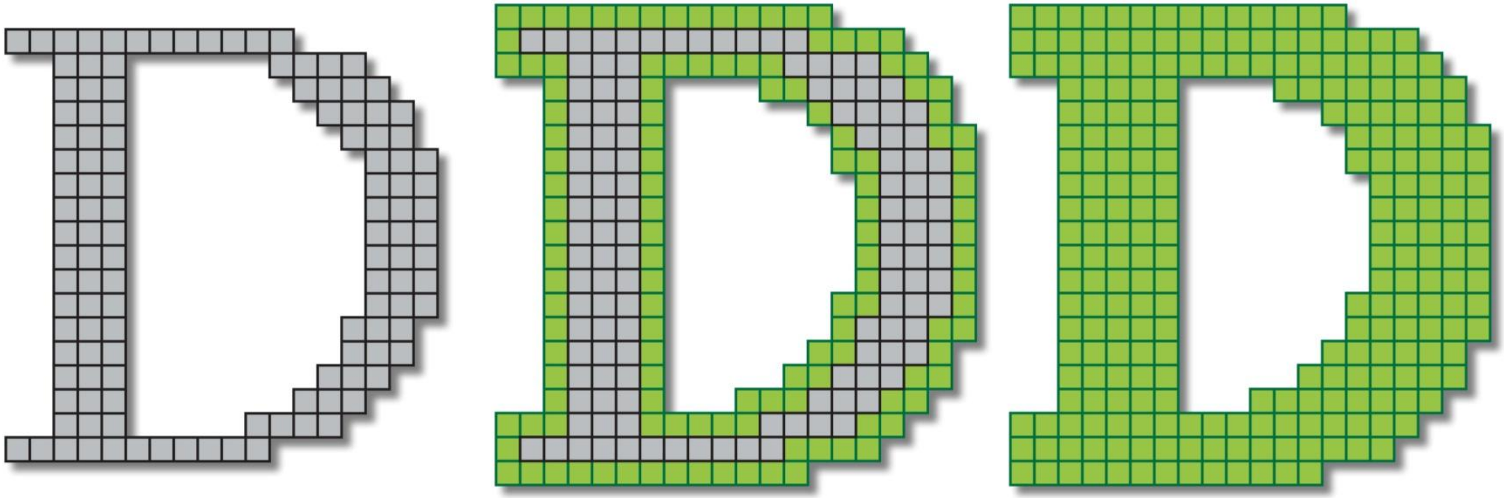
Digital Image Processing

Lecture # 8
Image Morphology & Texture

Dilation



Dilation

Definition 1:

The dilation of two sets A and B is defined as:

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \emptyset\}$$

i.e. when the reflection of set *B* about its origin is shifted by *z*, the dilation of *A* by *B* is the set of all displacements such that overlap *A* by at least one element

We will only consider symmetric SEs so reflection will have no effect

Dilation

Definition 2:

Dilation of image *f* by structuring element *s* is given by

$f \oplus s$

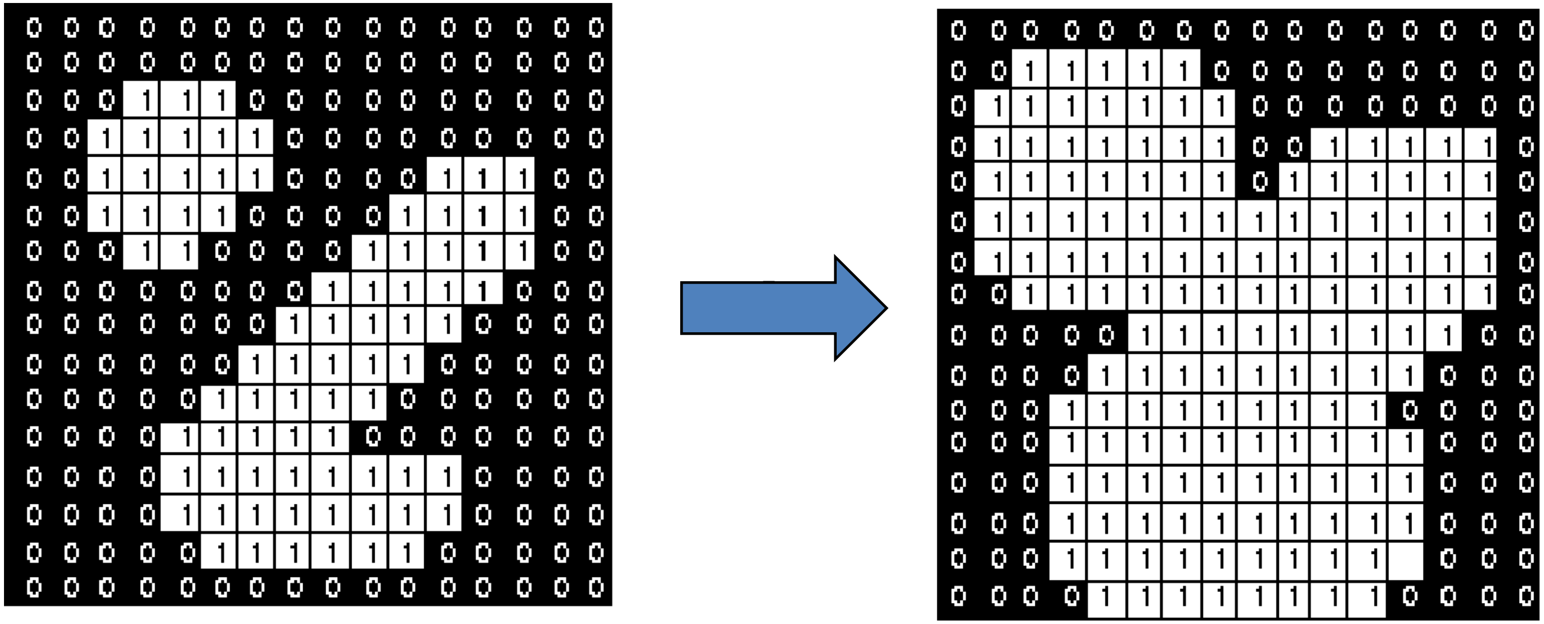
The structuring element s is positioned with its origin at *(x, y)* and the new pixel value is determined using the rule:

$$g(x, y) = \begin{cases} 1 \text{ if } s \text{ hits } f \\ 0 \text{ otherwise} \end{cases}$$

Dilation – How to compute

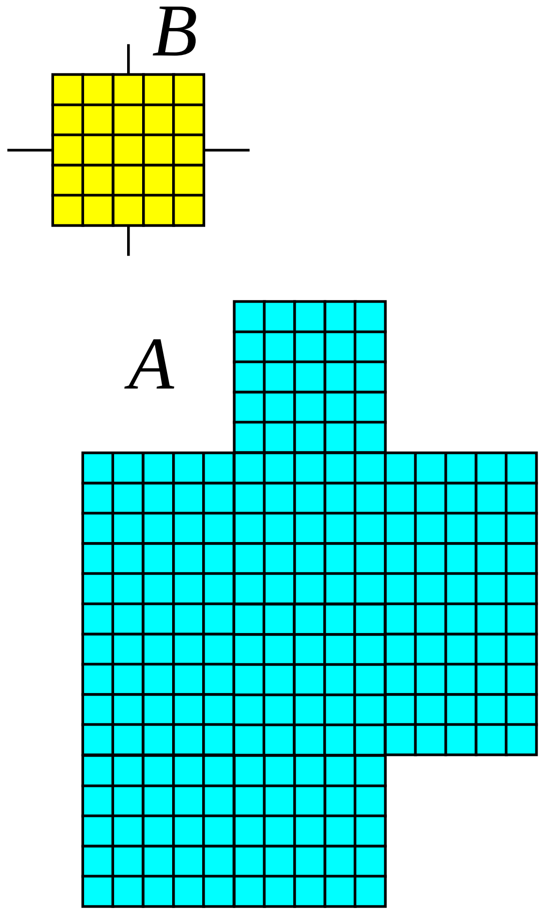
- For each background pixel (which we will call the *input pixel*)
  - Superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position
  - If *at least one* pixel in the structuring element coincides with a foreground pixel in the image underneath, then the input pixel is set to the foreground value
  - If all the corresponding pixels in the image are background, however, the input pixel is left at the background value

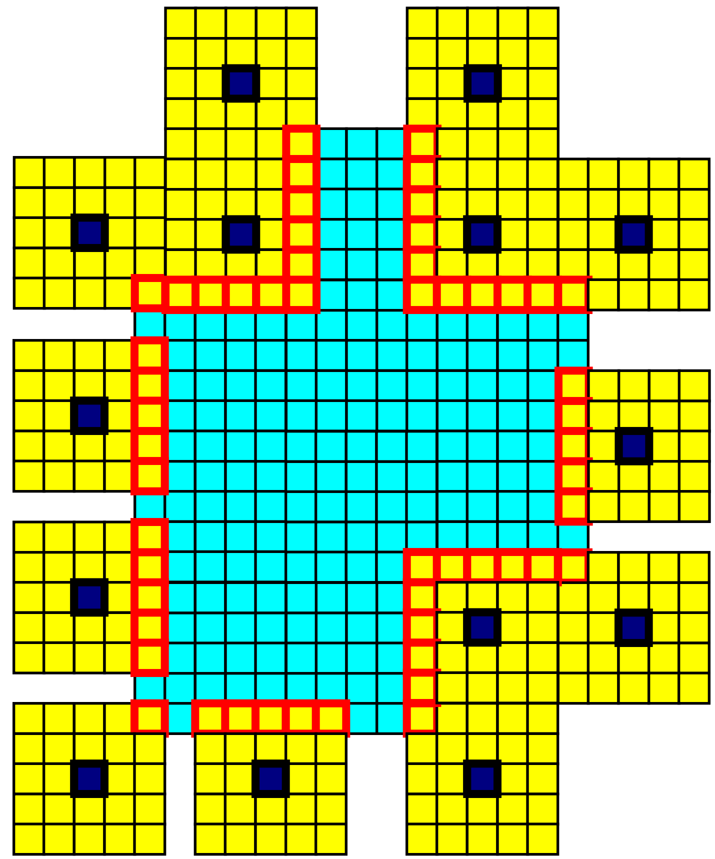
Dilation: Example



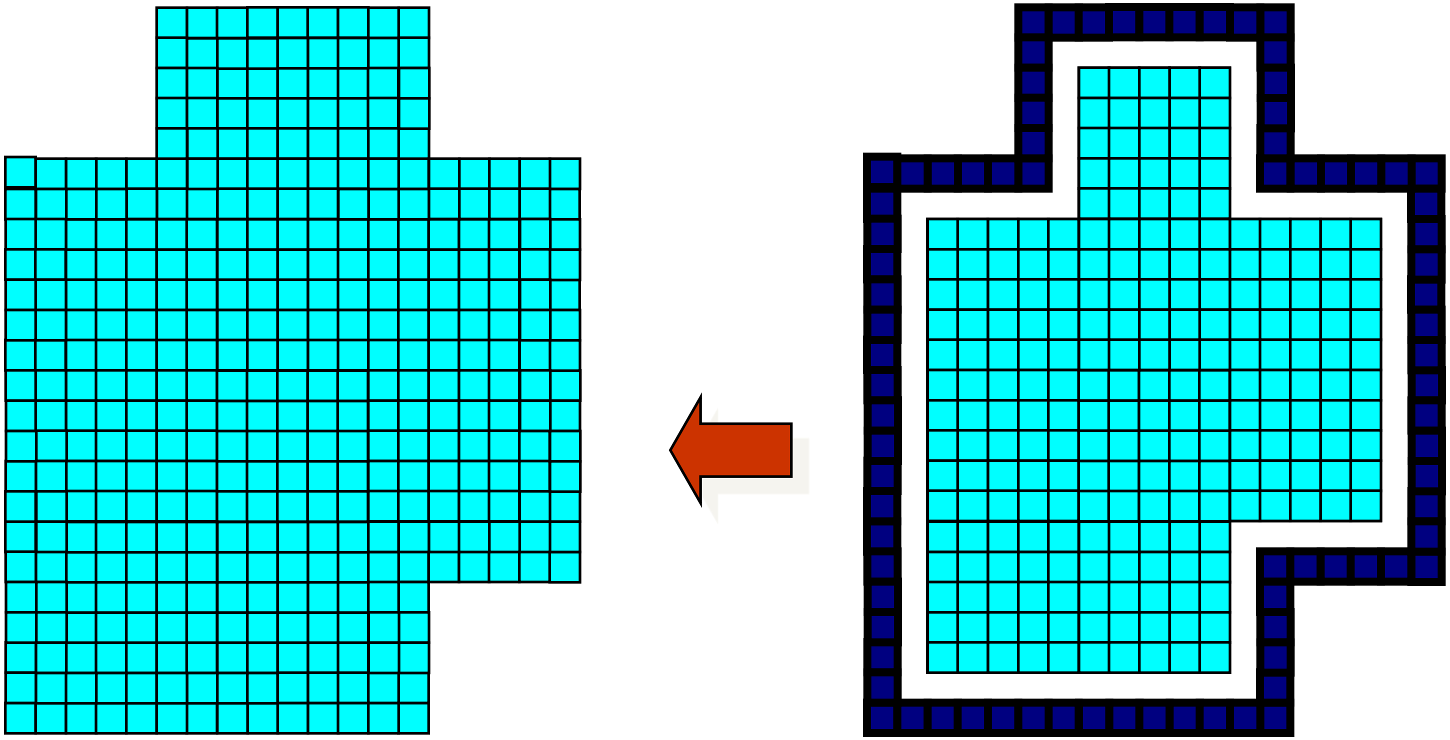
Effect of dilation using a 3×3 square structuring element

Dilation

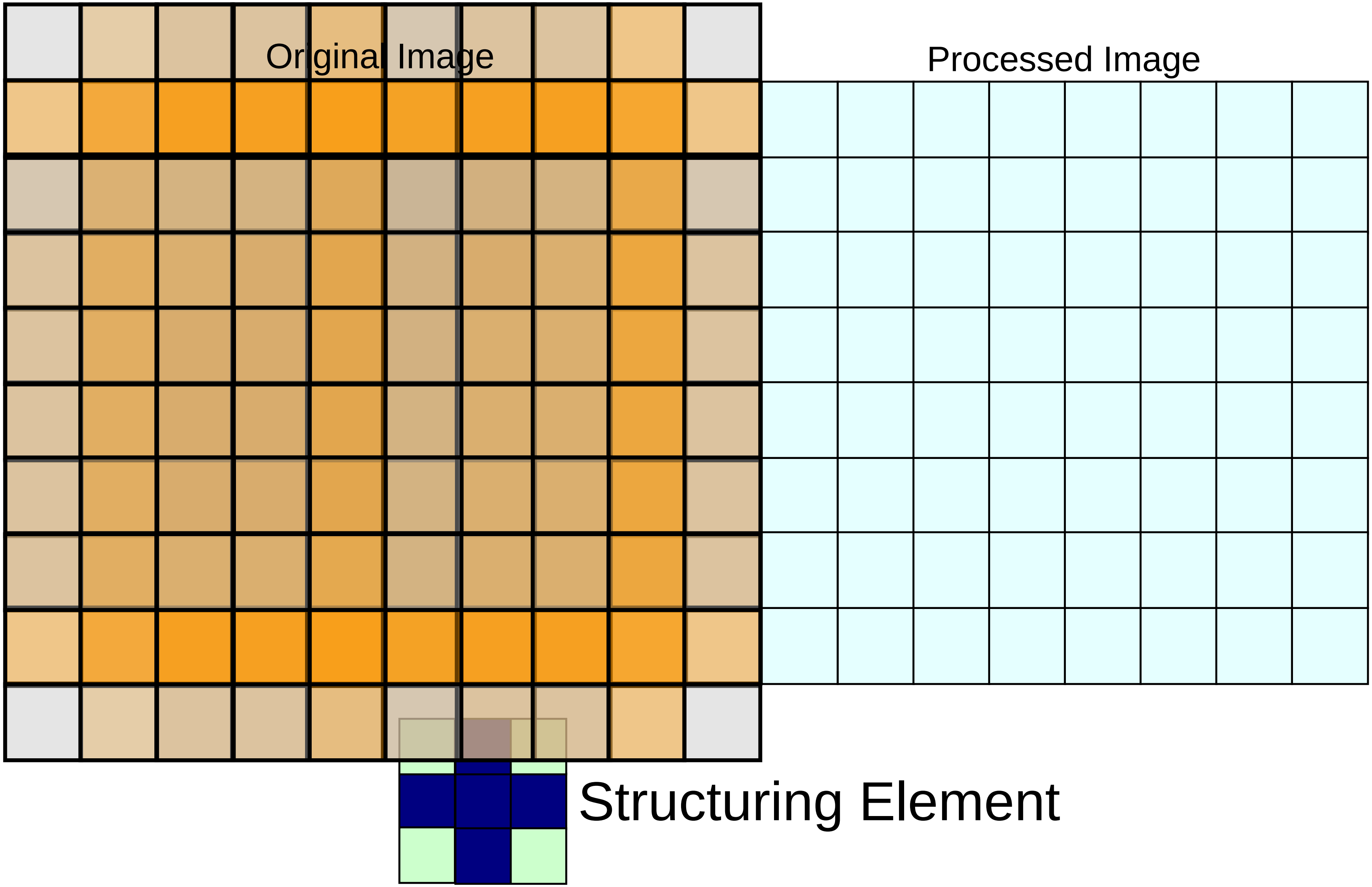




Dilation

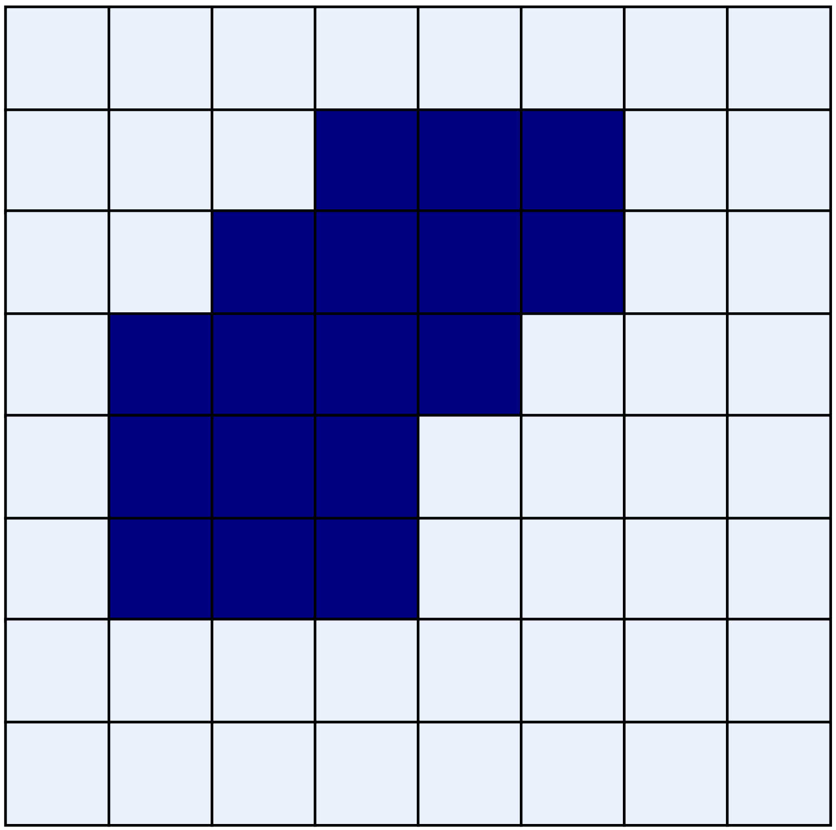


Dilation: Example

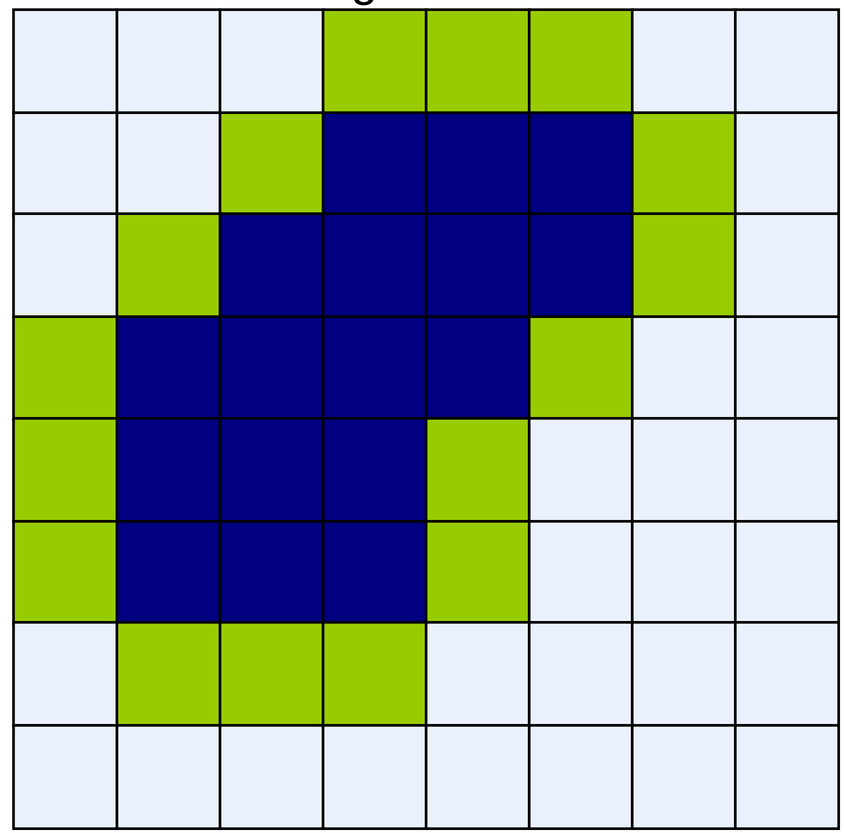


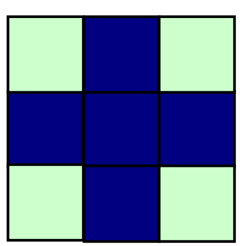
Dilation: Example

Original Image



Processed Image With Dilated Pixels





Structuring Element

Dilation

- Effects
  - Expands the size of foreground(1-valued) objects
  - Smoothes object boundaries
  - Closes holes and gaps

- Rule for Dilation

  In a binary image, if any of the pixel (in the neighborhood defined by structuring element) is 1, then output is 1

Dilation: Example 1

Original image

Dilation by 3*3 square structuring element

Dilation by 5*5 square structuring element

Note: In these examples a 1 refers to a black pixel!

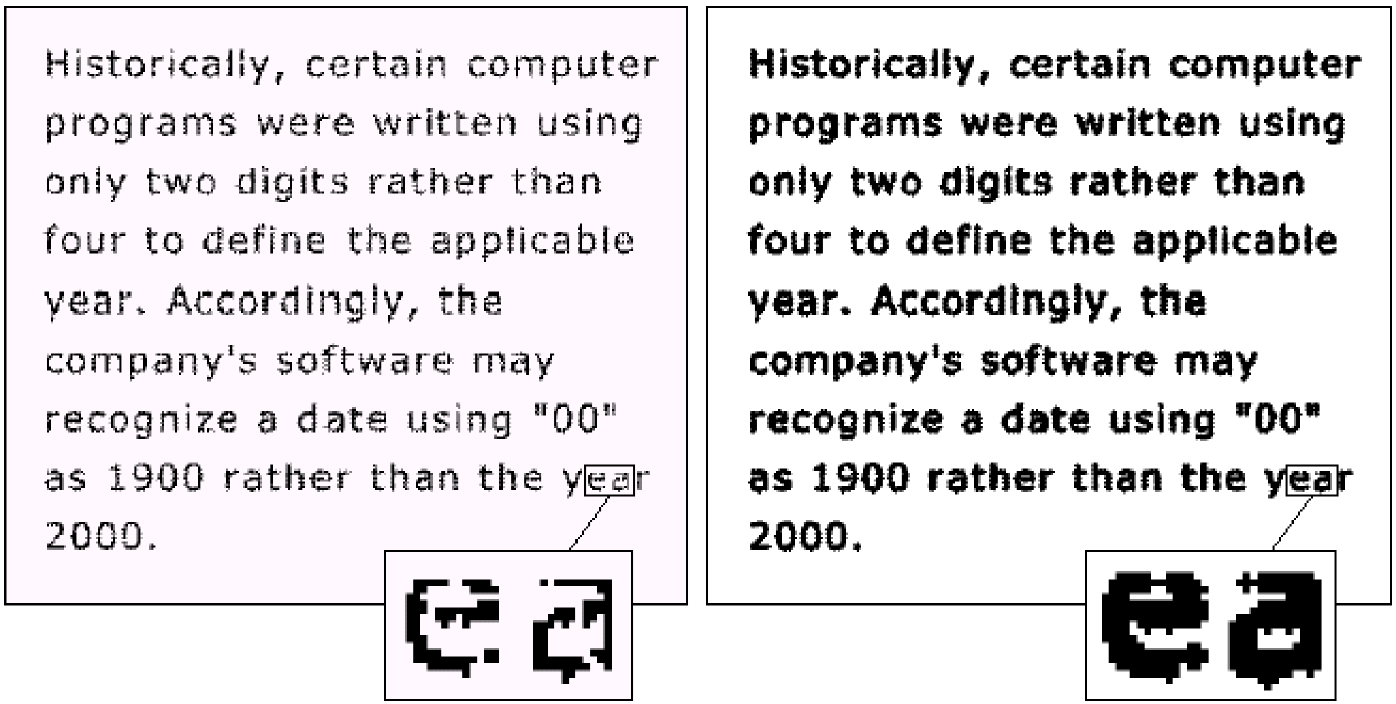
Dilation: Example 2

Original (178x178)

dilation with
3x3 structuring element

dilation with
7x7 structuring element

Dilation: Example 3



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

Dilation

Dilation can repair breaks

Dilation can repair intrusions

Watch out: Dilation enlarges objects

Duality relationship between Dilation and Erosion

- Dilation and erosion are duals of each other:

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

- For a symmetric structuring element:

$$(A \ominus B)^c = A^c \oplus B$$

It means that we can obtain erosion of an image A by B simply by dilating its background (i.e. A^c) with the same structuring element and complementing the result.

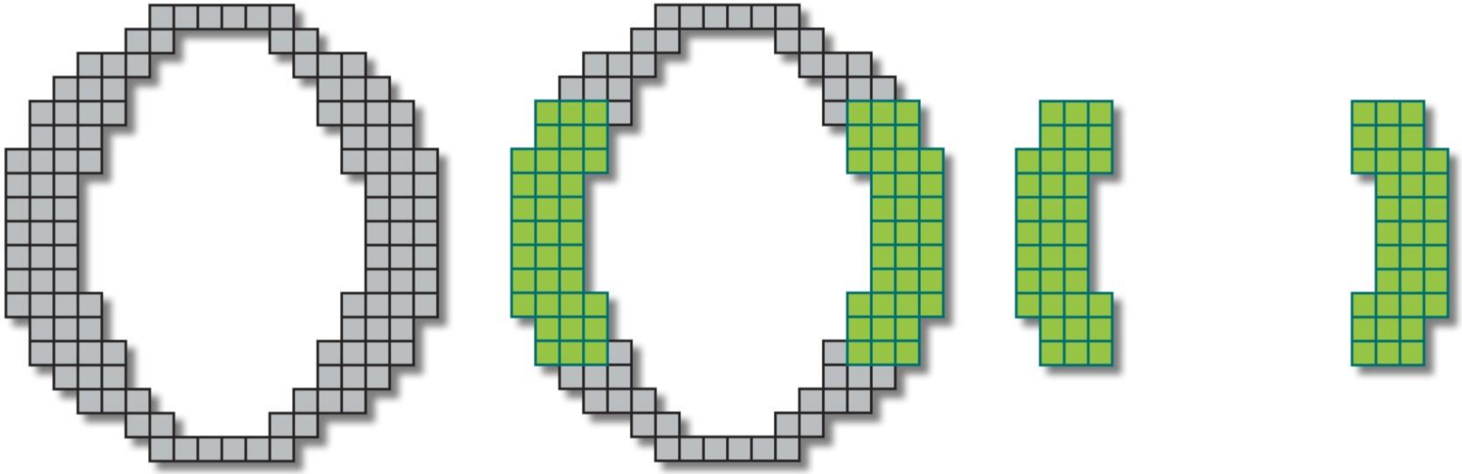
Compound Operations

- More interesting morphological operations can be performed by performing combinations of erosions and dilations

The most widely used of these *compound operations* are:

- Opening
- Closing

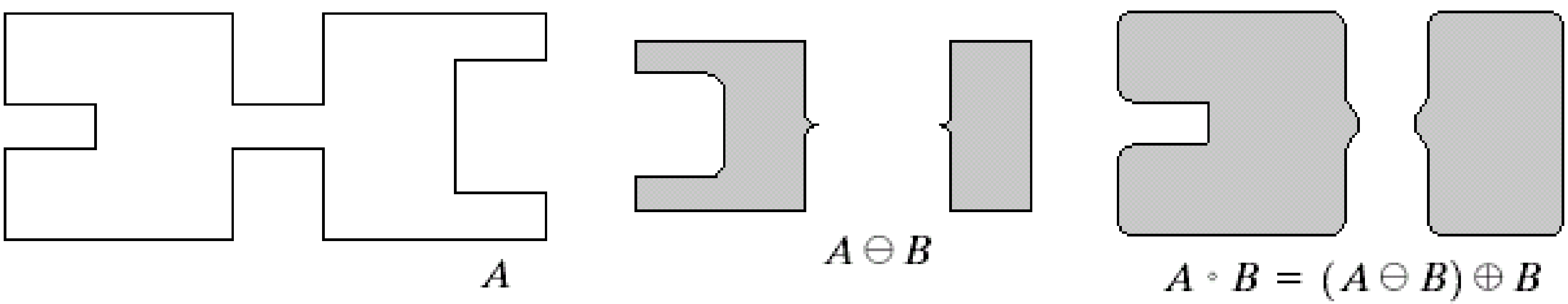
Opening



Opening

The opening of image *f* by structuring element *s*, denoted by $f \circ s$ is simply an erosion followed by a dilation

$$f \circ s = (f \ominus s) \oplus s$$



Original shape | After erosion | After dilation (opening)

Opening: Example

Original Image

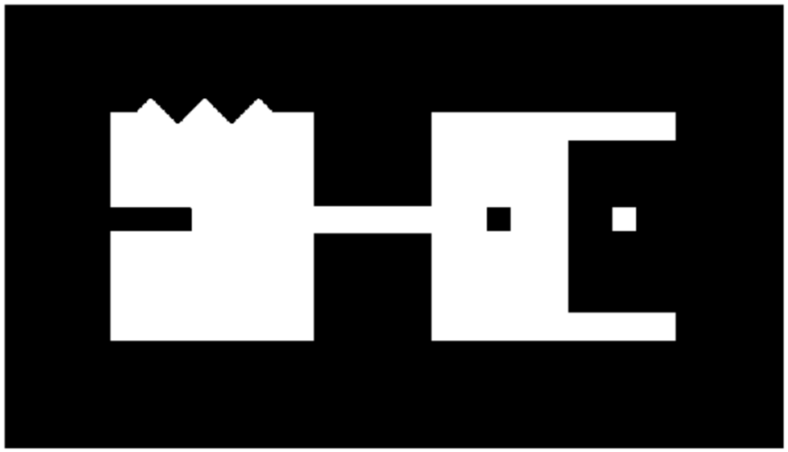
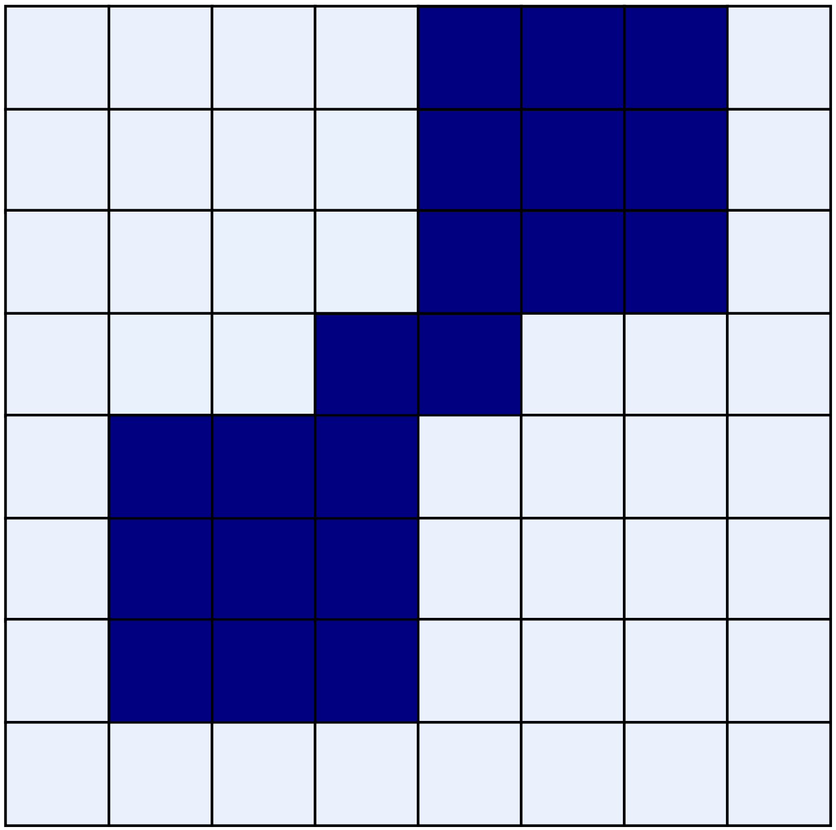


Image After Opening

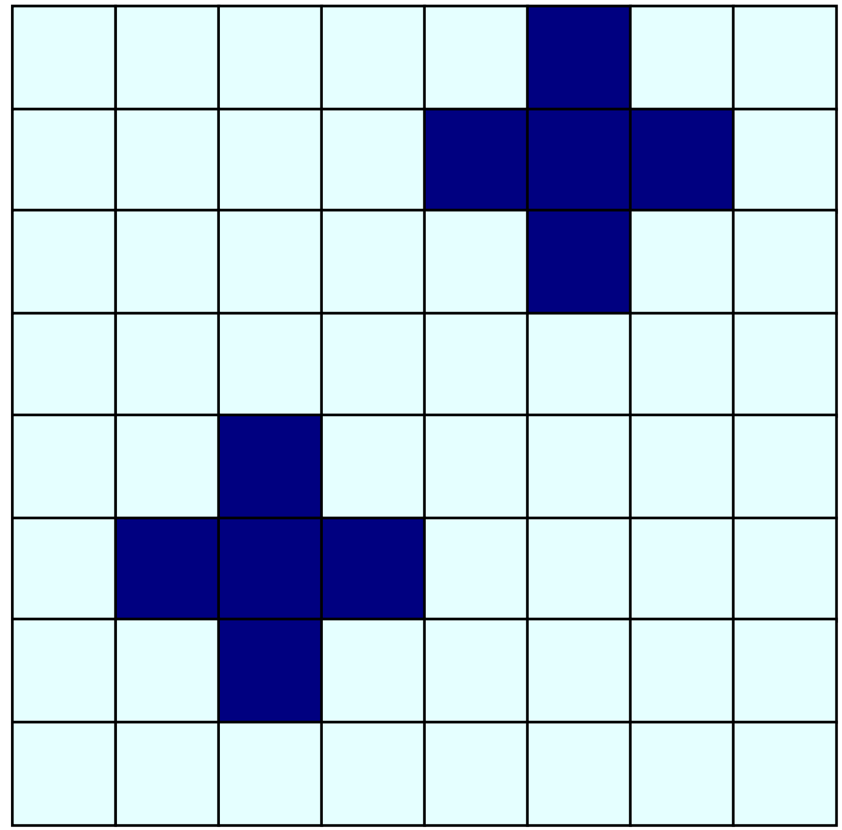
Opening
Breaks narrow joints
Removes ‘Salt’ noise

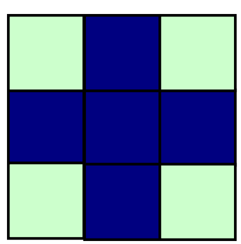
Opening: Example

Original Image



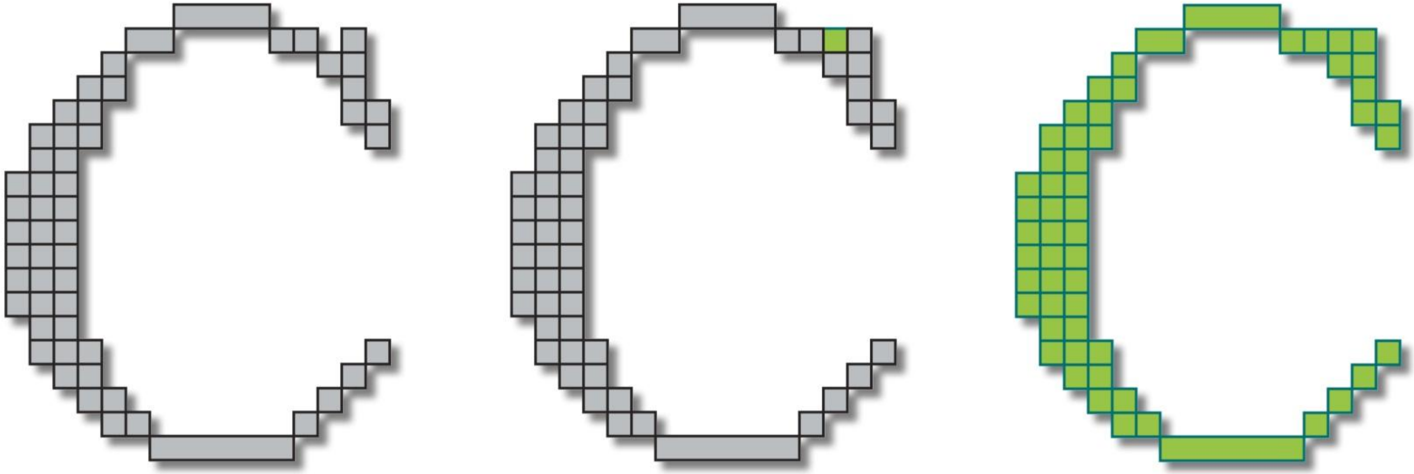
Processed Image





Structuring Element

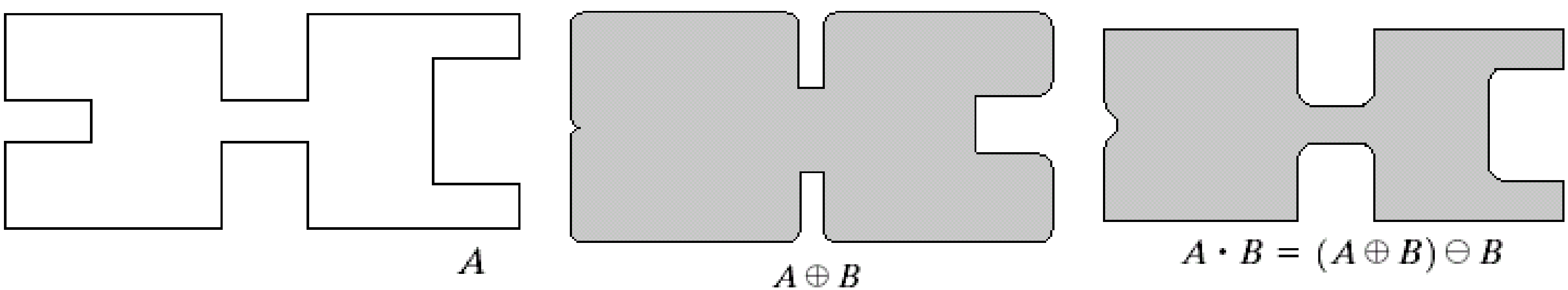
Closing



Closing

The closing of image *f* by structuring element *s*, denoted by *f* • *s* is simply a dilation followed by an erosion

$$f \bullet s = (f \oplus s) \ominus s$$



Original shape

After dilation

After erosion (closing)

Closing: Example

Original Image

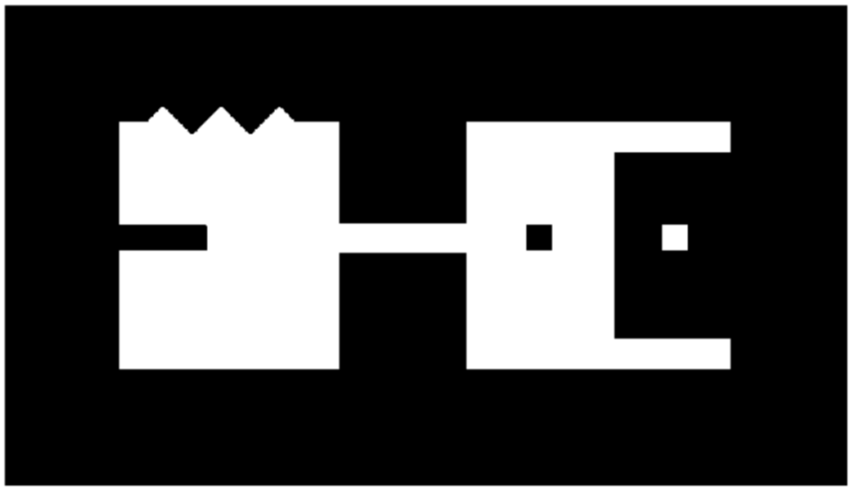


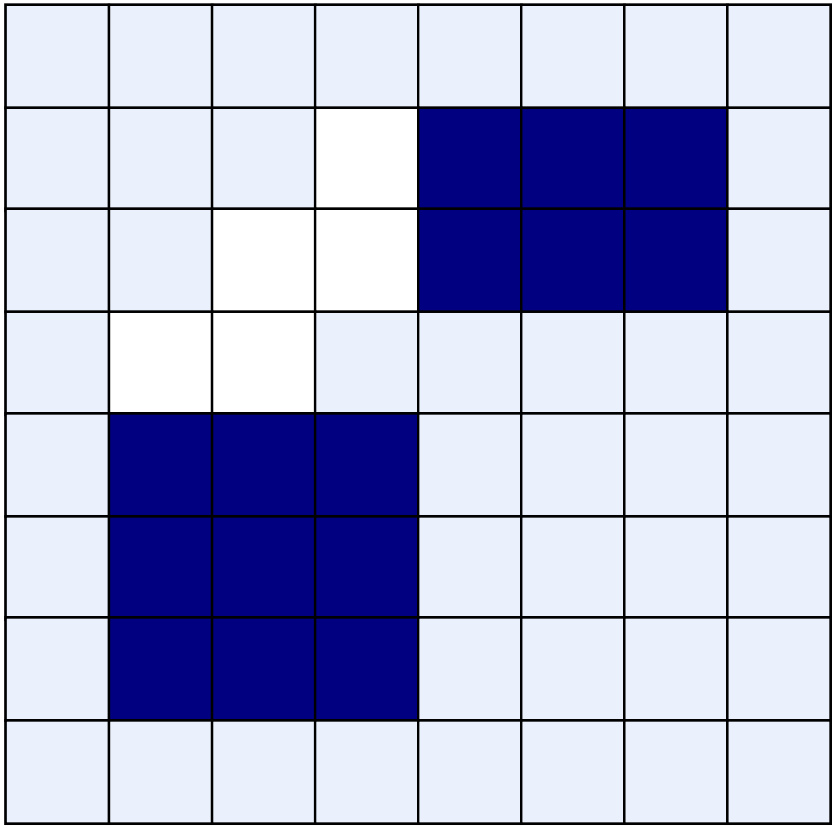
Image After Closing

Closing

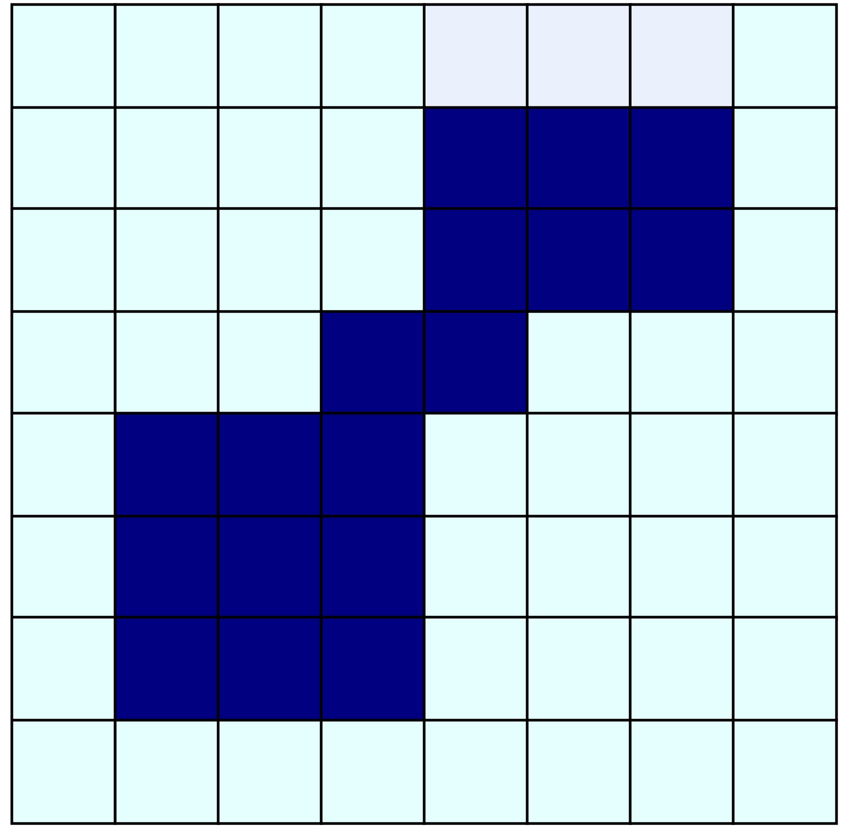
- Eliminates small holes
- Fills gaps
- Removes ‘Pepper’ noise

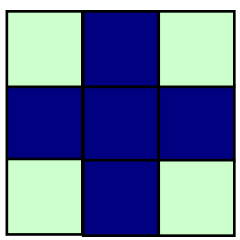
Closing: Example

Original Image



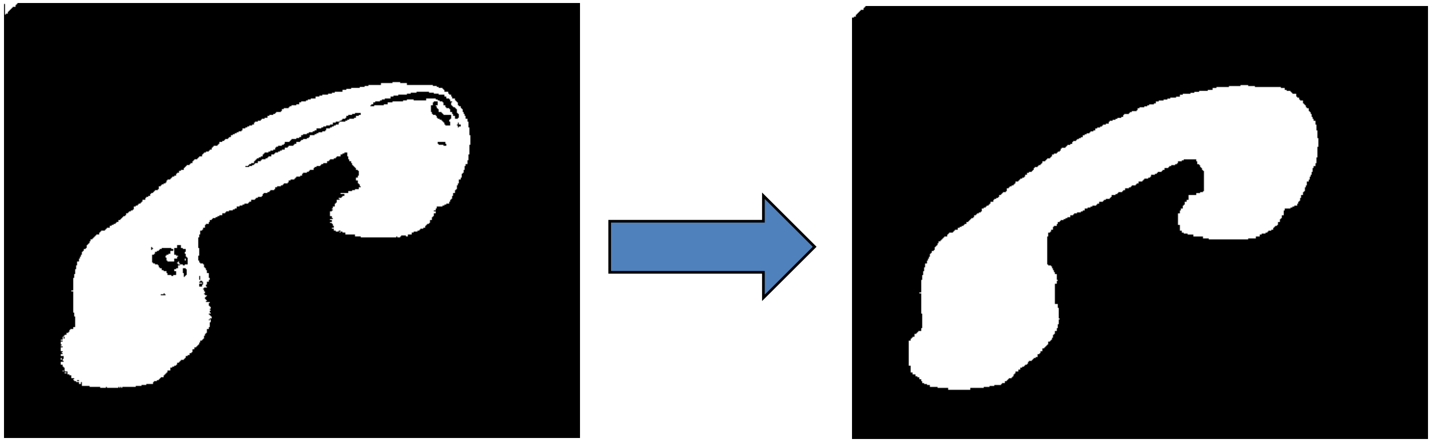
Processed Image





Structuring Element

Closing



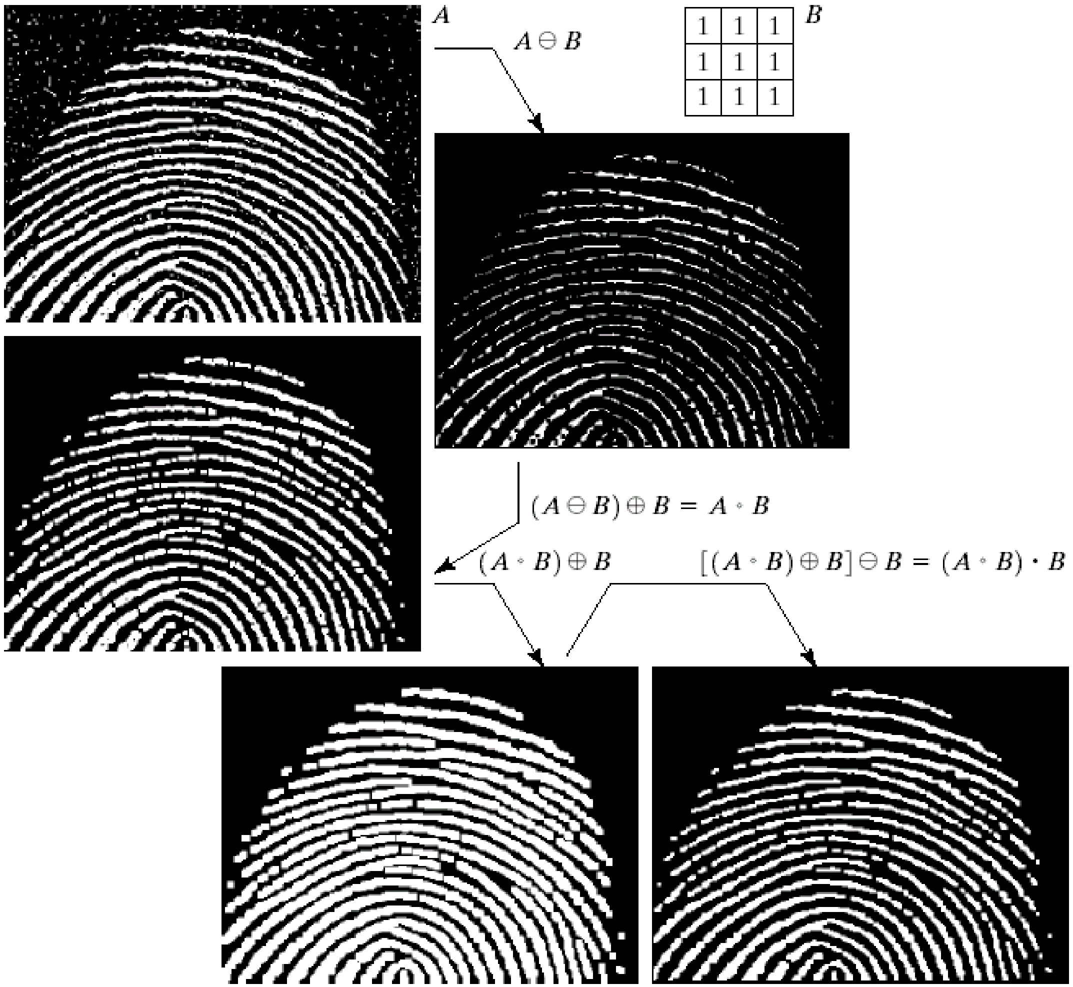
Opening & Closing

- Opening and closing are duals of each others

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

$$(A \circ B)^c = (A^c \bullet \hat{B})$$

Morphological Processing Example



a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

Morphological Algorithms

Using the simple technique we have looked at so far we can begin to consider some more interesting morphological algorithms

We will look at:

- Boundary extraction
- Region filling
- Extraction of connected components

There are lots of others as well though:

- Thinning/thickening
- Skeletonization

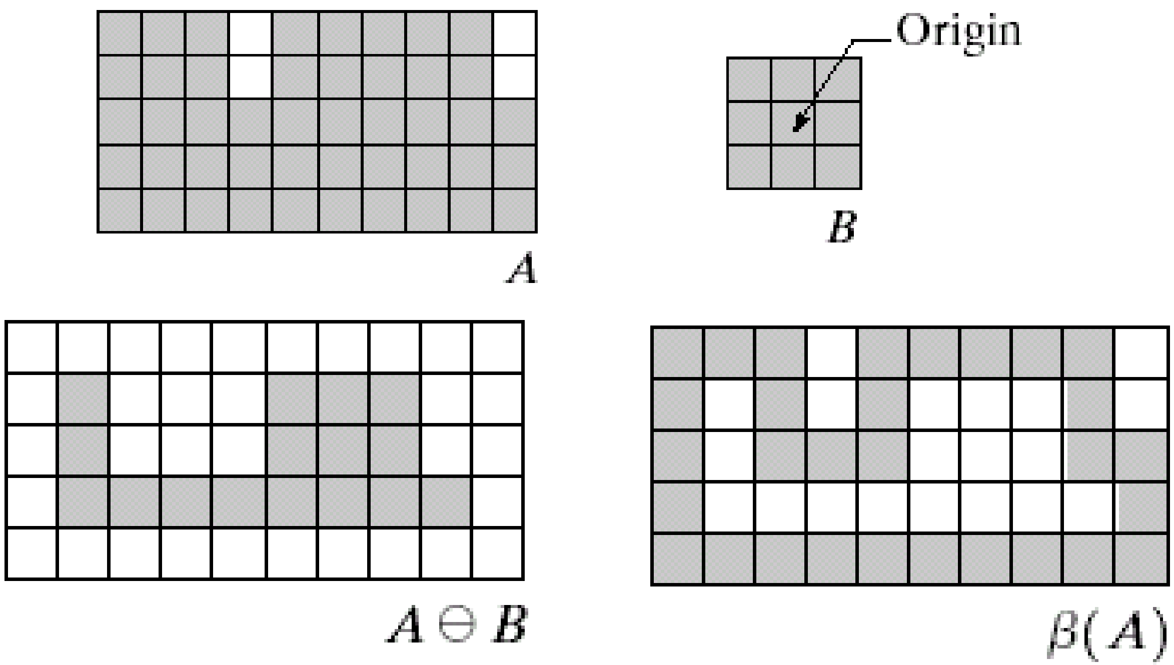
Boundary Extraction

The boundary of set A denoted by *β(A)* is obtained by first eroding *A* by a suitable structuring element *B* and then taking the difference between *A* and its erosion.

$$\beta(A) = A - (A \ominus B)$$

Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$



Boundary Extraction

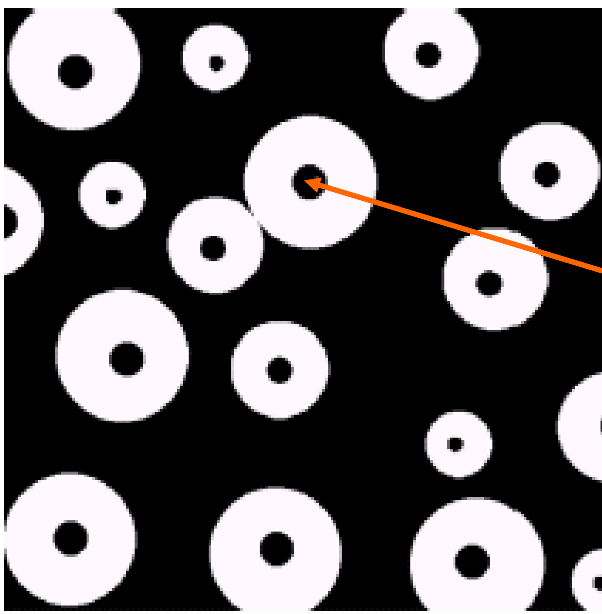
A simple image and the result of performing boundary extraction using a square 3*3 structuring element

Original Image

Extracted Boundary

Region (hole) Filling

Given a pixel inside a boundary, *region filling* attempts to fill that boundary with object pixels (1s)



Given a point inside here, can we fill the whole circle?

Region Filling

Let A is a set containing a subset whose elements are 8-connected boundary points of a region, enclosing a background region i.e. hole

If all boundary points are labeled 1 and non boundary points are labeled 0, the following procedure fills the region:

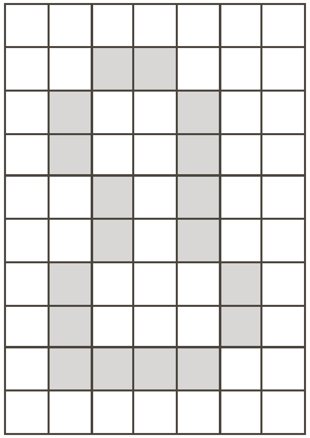
Inside the boundary

- Start from a known point p and taking $X_0 = p$,
- Then taking the next values of X_k as:

$$X_k = (X_k \oplus B) \bigcap A^c \qquad k = 1, 2, 3, \cdots$$

B is suitable structuring element

- Terminate iterations if $X_{k+1} = X_k$
- The set union of X_k and A contains the filled set and its boundaries.



A

Region Filling

0	0	0	0	0	0	0
0	1	1	1	1	1	0
0	1	0	0	0	1	0
0	1	0	0	0	1	0
0	1	0	0	0	1	0
0	1	1	1	1	1	0
0	0	0	0	0	0	0

A

1	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	1	1	1	0	1
1	0	1	1	1	0	1
1	0	1	1	1	0	1
1	0	0	0	0	0	1
1	1	1	1	1	1	1

A^c

0	1	0
1	1	1
0	1	0

B

Region Filling

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

X_0

0	1	0
1	1	1
0	1	0

B

0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	1	1	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$(X_0 \oplus B)$

$$X_{k+1} = (X_k \oplus B) \cap A^c$$

$$k = 1, 2, 3, \cdots$$

Region Filling

0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	1	1	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$X_1 = (X_0 \oplus B)$

1	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	1	1	1	0	1
1	0	1	1	1	0	1
1	0	1	1	1	0	1
1	0	0	0	0	0	1
1	1	1	1	1	1	1

A^c

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	1	1	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$X_1 = (X_0 \oplus B) \cap A^c$

$$X_{k+1} = (X_k \oplus B) \cap A^c$$

$$k = 1, 2, 3, \cdots$$

Region Filling

$$X_{k+1} = (X_k \oplus B) \cap A^c$$
$$k = 1, 2, 3, \cdots$$

NOTE:
The intersection of dilation and the complement of A limits the result to inside the region of interest

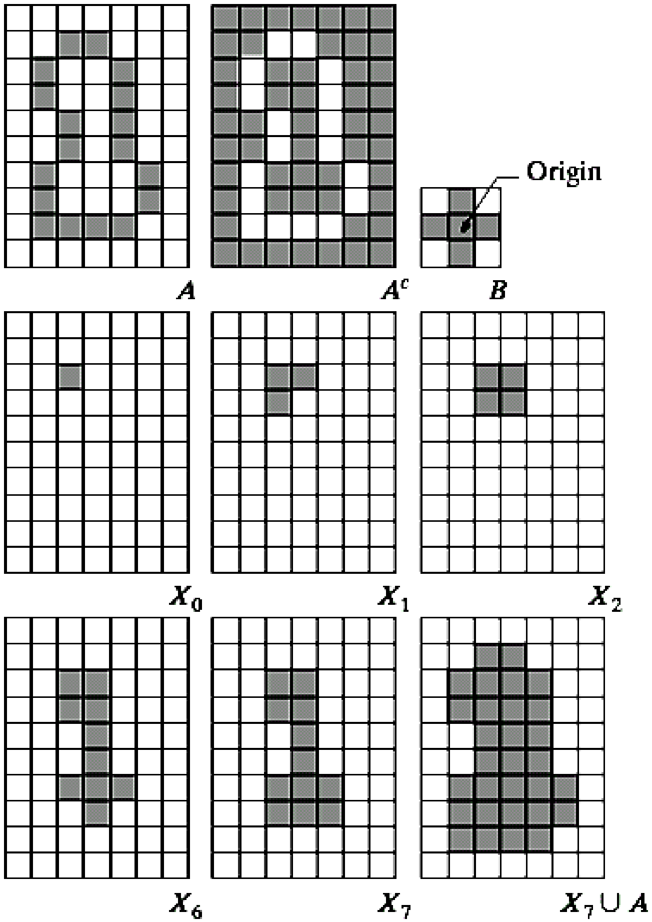
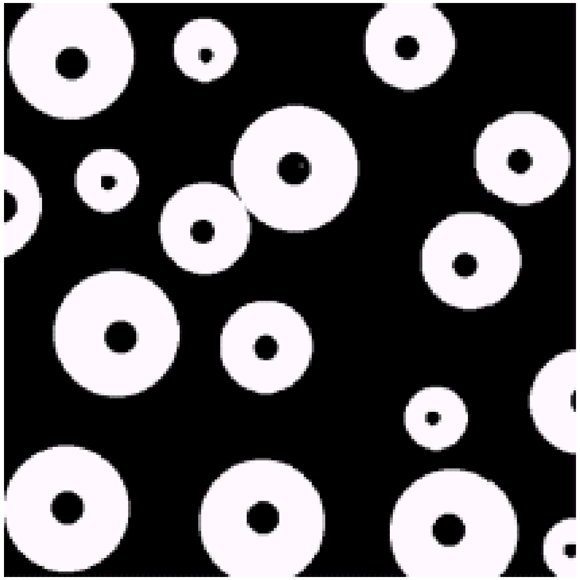
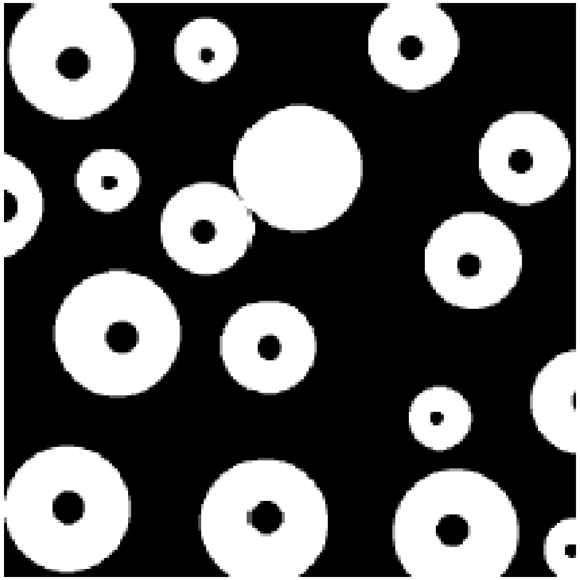


FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

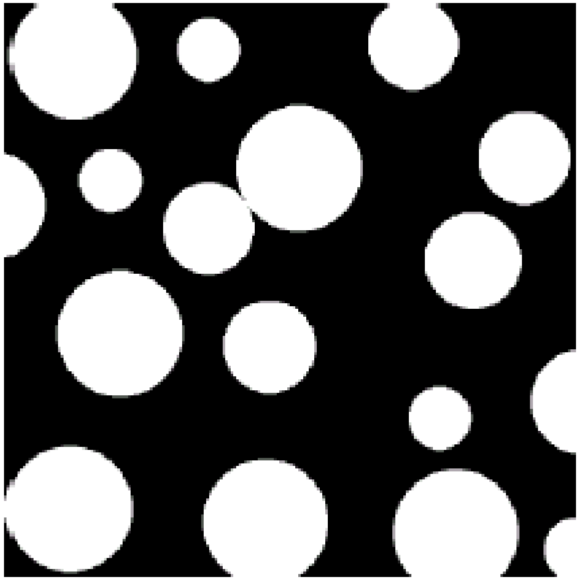
Region Filling: Example



Original Image



One Region Filled



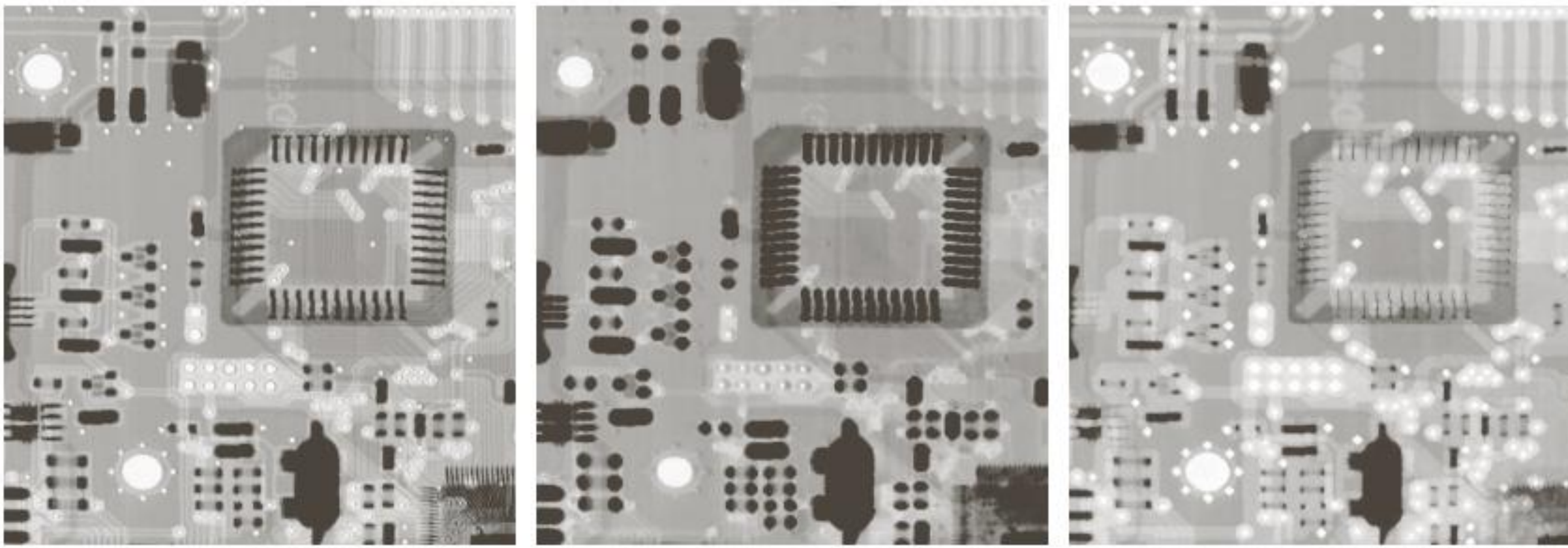
All Regions Filled

Gray Level Image Morphological Operations

Dilation & Erosion

$$(f \oplus b)(s,t) = max\{f(s-x, t-y)\}$$

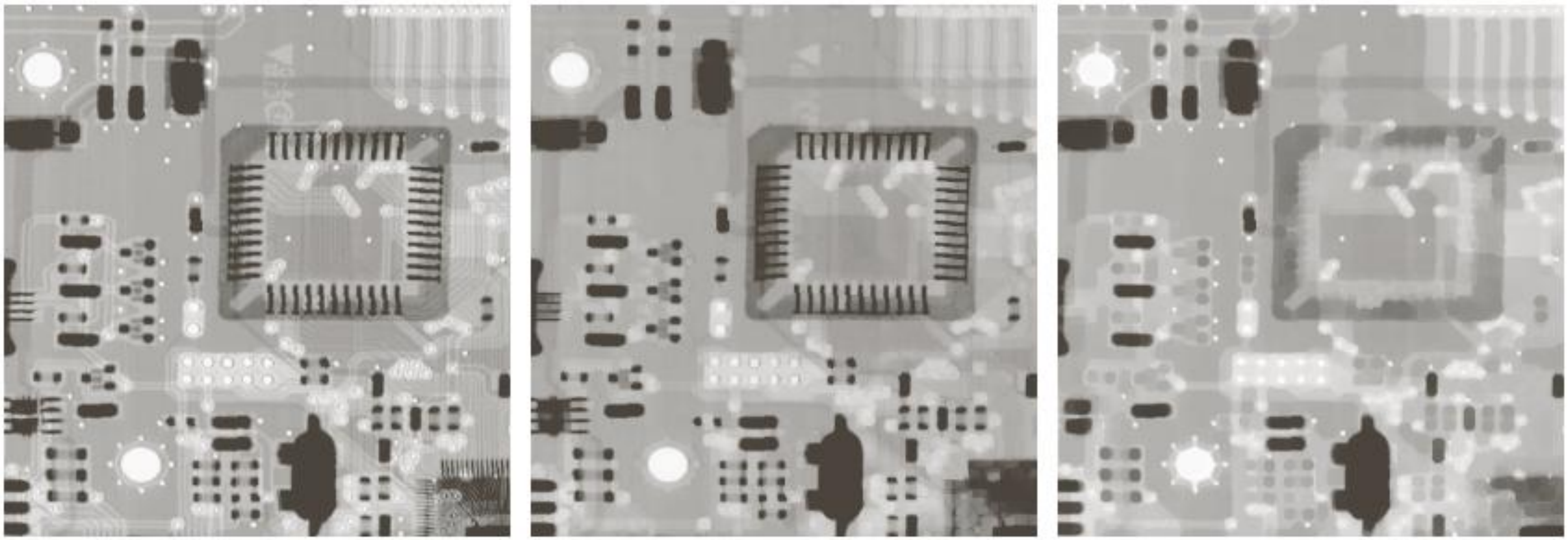
$$(f \ominus b)(s,t) = min\{f(s-x, t-y)\}$$



a b c

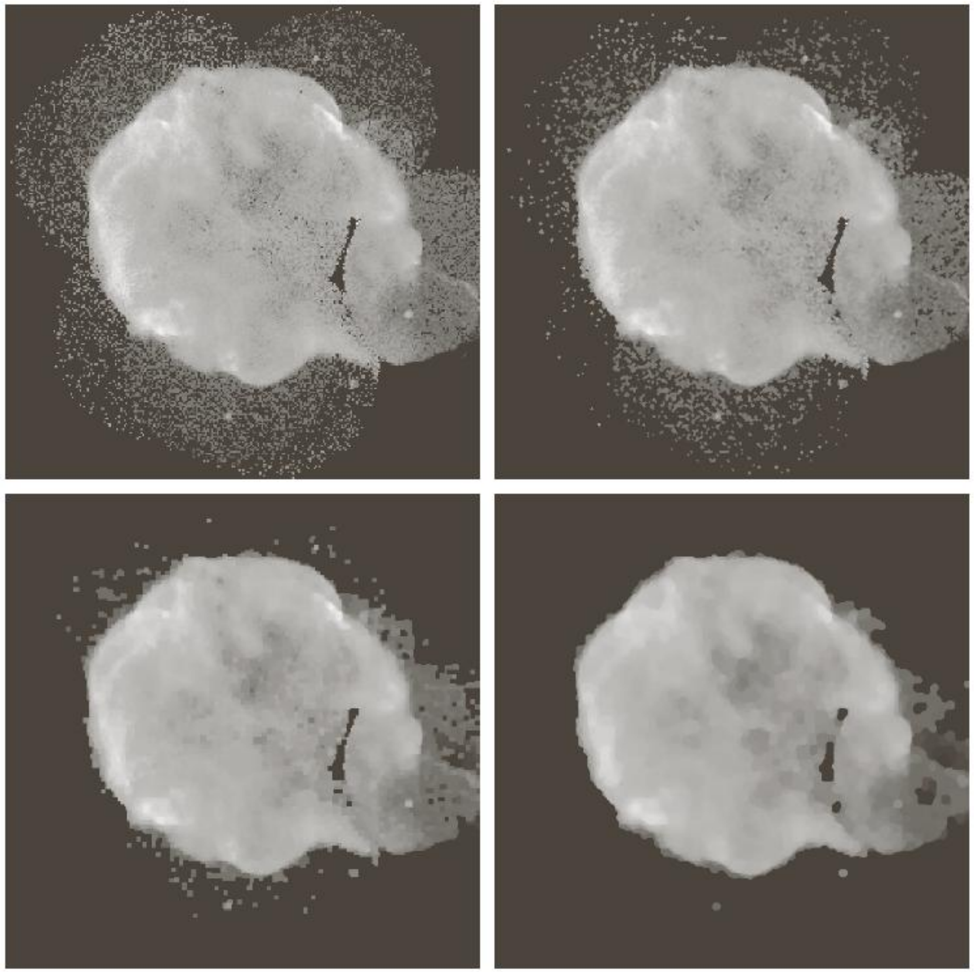
FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Opening & Closing



a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

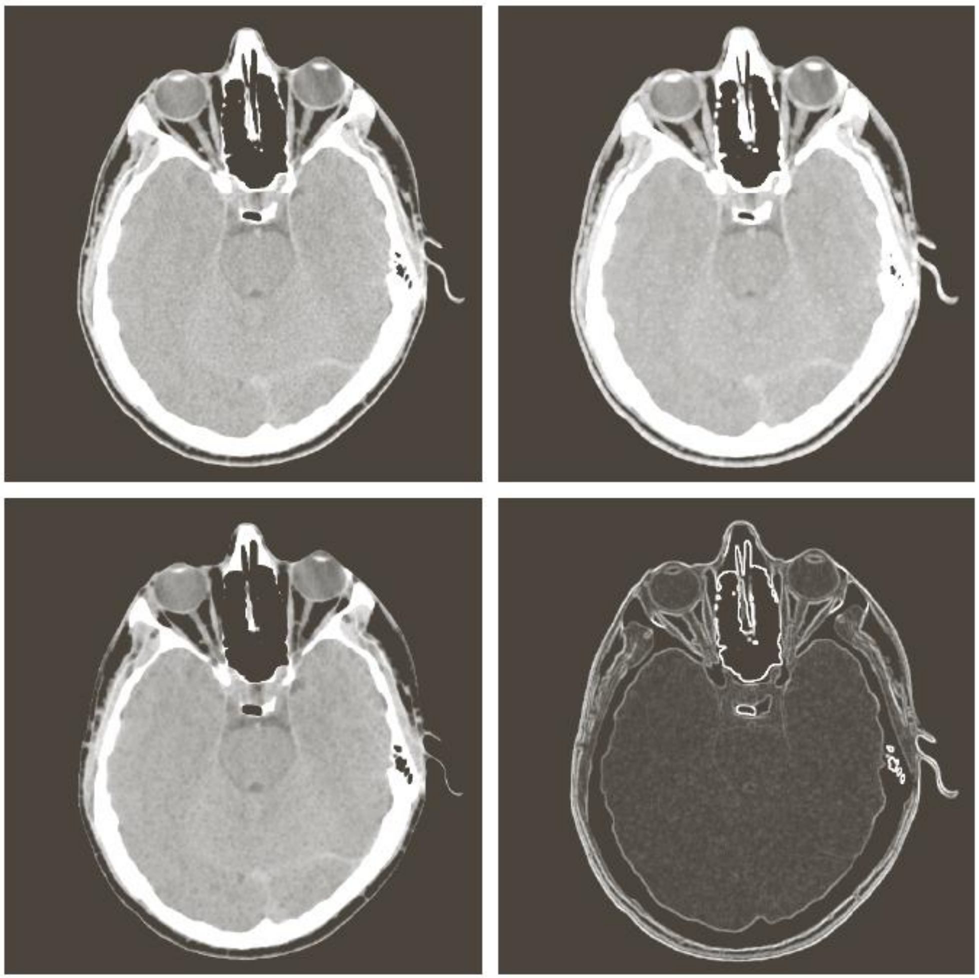


a b
c d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

Morphological Gradient

$$g = (f \oplus b) - (f \ominus b)$$

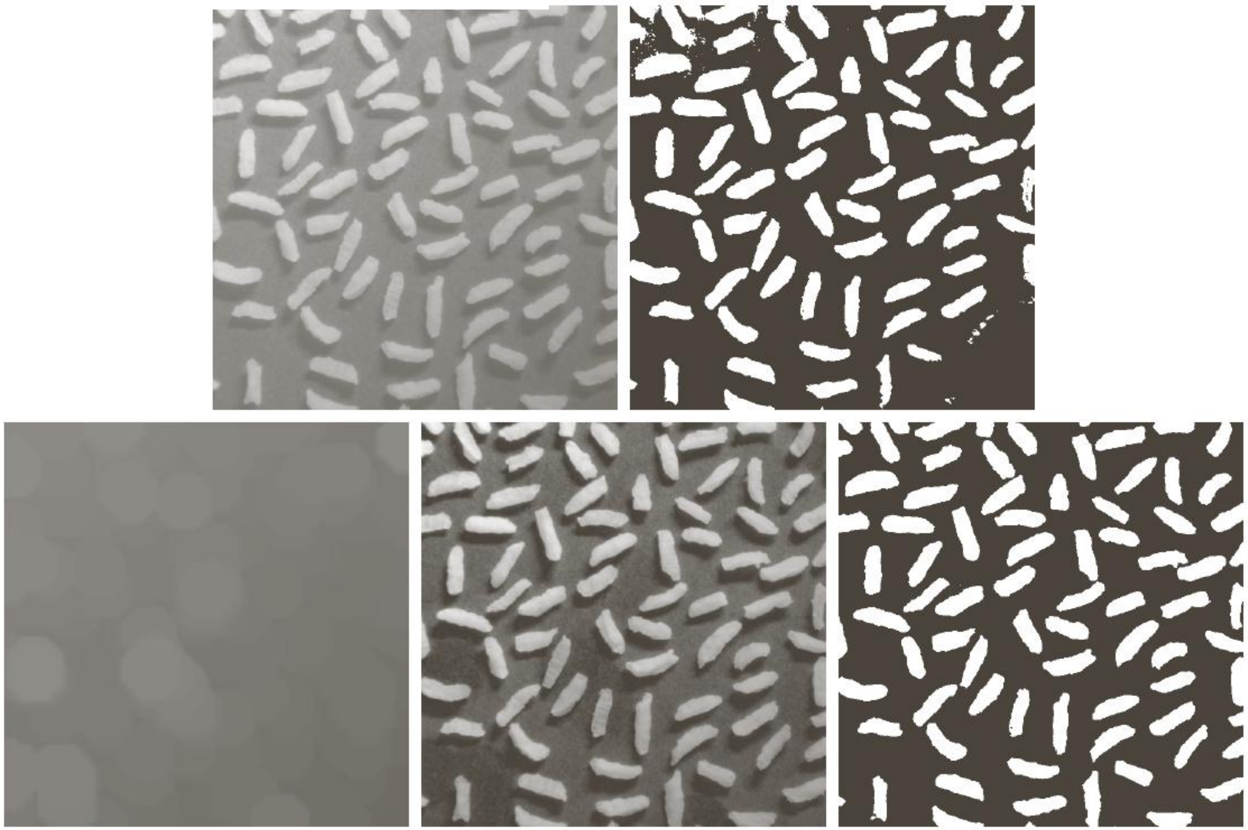


a b
c d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top Hat & Bottom Hat Transformations

$$g_{top} = f - (f \circ b) \qquad g_{bot} = f - (f \bullet b)$$



a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

Texture Based Descriptors

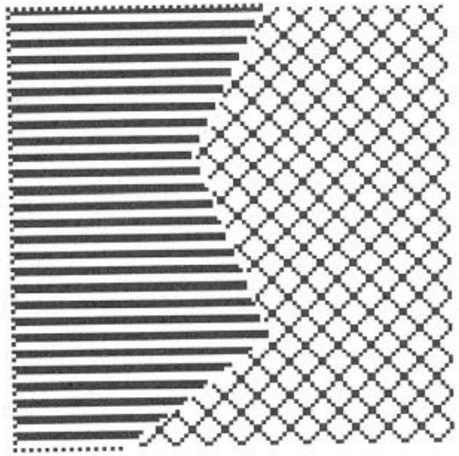
Texture

- Organized patterns of quite regular subelements called textons.
- Texture is a property of sufficiently large regions

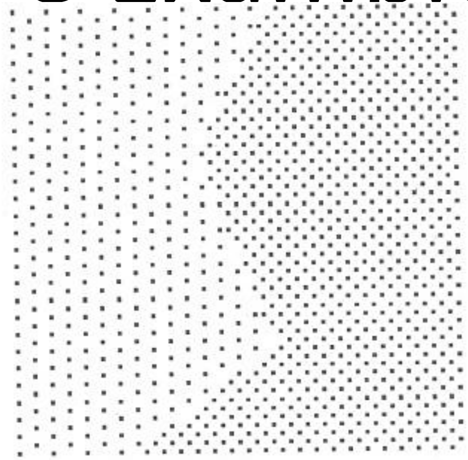
Applications:

- Texture based segmentation
- Texture synthesis
- Texture analysis and texture based matching
- Shape (surface orientation) from texture

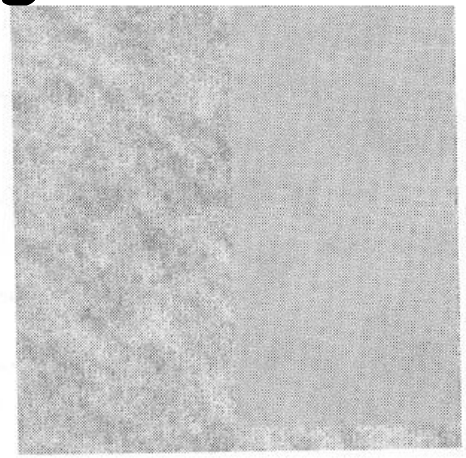
Texture Examples



Test image T1
(a)



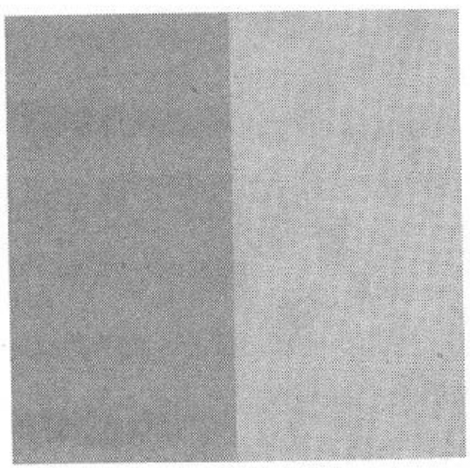
Test image T2
(b)



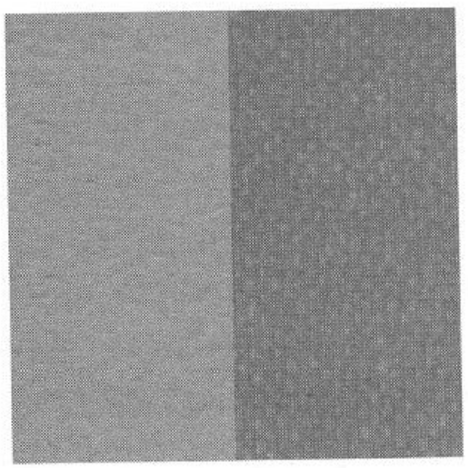
Test image T3
(c)

(a,b): Artificial textures

(c,d,e): Naturally occurring textures



Test image T4
(d)



Test image T5
(e)

Representing textures

Statistical

yields characterization of textures as smooth, coarse grainy, etc.

Spectral

are based on Fourier spectrum and are primarily used to detect the global periodicity in an image by identifying high energy narrow peaks in the spectrum.

Statistical approaches

- Based on the histogram measures of image
- Based on the Grey Level Co-occurrence Matrix (GLCM) and related measurement

Histogram based texture description

Using statistical moments of grey level histogram of the image or region

Let $p(z_i)$ is the histogram of the grey levels z_i of an image

The nth moment about the mean is given by:

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

Where mean is

$$m = \sum_{i=0}^{L-1} z_i p(z_i)$$

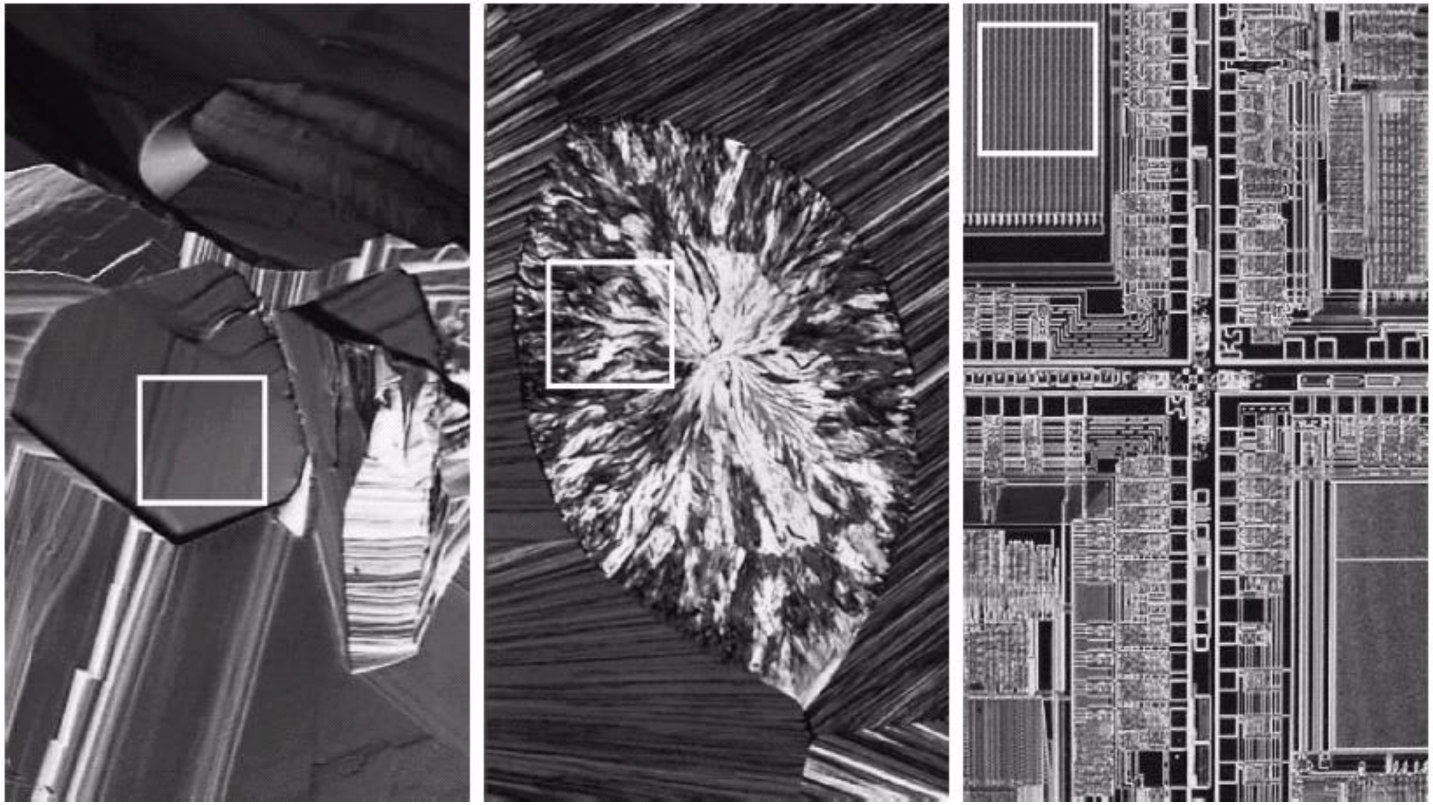
The variance is the second moment and is given by

$$\sigma^2(z) = \mu_2(z) = \sum_{i=0}^{L-1} (z_i - m)^2 p(z_i)$$

Histogram based texture description

- For texture description the following parameters are useful
  - Variance and related measures: descriptor of relative smoothness, use normalized variance $R = 1 - \frac{1}{1+\sigma^2(z)}$
  - Skewness of histogram $\mu_3(z) = \sum_{i=0}^{L-1} (z_i - m)^3 p(z_i)$
  - Relative flatness of histogram $\mu_4(z) = \sum_{i=0}^{L-1} (z_i - m)^4 p(z_i)$
  - Uniformity $U = \sum_{i=0}^{L-1} p^2(z_i)$
  - Average Entropy $e = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$

Histogram based texture description (example)



Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

GLCMs

- For texture description the following parameters of GLCM are measured and analyzed

  Maximum probability

  $$\max_{i,j}(c_{ij})$$

  Contrast

  $$\sum_i \sum_j (i-j)^2 c_{ij}$$

  Uniformity

  $$\sum_i \sum_j c_{ij}^2$$

  Entropy

  $$-\sum_i \sum_j c_{ij} \log_2 c_{ij}$$

FIGURE 11.29 How to generate a co-occurrence matrix.

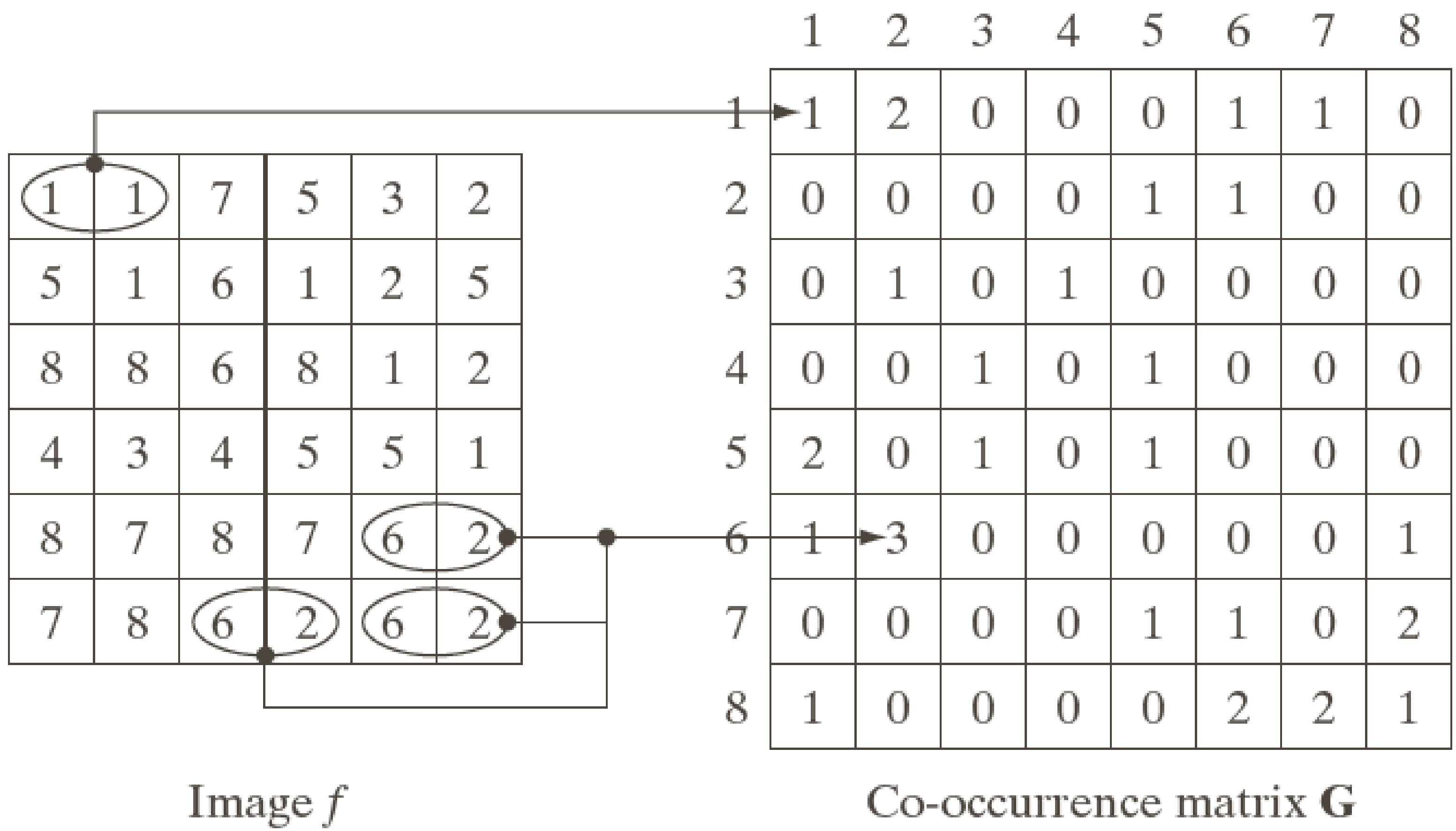
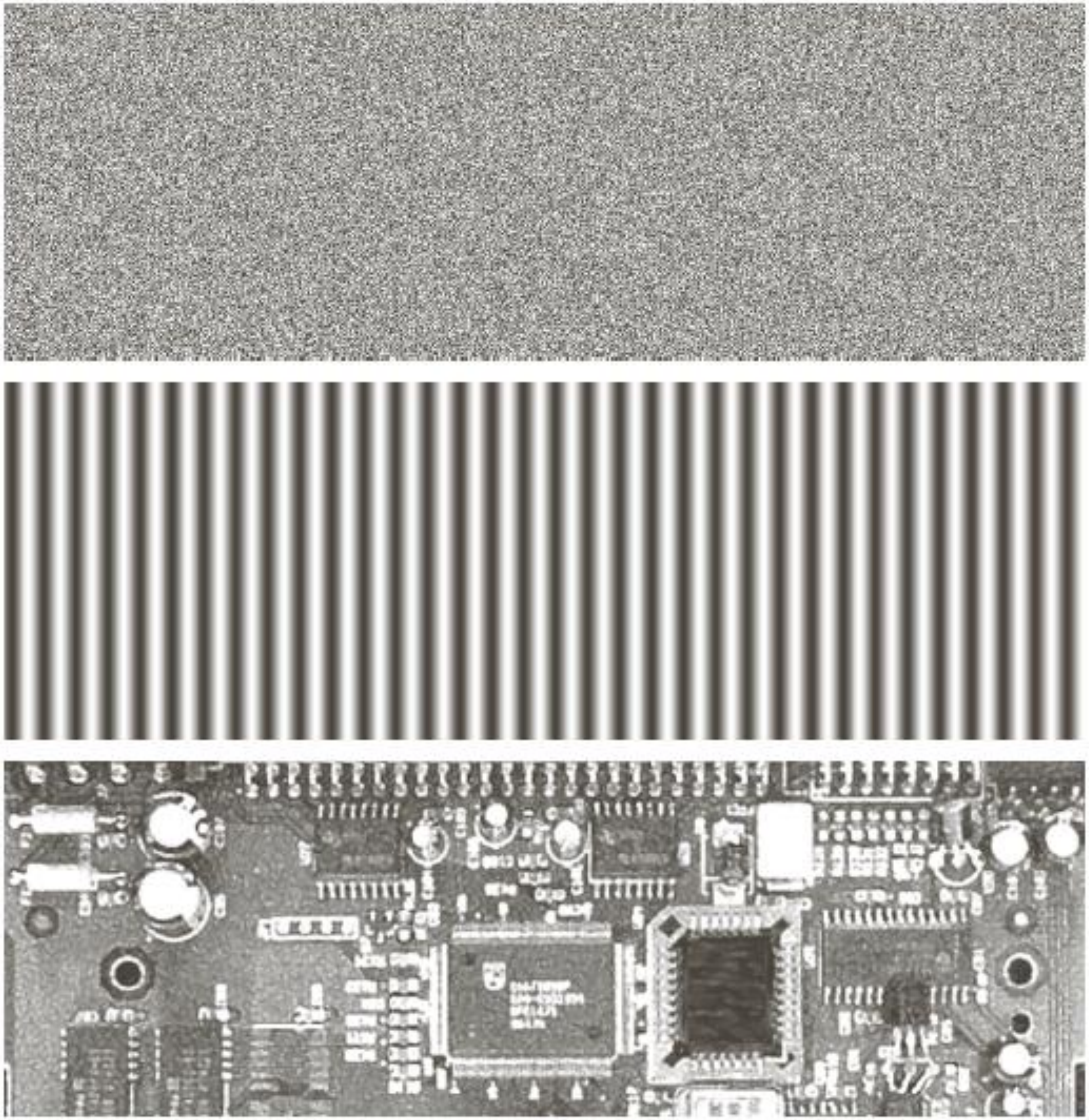


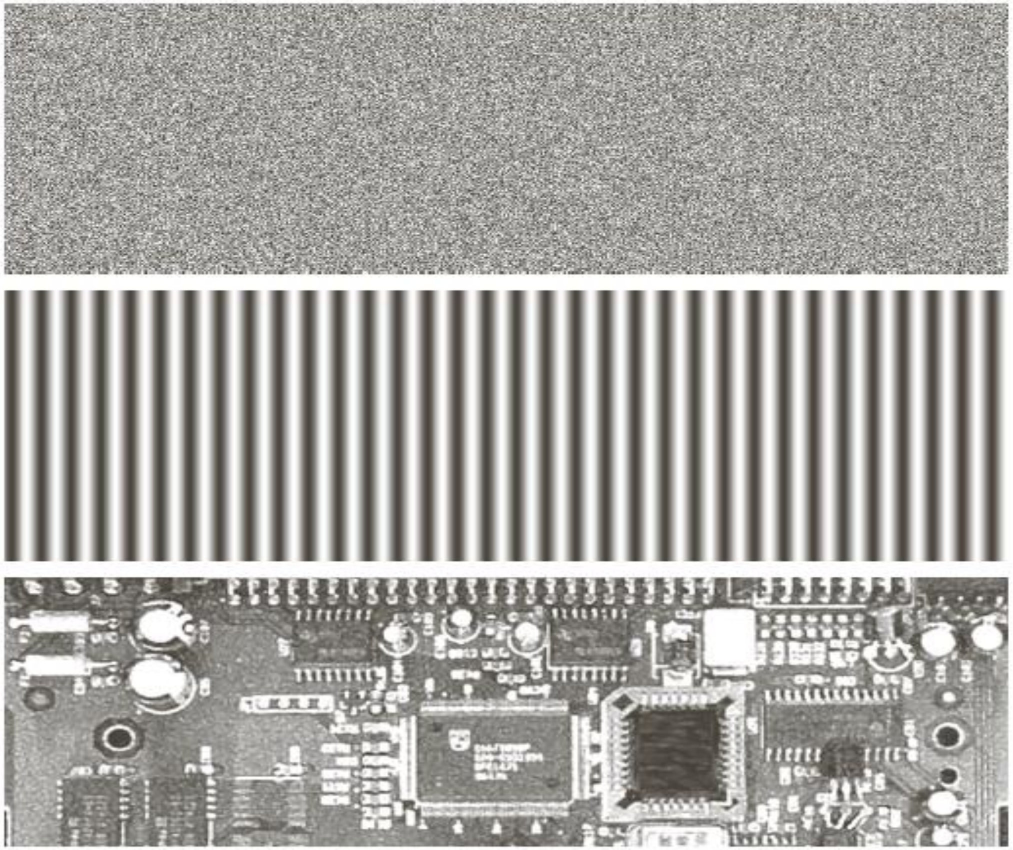
TABLE 11.3 Descriptors used for characterizing co-occurrence matrices of size $K \times K$. The term p_{ij} is the ijth term of $\mathbf{G}$ divided by the sum of the elements of $\mathbf{G}$.

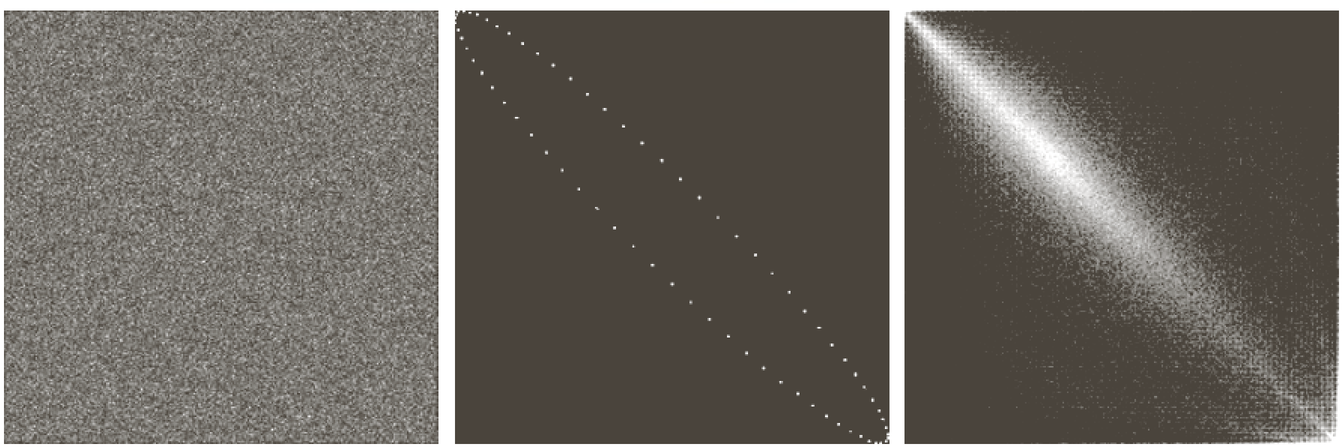
Descriptor	Explanation	Formula
Maximum probability	Measures the strongest response of **G**. The range of values is [0, 1].	$\max_{i,j}(p_{ij})$
Correlation	A measure of how correlated a pixel is to its neighbor over the entire image. Range of values is 1 to −1, corresponding to perfect positive and perfect negative correlations. This measure is not defined if either standard deviation is zero.	$\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{(i - m_r)(j - m_c)p_{ij}}{\sigma_r \sigma_c}$ $\sigma_r \neq 0; \sigma_c \neq 0$
Contrast	A measure of intensity contrast between a pixel and its neighbor over the entire image. The range of values is 0 (when **G** is constant) to $(K - 1)^2$.	$\sum_{i=1}^{K}\sum_{j=1}^{K}(i - j)^2 p_{ij}$
Uniformity (also called Energy)	A measure of uniformity in the range [0, 1]. Uniformity is 1 for a constant image.	$\sum_{i=1}^{K}\sum_{j=1}^{K} p_{ij}^2$
Homogeneity	Measures the spatial closeness of the distribution of elements in **G** to the diagonal. The range of values is [0, 1], with the maximum being achieved when **G** is a diagonal matrix.	$\sum_{i=1}^{K}\sum_{i=1}^{K}\frac{p_{ij}}{1 + \lvert i - j \rvert}$
Entropy	Measures the randomness of the elements of **G**. The entropy is 0 when all p_{ij}'s are 0 and is maximum when all p_{ij}'s are equal. The maximum value is $2 \log_2 K$. (See Eq. (11.3-9) regarding entropy).	$-\sum_{i=1}^{K}\sum_{i=1}^{K} p_{ij} \log_2 p_{ij}$



a
b
c

FIGURE 11.30 Images whose pixels have (a) random, (b) periodic, and (c) mixed texture patterns. Each image is of size 263×800 pixels.





a b c

FIGURE 11.31 256 × 256 co-occurrence matrices, $\mathbf{G}_1$, $\mathbf{G}_2$, and $\mathbf{G}_3$, corresponding from left to right to the images in Fig. 11.30.

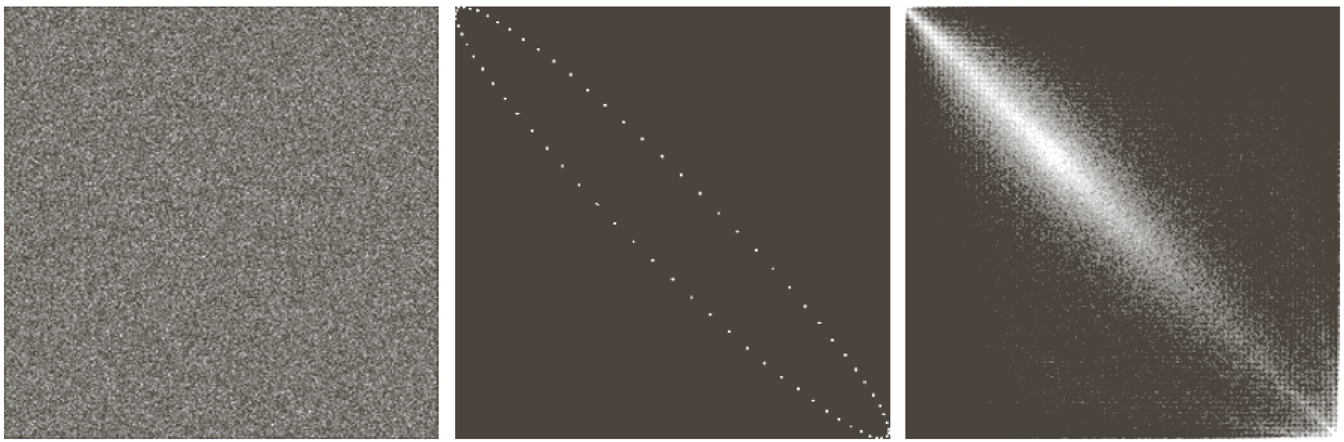


TABLE 11.4
Descriptors evaluated using the co-occurrence matrices displayed in Fig. 11.31.

Normalized Co-occurrence Matrix	Descriptor					
	Max Probability	Correlation	Contrast	Uniformity	Homogeneity	Entropy
$\mathbf{G}_1/n_1$	0.00006	−0.0005	10838	0.00002	0.0366	15.75
$\mathbf{G}_2/n_2$	0.01500	0.9650	570	0.01230	0.0824	6.43
$\mathbf{G}_3/n_3$	0.06860	0.8798	1356	0.00480	0.2048	13.58

Readings from Book (3rd Edn.)

- Morphological Operations (Chapter – 09)
- Reading Assignment
  - Connected Component
- Texture (Chapter-11)

Reading Assignment:

- Table-11.3, 11.4

Acknowledgements

- Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- Computer Vision for Computer Graphics, Mark Borg

Material in these slides has been taken from, the following resources