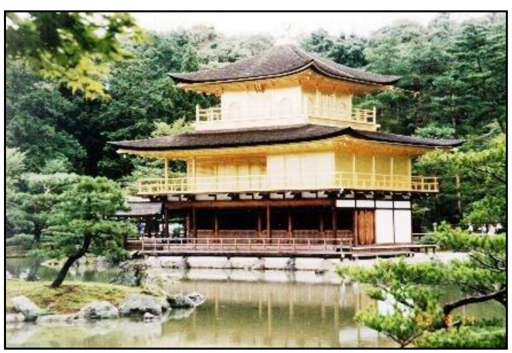


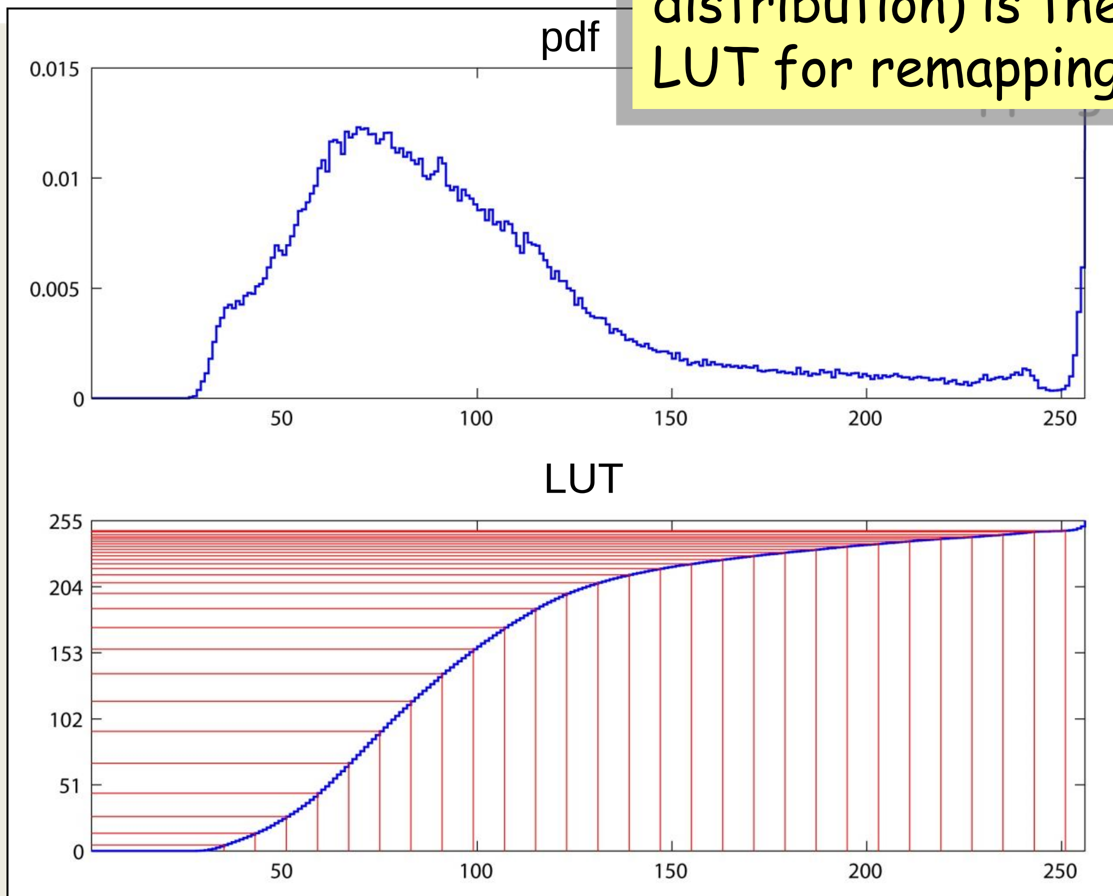


The CDF (cumulative distribution) is the LUT for remapping.

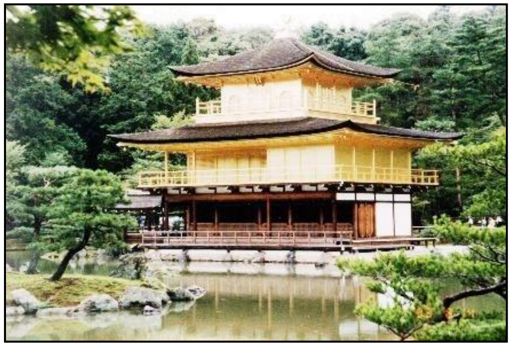
Histogram Equalization

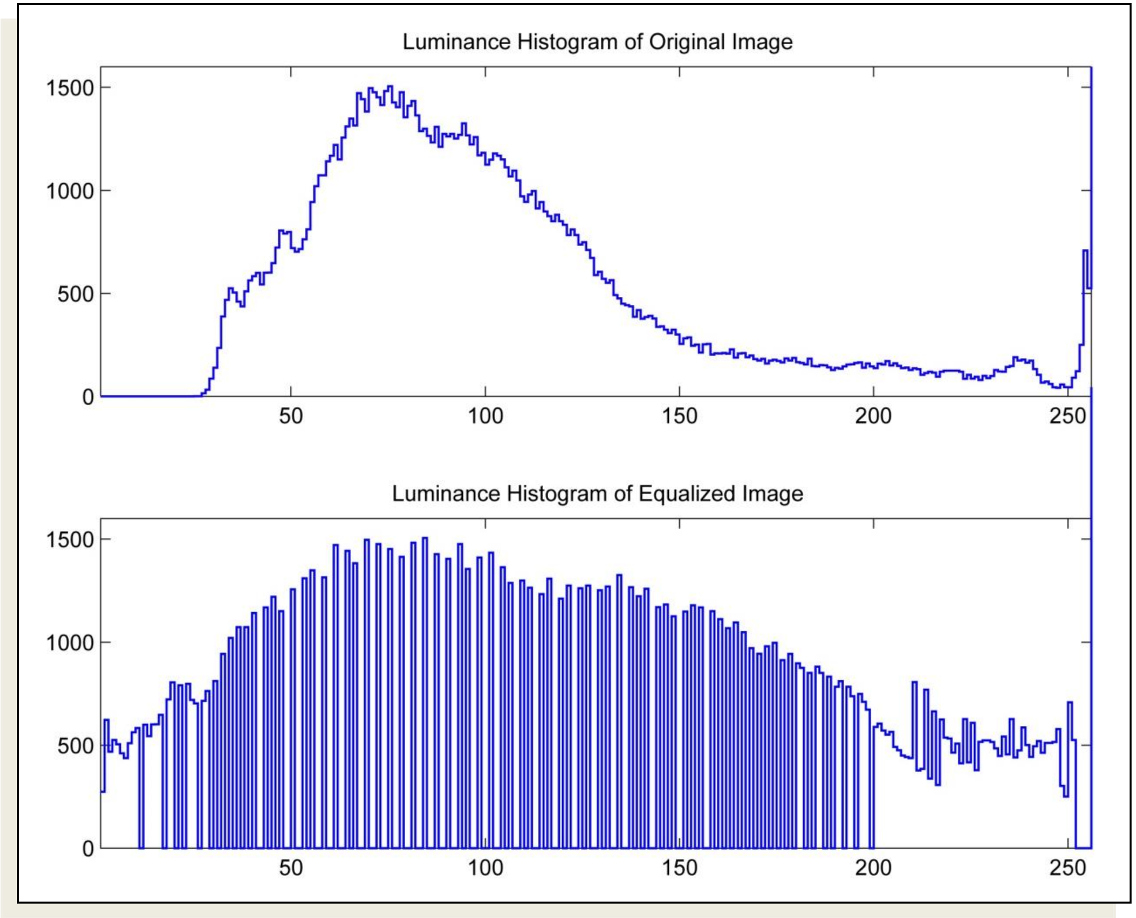
The CDF (cumulative distribution) is the LUT for remapping.



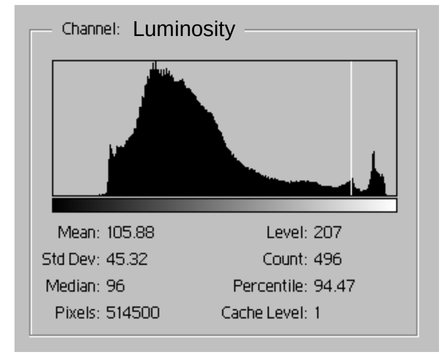


Histogram Equalization

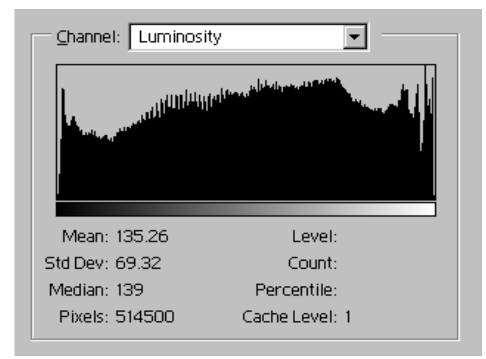




Histogram Equalization



before



after

$$J(r,c) = 255 \cdot P_I\left[I(r,c)\right].$$

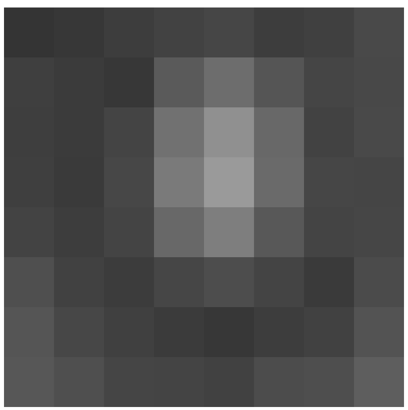
HISTOGRAM EQUALIZATION IMPLEMENTATION

0	0	0	0	0
1	1	1	1	4
4	5	6	6	6
8	8	8	8	9

2	2	2	2	2
4	4	4	4	5
5	5	7	7	7
9	9	9	9	9

Gray levels	0	1	2	3	4	5	6	7	8	9
Counts ($h(r_k)$)	5	4	0	0	2	1	3	0	4	1
	r_0	r_1			r_2	r_3	r_4		r_5	r_6
Normalized h ($P(r_k)$)	5/20	4/20	0	0	2/20	1/20	3/20	0	4/20	1/20
cdf $F(r_k)$	5/20	9/20			**11/20**	**12/20**	**15/20**		**19/20**	**20/20**
$s_k = \text{round}(9 \cdot F(r_k))$	2	4			5	5	7		9	9
	s_0	s_1			s_2	s_3	s_4		s_5	s_6

Histogram Equalization: Example



$$
\begin{bmatrix}
52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\
63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\
62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\
63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\
67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\
79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\
85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\
87 & 79 & 69 & 68 & 65 & 76 & 78 & 94
\end{bmatrix}
$$

An 8x8 image

Histogram Equalization: Example

Fill in the following table/histogram

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52		64		72		85		113	
55		65		73		87		122	
58		66		75		88		126	
59		67		76		90		144	
60		68		77		94		154	
61		69		78		104			
62		70		79		106			
63		71		83		109			

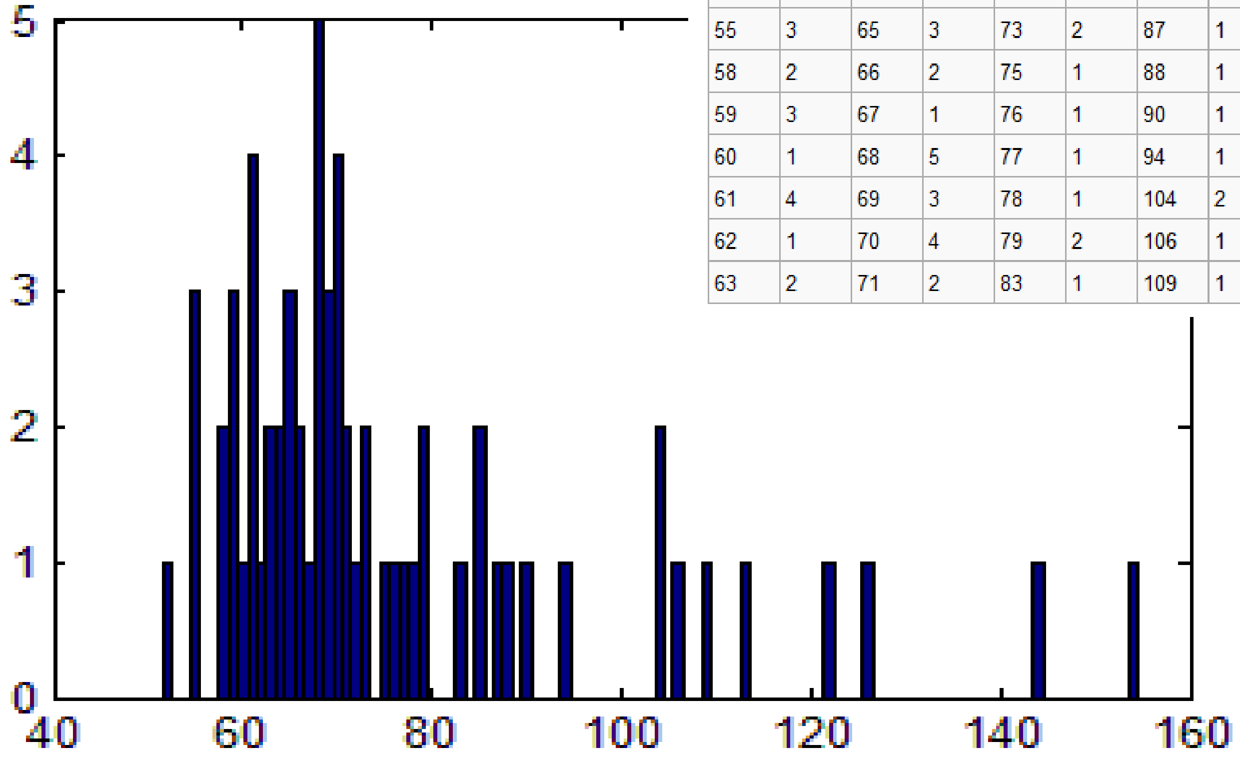
Image Histogram (Non-zero values)

Histogram Equalization: Example

Image Histogram (Non-zero values shown)

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Histogram Equalization: Example



Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Histogram Equalization: Example

Image Histogram/Prob Mass Function

Value	Count	Value	Count	Value	Count	Value	Count	Value	Count
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

Cumulative Distribution Function (cdf)

Value	cdf	Value	cdf	Value	cdf	Value	cdf	Value	cdf
52		64		72		85		113	
55		65		73		87		122	
58		66		75		88		126	
59		67		76		90		144	
60		68		77		94		154	
61		69		78		104			
62		70		79		106			
63		71		83		109			

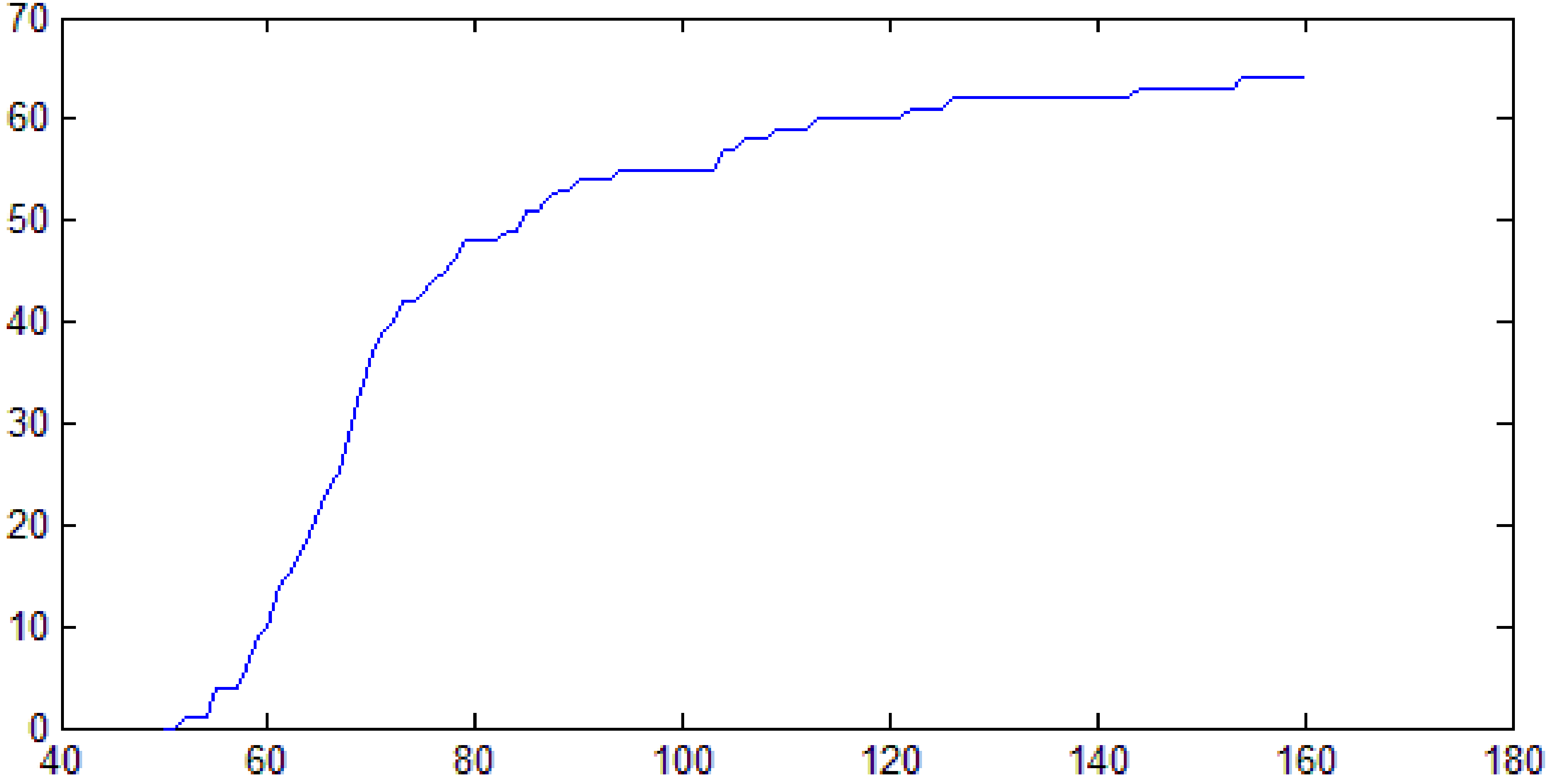
Histogram Equalization: Example

Cumulative Distribution Function (cdf)

Value	cdf	Value	cdf	Value	cdf	Value	cdf	Value	cdf
52	1	64	19	72	40	85	51	113	60
55	4	65	22	73	42	87	52	122	61
58	6	66	24	75	43	88	53	126	62
59	9	67	25	76	44	90	54	144	63
60	10	68	30	77	45	94	55	154	64
61	14	69	33	78	46	104	57		
62	15	70	37	79	48	106	58		
63	17	71	39	83	49	109	59		

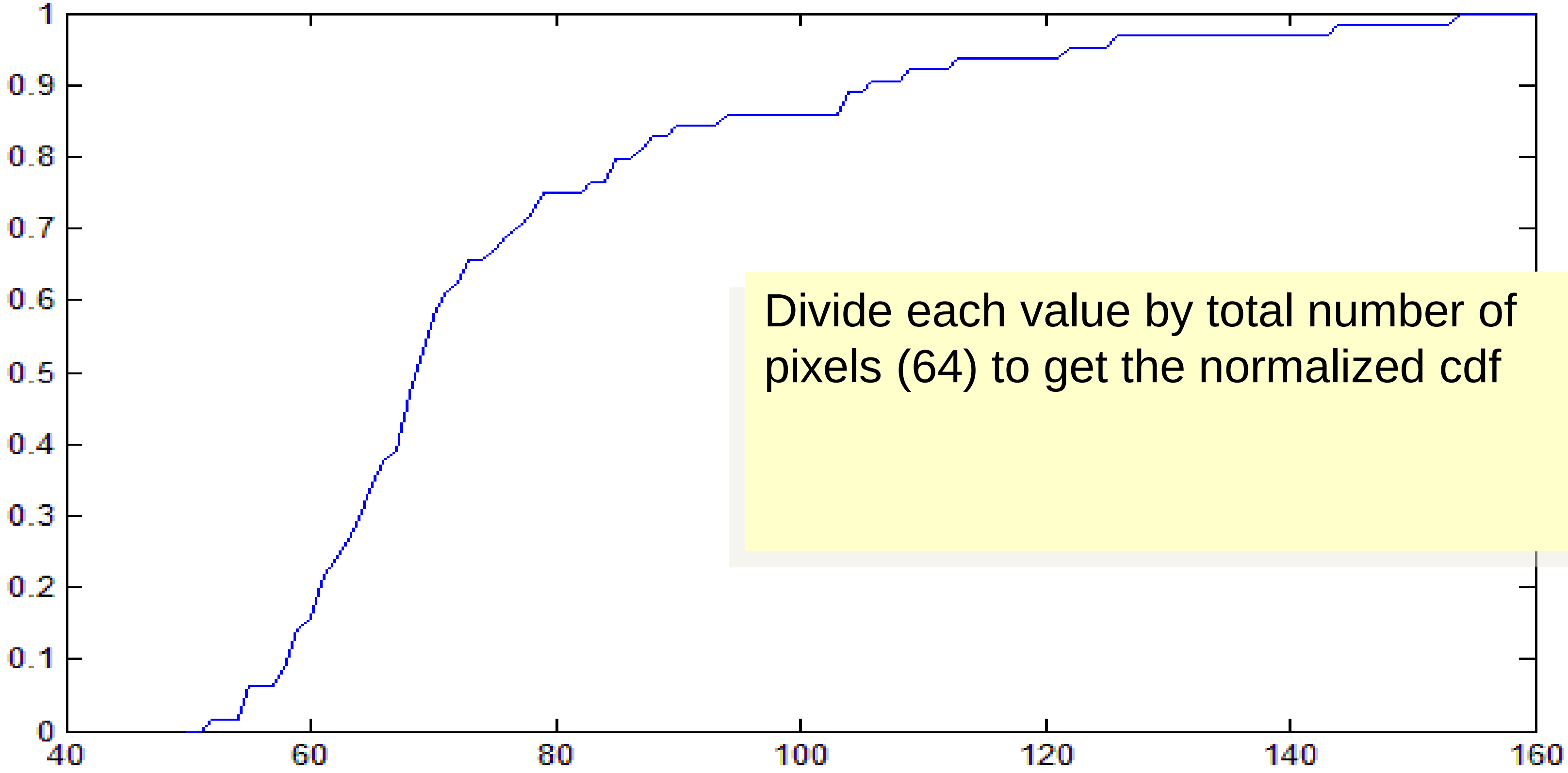
Histogram Equalization: Example

Cumulative Distribution Function (cdf)



Histogram Equalization: Example

Normalized Cumulative Distribution Function (cdf)



Divide each value by total number of pixels (64) to get the normalized cdf

Histogram Equalization: Example

Value	cdf	Value	cdf	Value	cdf	Value	cdf	Value	cdf
52	1	64	19	72	40	85	51	113	60
55	4	65	22	73	42	87	52	122	61
58	6	66	24	75	43	88	53	126	62
59	9	67	25	76	44	90	54	144	63
60	10	68	30	77	45	94	55	154	64
61	14	69	33	78	46	104	57		
62	15	70	37	79	48	106	58		
63	17	71	39	83	49	109	59		

$$J(r,c) = 255 \cdot P_I\left[I(r,c)\right].$$

If cdf is normalized

$$s = round(255.cdf(r))$$

If cdf is NOT normalized

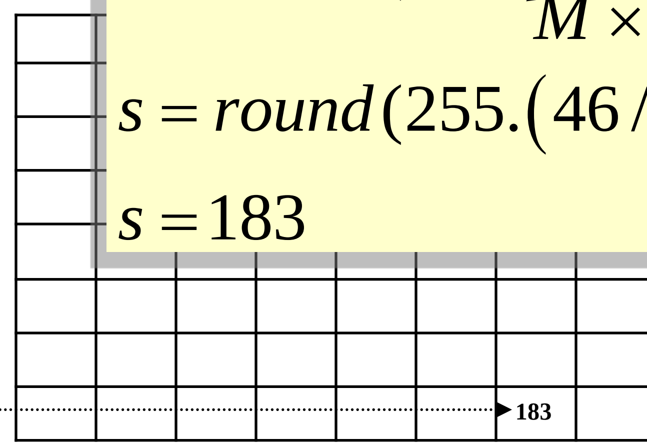
$$s = round(255.\frac{cdf(r)}{M \times N})$$

$$s = round(255.(46/64))$$

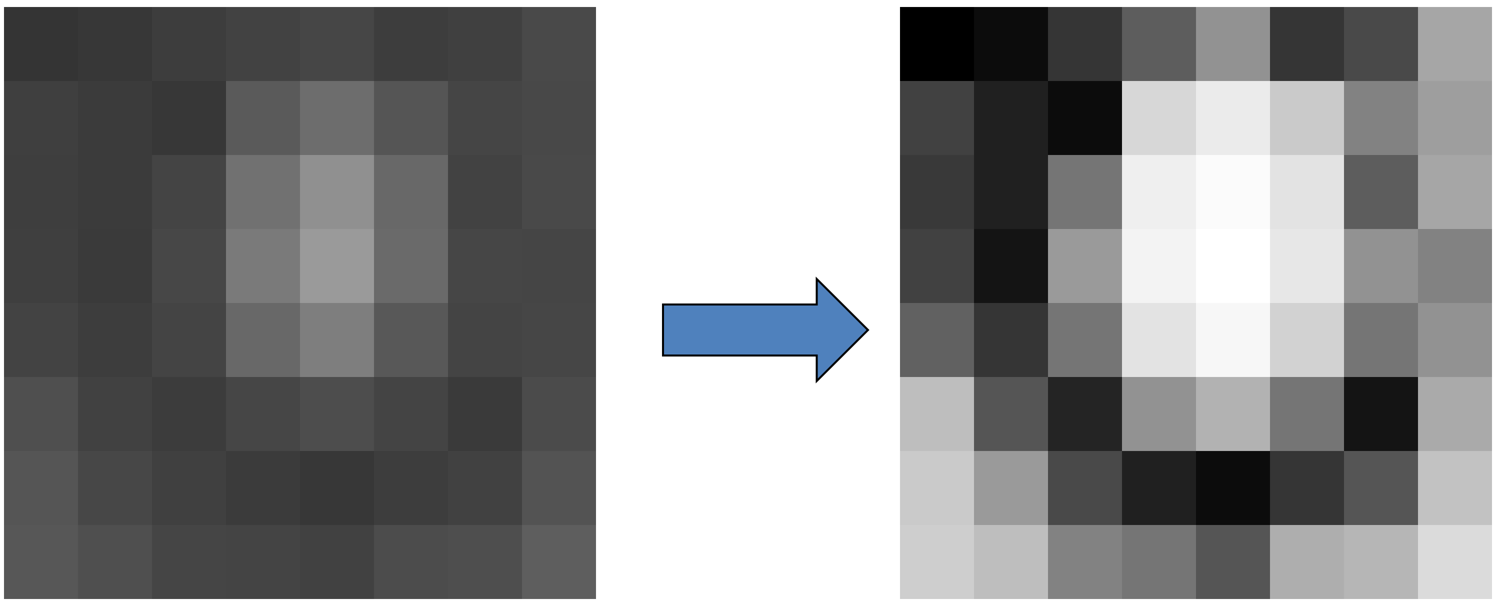
$$s = 183$$

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 64 & 73 \\ 63 & 59 & 55 & 90 & 109 & 85 & 69 & 72 \\ 62 & 59 & 68 & 113 & 144 & 104 & 66 & 73 \\ 63 & 58 & 71 & 122 & 154 & 106 & 70 & 69 \\ 67 & 61 & 68 & 104 & 126 & 88 & 68 & 70 \\ 79 & 65 & 60 & 70 & 77 & 68 & 58 & 75 \\ 85 & 71 & 64 & 59 & 55 & 61 & 65 & 83 \\ 87 & 79 & 69 & 68 & 65 & 76 & 78 & 94 \end{bmatrix}$$

Original Image

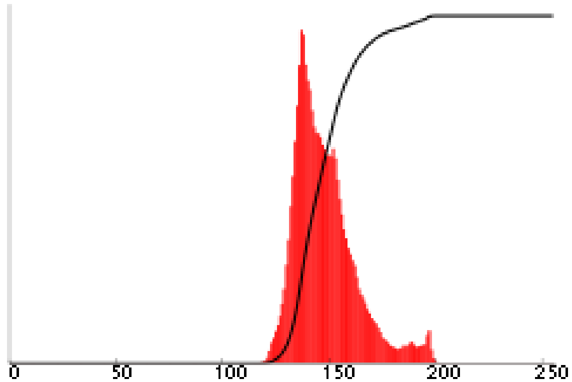


Histogram Equalization: Example



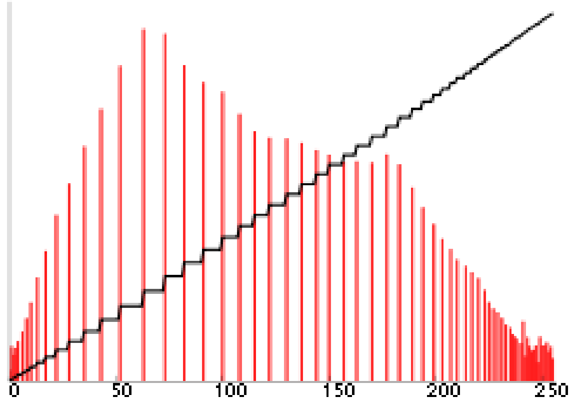
Histogram Equalization: Example

Original Image



Corresponding histogram (red) and cumulative histogram (black)

Image after histogram equalization

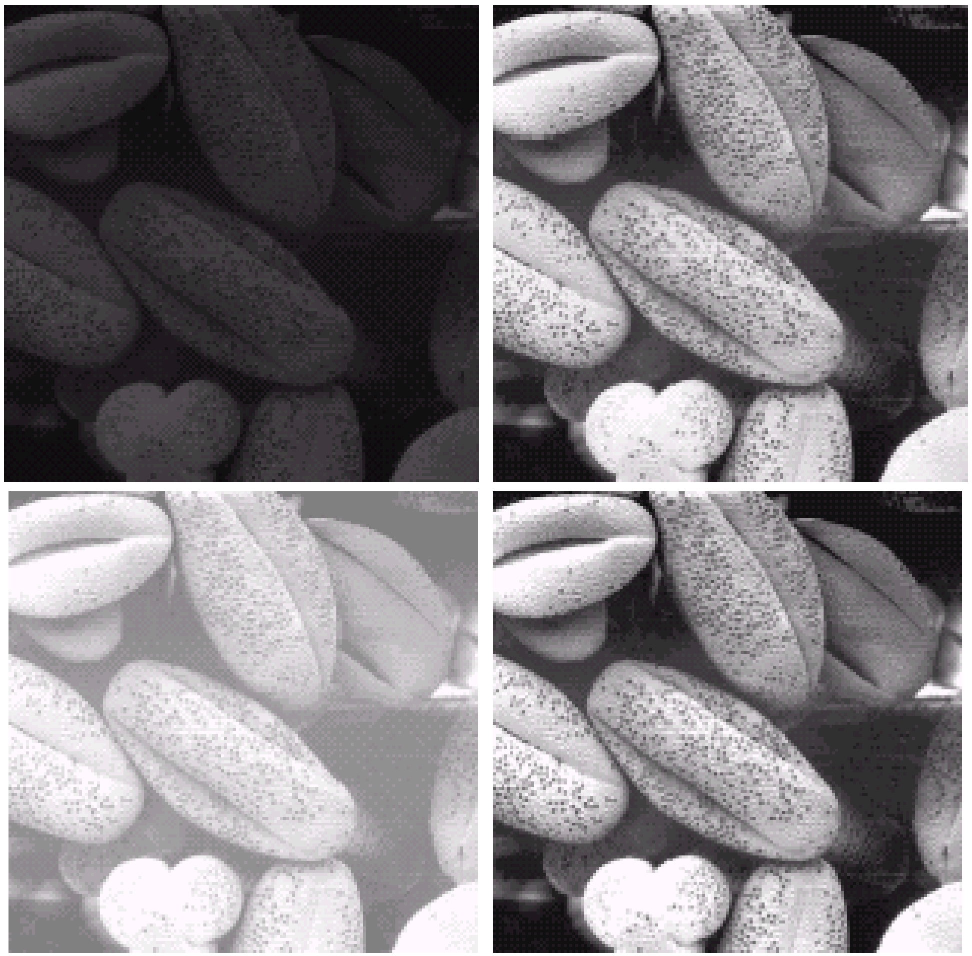


Corresponding histogram (red) and cumulative histogram (black)

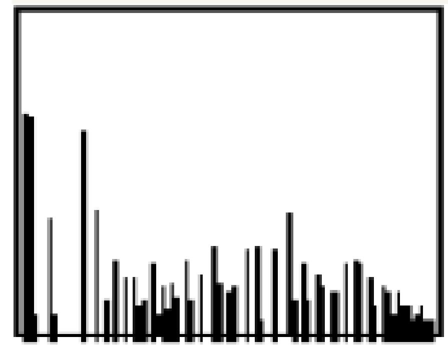
Histogram Equalization: Example

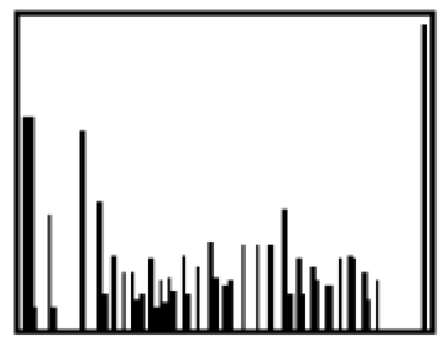
Dark image

Bright image



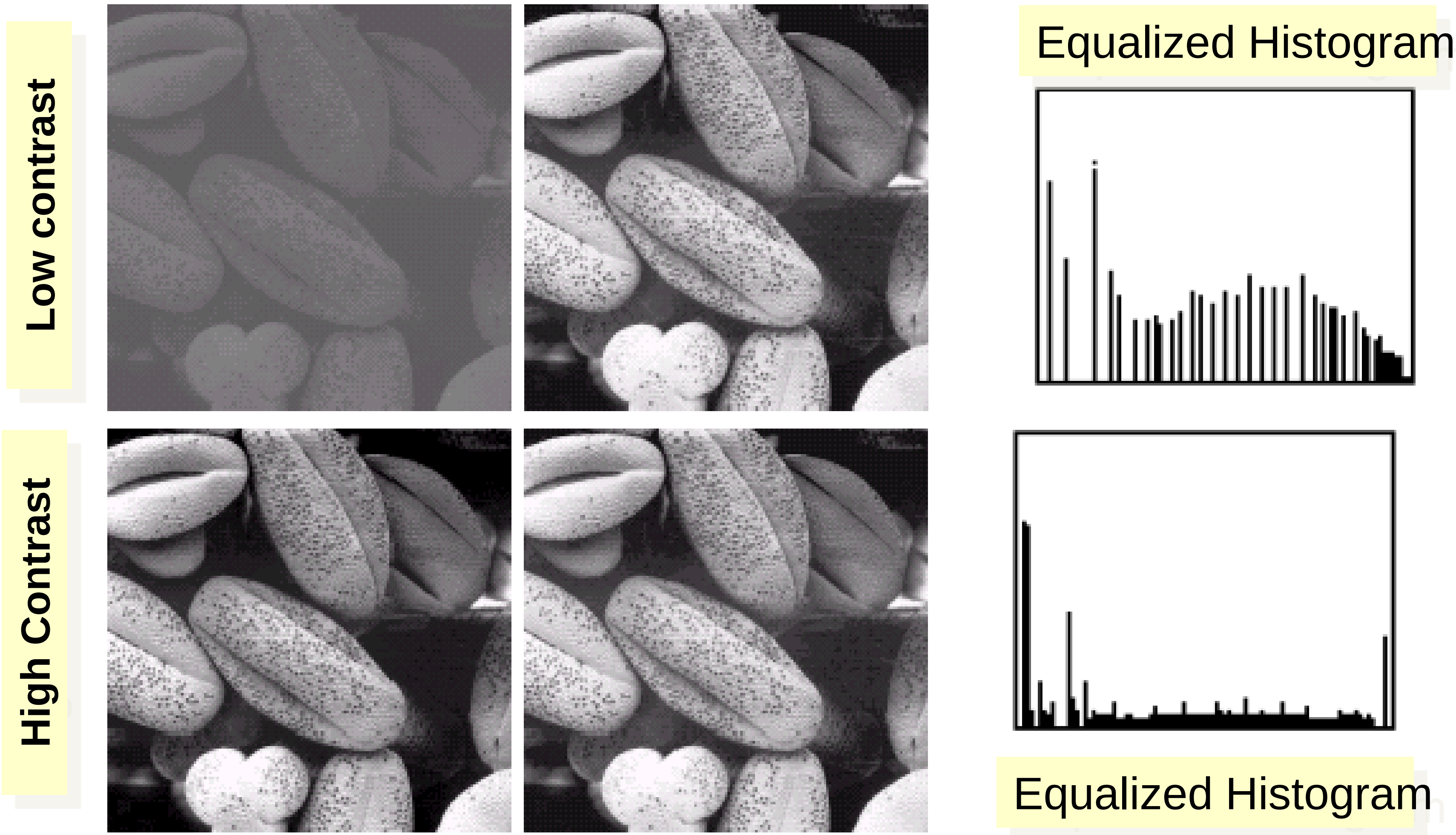
Equalized Histogram





Equalized Histogram

Histogram Equalization: Example



HISTOGRAM MATCHING (SPECIFICATION)

- HISTOGRAM EQUALIZATION DOES NOT ALLOW INTERACTIVE IMAGE ENHANCEMENT AND GENERATES ONLY ONE RESULT: AN APPROXIMATION TO A UNIFORM HISTOGRAM.
- SOMETIMES THOUGH, WE NEED TO BE ABLE TO SPECIFY PARTICULAR HISTOGRAM SHAPES CAPABLE OF HIGHLIGHTING CERTAIN GRAY-LEVEL RANGES.

HISTOGRAM SPECIFICATION

- THE PROCEDURE FOR HISTOGRAM-SPECIFICATION BASED ENHANCEMENT IS:
  - EQUALIZE THE LEVELS OF THE ORIGINAL IMAGE USING:

$$s = T(r_k) = \sum_{j=0}^{k} \frac{n_j}{n}$$

n: total number of pixels,

nj: number of pixels with gray level rj,

L: number of discrete gray levels

HISTOGRAM SPECIFICATION

– SPECIFY THE DESIRED DENSITY FUNCTION AND OBTAIN THE TRANSFORMATION FUNCTION G(z):

$$v_k = G(z_k) = \sum_{i=0}^{k} p_z(z_i) = s_k$$

p_z: specified desirable PDF for output

HISTOGRAM SPECIFICATION

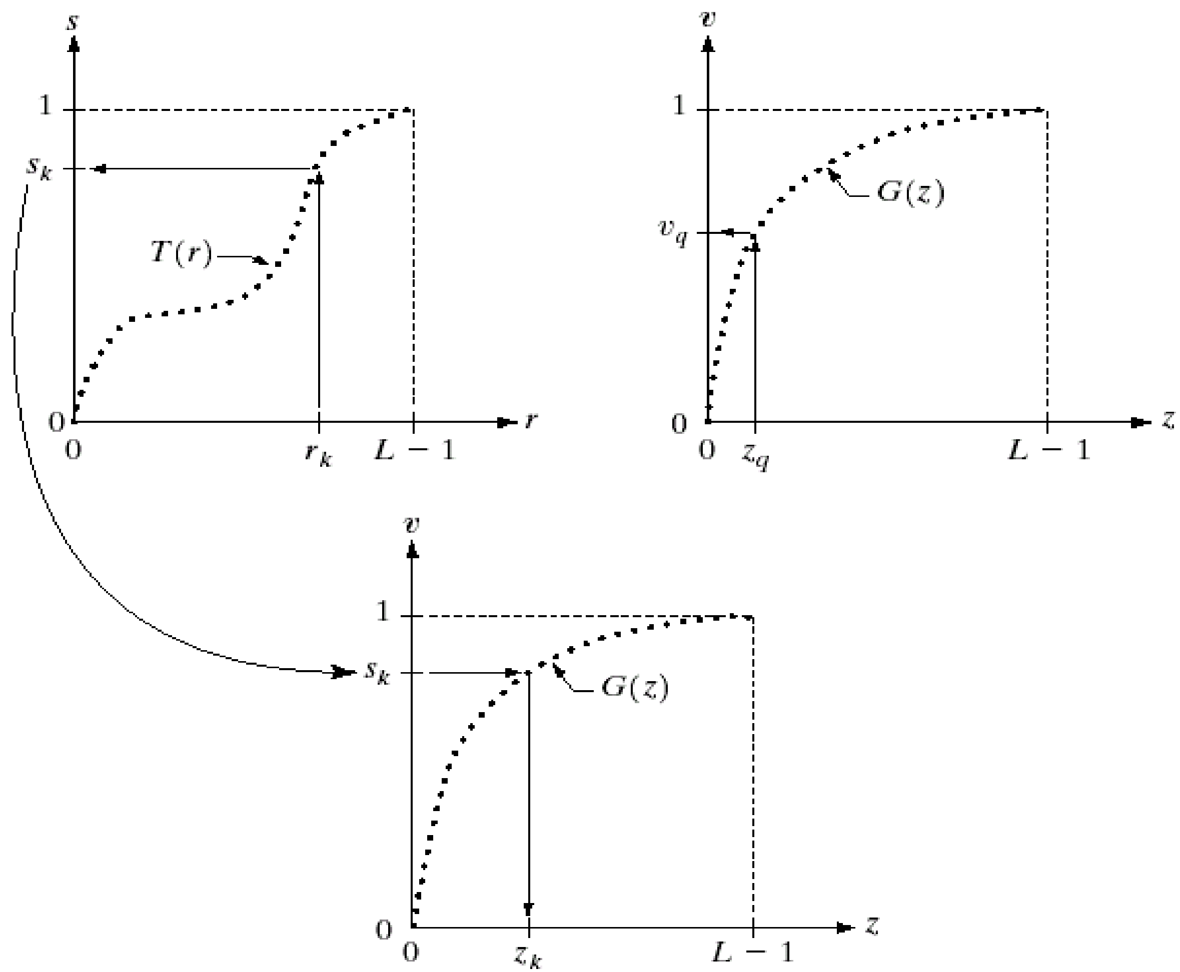
- THE NEW, PROCESSED VERSION OF THE ORIGINAL IMAGE CONSISTS OF GRAY LEVELS CHARACTERIZED BY THE SPECIFIED DENSITY $p_z(z)$.

In essence: $z = G^{-1}(s) \rightarrow \quad z = G^{-1}[T(r)]$

MAPPINGS

a b
c

FIGURE 3.19
(a) Graphical interpretation of mapping from r_k to s_k via $T(r)$.
(b) Mapping of z_q to its corresponding value v_q via $G(z)$.
(c) Inverse mapping from s_k to its corresponding value of z_k.



HISTOGRAM SPECIFICATION

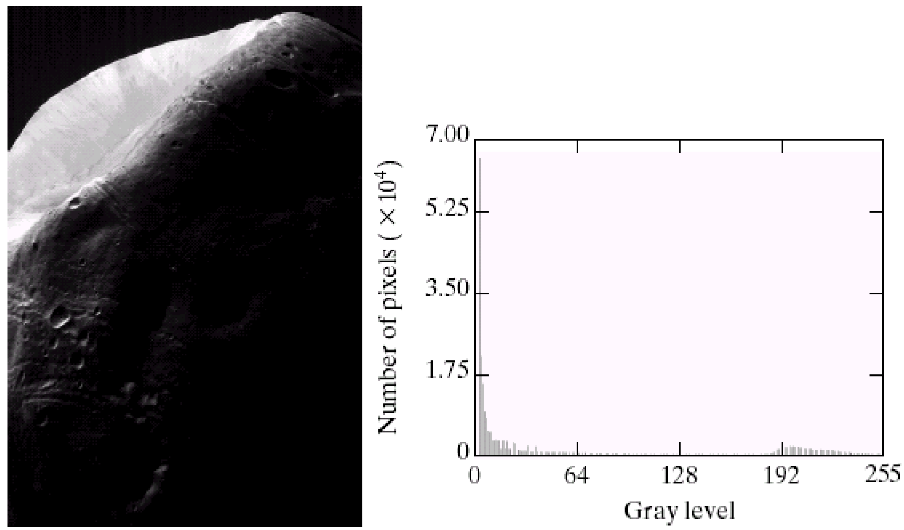
- OBTAIN THE HISTOGRAM OF THE GIVEN IMAGE
- MAP EACH LEVEL r_K TO A LEVEL S_K
- OBTAIN THE TRANSFORMATION FUNCTION G FROM THE GIVEN P_Z (Z)
- PRECOMPUTE Z_K FOR EACH VALUE OF S_K
- FOR EACH PIXEL IN THE ORIGINAL IMAGE, IF THE VALUE OF THAT PIXEL IS r_k MAP THIS VALUE TO ITS CORRESPONDING LEVEL S_K, THEN MAP LEVEL S_K INTO THE FINAL VALUE Z_K

HISTOGRAM SPECIFICATION

k	n_k	$p_r(r_k)$	s_k	$p_z(z_k)$	v_k	n_k
0	790	0.19	0.19	0	0	0
1	1023	0.25	0.44	0	0	0
2	850	0.21	0.65	0	0	0
3	656	0.16	0.81	0.15	0.15	790
4	329	0.08	0.89	0.2	0.35	1023
5	245	0.06	0.95	0.3	0.65	850
6	122	0.03	0.98	0.2	0.85	985
7	81	0.02	1.0	0.15	1.0	448

A 64X64 (4096 PIXELS) IMAGE WITH 8 GRAY LEVELS

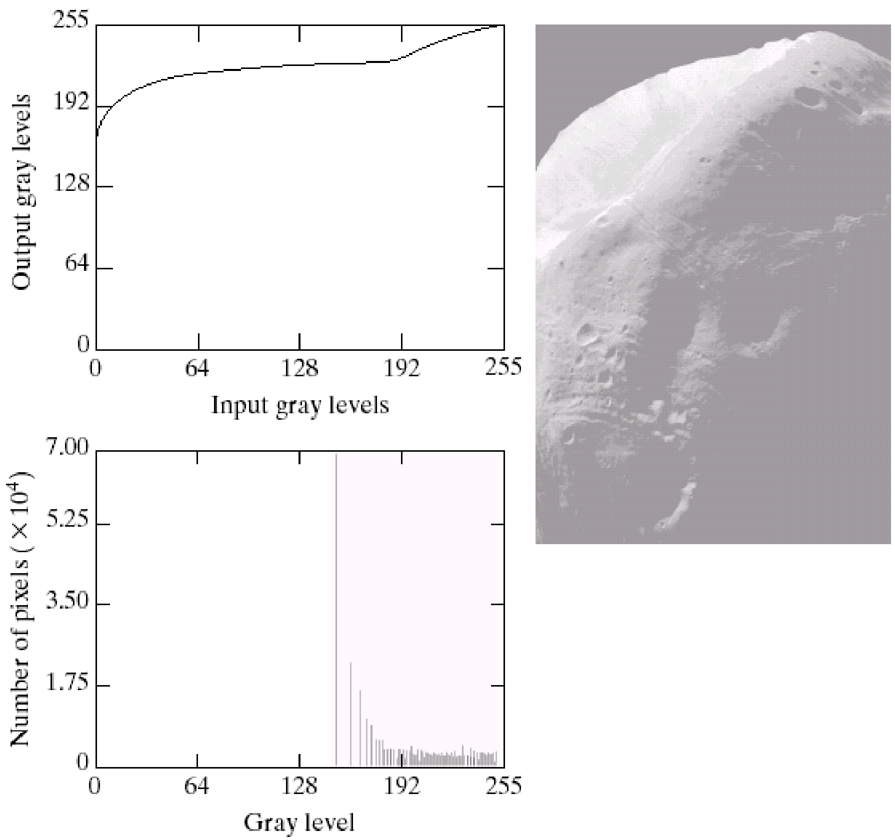
IMAGE ENHANCEMENT IN THE SPATIAL DOMAIN



a b

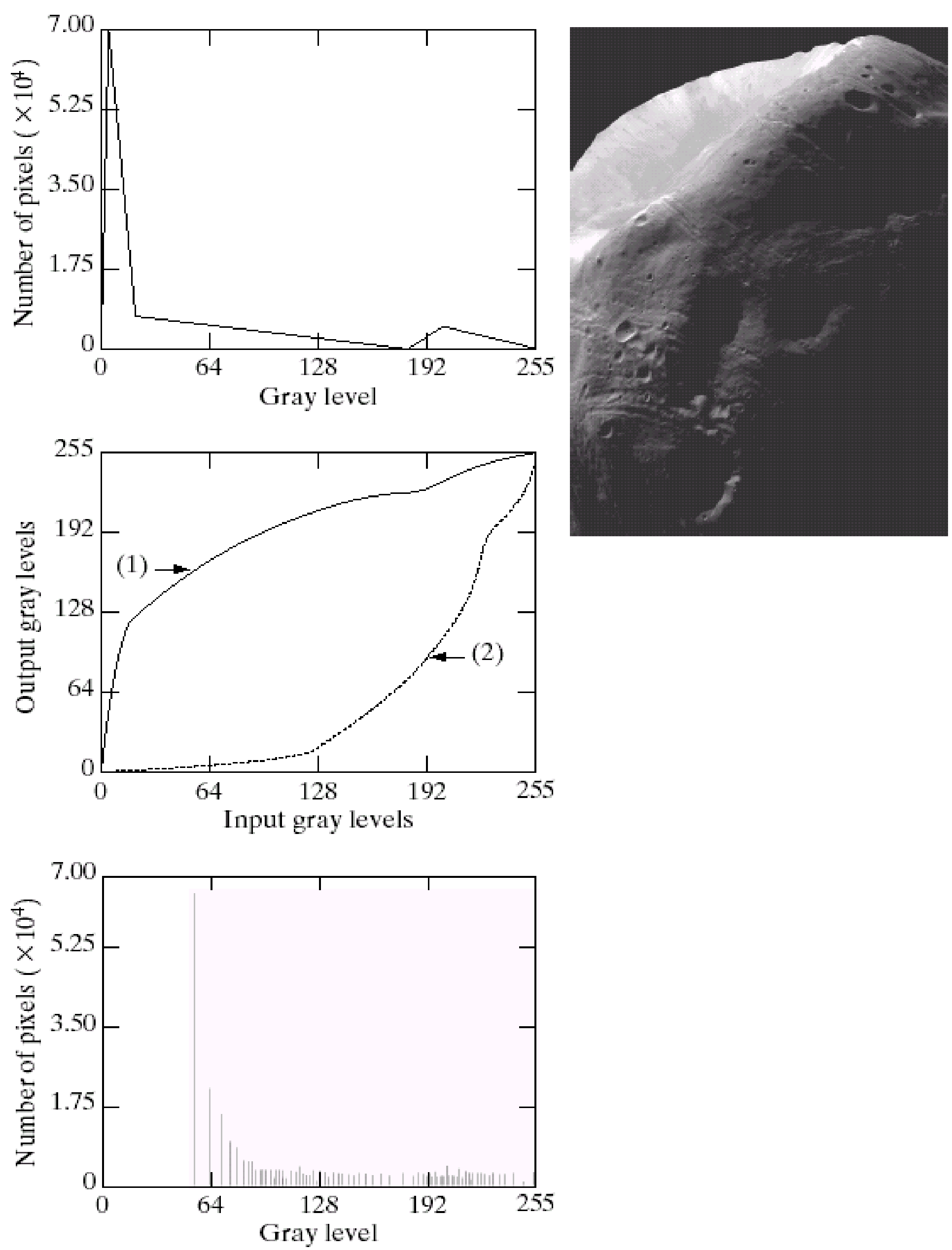
FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

IMAGE ENHANCEMENT IN THE SPATIAL DOMAIN



a b
c

FIGURE 3.21
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).



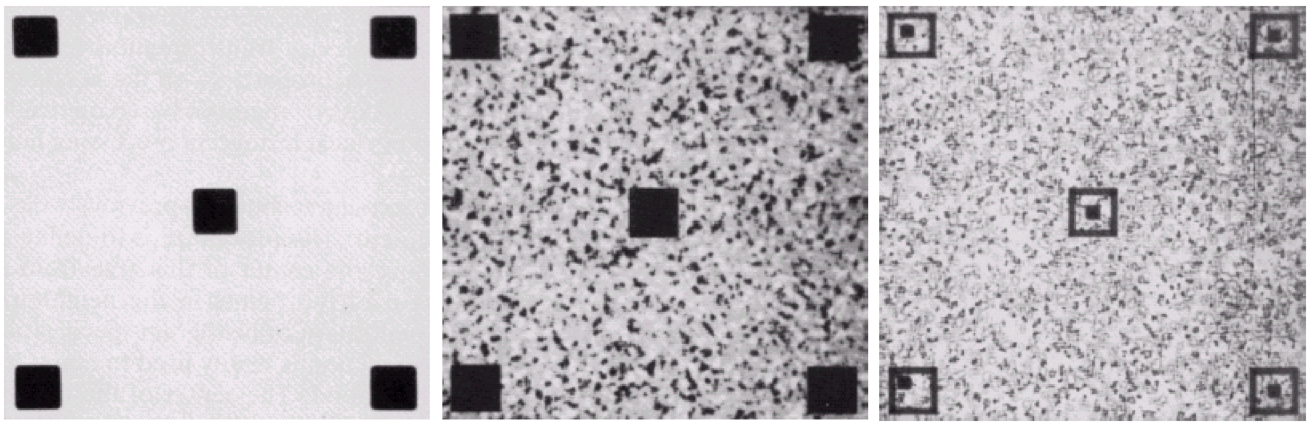
a c
b
d

FIGURE 3.22
(a) Specified histogram.
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

GLOBAL/LOCAL HISTOGRAM EQUALIZATION

- IT MAY BE NECESSARY TO ENHANCE DETAILS OVER SMALL AREAS IN THE IMAGE
- THE NUMBER OF PIXELS IN THESE AREAS MAY HAVE NEGLIGIBLE INFLUENCE ON THE COMPUTATION OF A GLOBAL TRANSFORMATION WHOSE SHAPE DOES NOT NECESSARILY GUARANTEE THE DESIRED LOCAL ENHANCEMENT
- DEVISE TRANSFORMATION FUNCTIONS BASED ON THE GRAY LEVEL DISTRIBUTION IN THE NEIGHBORHOOD OF EVERY PIXEL IN THE IMAGE
- THE PROCEDURE IS:
  - DEFINE A SQUARE (OR RECTANGULAR) NEIGHBORHOOD AND MOVE THE CENTER OF THIS AREA FROM PIXEL TO PIXEL.
  - AT EACH LOCATION, THE HISTOGRAM OF THE POINTS IN THE NEIGHBORHOOD IS COMPUTED AND EITHER A HISTOGRAM EQUALIZATION OR HISTOGRAM SPECIFICATION TRANSFORMATION FUNCTION IS OBTAINED.
  - THIS FUNCTION IS FINALLY USED TO MAP THE GRAY LEVEL OF THE PIXEL CENTERED IN THE NEIGHBORHOOD.
  - THE CENTER IS THEN MOVED TO AN ADJACENT PIXEL LOCATION AND THE PROCEDURE IS REPEATED.

GLOBAL/LOCAL HISTOGRAM EQUALIZATION



a b c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.

USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT (Global)

- LET r REPRESENT A GRAY LEVEL IN THE IMAGE [0, L-1], AND LET $p(r_i)$ DENOTE THE NORMALIZED HISTOGRAM COMPONENT CORRESPONDING TO THE i^{th} VALUE OF r.
- THE n^{th} MOMENT OF r ABOUT ITS MEAN IS DEFINED AS

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n \, p(r_i)$$

- WHERE m IS THE MEAN VALUE OF r (AVERAGE GRAY LEVEL)

$$m = \sum_{i=0}^{L-1} r_i \, p(r_i)$$

USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT (Global)

- THE SECOND MOMENT IS GIVEN BY

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 \, p(r_i)$$

- WHICH IS THE VARIANCE OF r
- MEAN AS A MEASURE OF AVERAGE GRAY LEVEL IN THE IMAGE
- VARIANCE AS A MEASURE OF AVERAGE CONTRAST

USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT (Local)

- LET (x,y) BE THE COORDINATES OF A PIXEL IN AN IMAGE, AND LET $S_{X,Y}$ DENOTE A NEIGBORHOOD OF SPECIFIED SIZE, CENTERED AT (x,y)
- THE MEAN VALUE m_{SXY} OF THE PIXELS IN $S_{X,Y}$ IS

$$m_{s_{xy}} = \sum_{(s,t)\in S_{xy}} r_{s,t}\, p(r_{s,t})$$

- THE GRAY LEVEL VARIANCE OF THE PIXELS IN REGION $S_{X,Y}$ IS GIVEN BY

$$\sigma_{S_{xy}}{}^2 = \sum_{(s,t)\in S_{xy}} \left[r_{s,t} - m_{s_{xy}}\right]^2 p(r_{s,t})$$

USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT

- THE GLOBAL MEAN AND VARIANCE ARE MEASURED OVER AN ENTIRE IMAGE AND ARE USEFUL FOR GROSS ADJUSTMENTS OF OVERALL INTENSITY AND CONTRAST.
- A USE OF THESE MEASURES IN LOCAL ENHANCEMENT IS, WHERE THE LOCAL MEAN AND VARIANCE ARE USED AS THE BASIS FOR MAKING CHANGES THAT DEPEND ON IMAGE CHARACTERISTICS IN A PREDEFINED REGION ABOUT EACH PIXEL IN THE IMAGE.

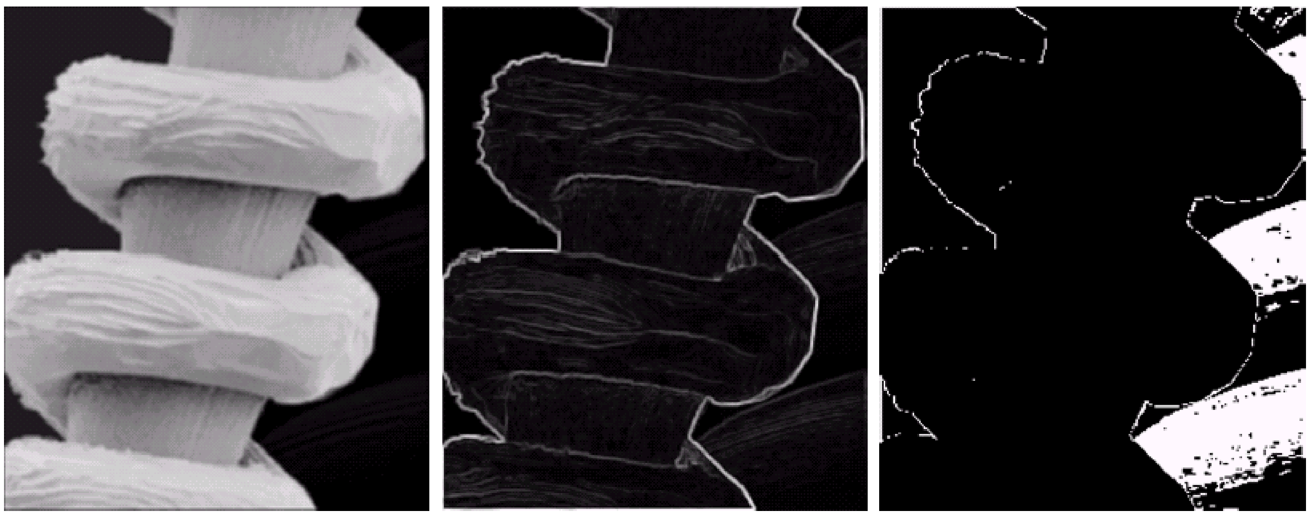
TUNGSTEN FILAMENT IMAGE

FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT

- A PIXEL AT POINT (x,y) IS CONSIDERED IF:
  - $m_{SXY} \leq k_0 M_G$, where k0 is a positive constant less than 1.0, and M_G is global mean
  - $\sigma_{sxy} \leq k_2 D_G$, where D_G is the global standard deviation and k_2 is a positive constant
  - $k_1 D_G \leq \sigma_{sxy}$,, with k1 < k2
- A PIXEL THAT MEETS ALL ABOVE CONDITIONS IS PROCESSED SIMPLY BY MULTIPLYING IT BY A SPECIFIED CONSTANT, E, TO INCREASE OR DECREASE THE VALUE OF ITS GRAY LEVEL RELATIVE TO THE REST OF THE IMAGE.
- THE VALUES OF PIXELS THAT DO NOT MEET THE ENHANCEMENT CONDITIONS ARE LEFT UNCHANGED.

IMAGE ENHANCEMENT IN THE SPATIAL DOMAIN



a b c

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

IMAGE ENHANCEMENT IN THE SPATIAL DOMAIN

FIGURE 3.26 Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

Readings from Book (3rd Edn.)

- 3.2 Basic Intensity Transformation Functions
- Bit Plane Slicing
- 3.3 Histogram

Acknowledgements

- Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2008

Material in these slides has been taken from, the following resources