Digital Image Processing

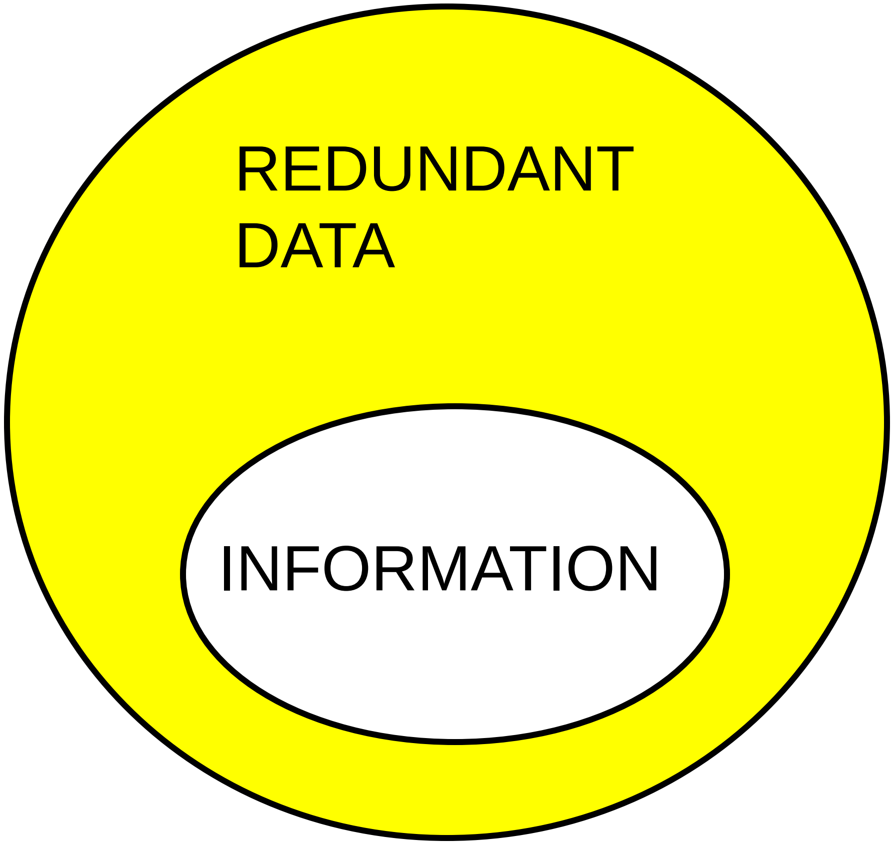
Lecture # 11
Image Compression

IMAGE COMPRESSION

IMAGE COMPRESSION

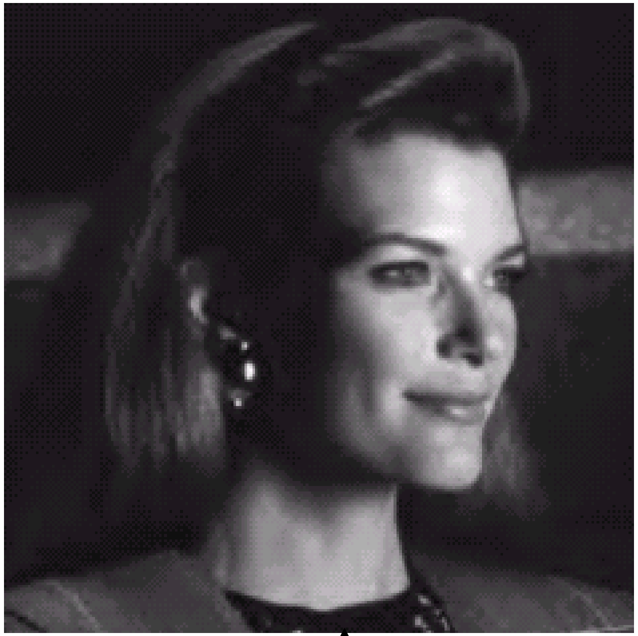
- Addresses the problem of reducing the amount of data required to represent a digital image
- The underlying basis of the reduction process is the removal of redundant data

Information vs Data



DATA = INFORMATION + REDUNDANT DATA

IMAGE COMPRESSION: CODING AND DECODING



original image
262144 Bytes

image encoder

compressed bit stream
00111000001001101…
(2428 Bytes)

image decoder

compression ratio (CR) = 108:1

LOSSY VS LOSSLESS COMPRESSION

- Lossless (Information preserving)
  - Images can be compressed and restored without any loss of information.
  - Application: Medical images
- Lossy
  - Perfect recovery is not possible but provides a large data compression.
  - Example: TV signals, teleconferencing

FUNDAMENTALS

- Raw image: A set of n1 bits
- Compressed image: A set of n2 bits.
- Compression ratio: $$C_R = \frac{n_1}{n_2}$$
- Relative Data Redundancy of first set: $$R_D = 1 - \frac{1}{C_R}$$
- Example: *n1*= 100KB and *n2* = 10Kb, then *CR* = 10 , and *RD* = 90%
- Special cases:
  - n1>>n2 → CR ≈ ∞, RD ≈ 1
  - n1 ≈ n2 → CR ≈ 1, RD ≈ 0

IMAGE COMPRESSION MODEL

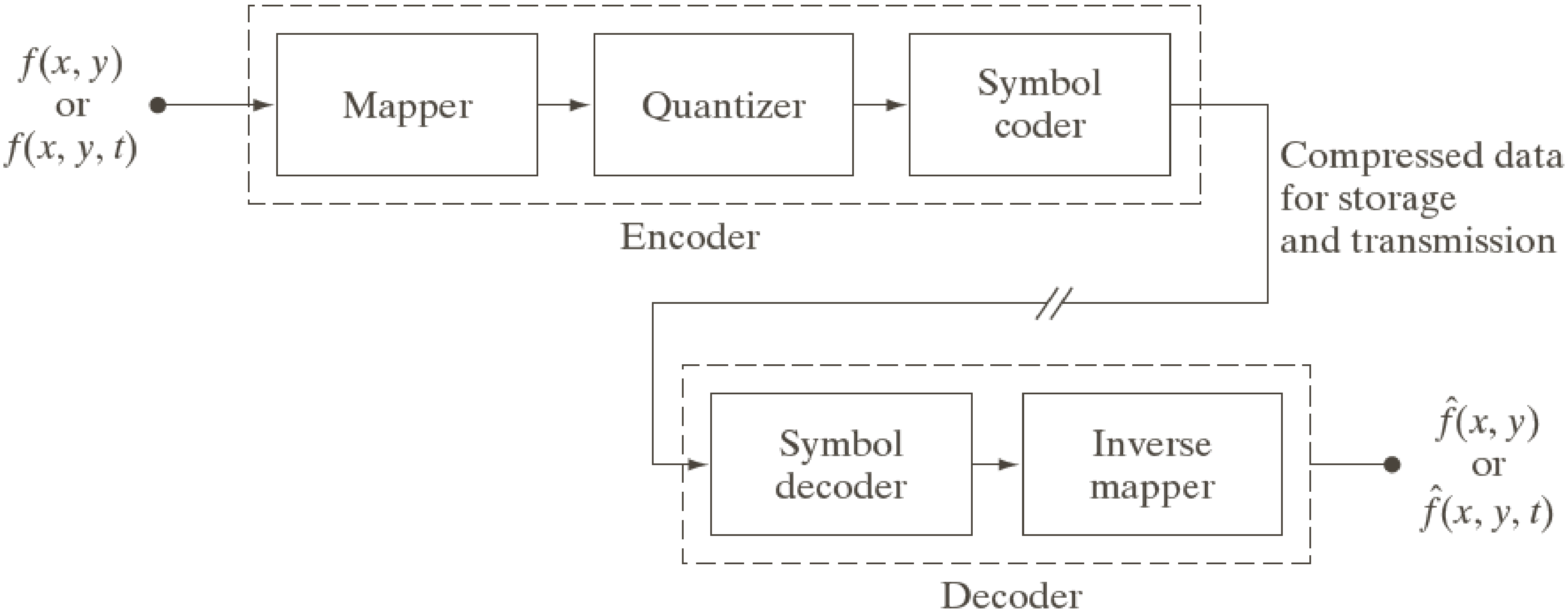


FIGURE 8.5 Functional block diagram of a general image compression system.

DATA REDUNDANCY

- Three basic data redundancies:
  - Coding redundancy
  - Spatial and Temporal redundancy
  - Irrelevant Information

a b c

FIGURE 8.1 Computer generated $256 \times 256 \times 8$ bit images with (a) coding redundancy, (b) spatial redundancy, and (c) irrelevant information. (Each was designed to demonstrate one principal redundancy but may exhibit others as well.)

CODING REDUNDANCY

- Type of coding (# of bits for each gray level)
- Image histogram:
  - *rk*: Represents the gray levels of an image
  - *pr*(*rk*): Probability of occurrence of *rk*

$$p_r(r_k) = \frac{n_k}{n} \qquad k = 0, 1, 2, \ldots, L-1$$

  - *l(rk)*: Number of bits used to represent each *rk* (after compression)
  - Lavg: Average # of bits required to represent each pixel:

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k) p_r(r_k)$$

CODING REDUNDANCY

- It makes sense to assign fewer bits to those rk for which pr(rk) are large in order to reduce the sum.
- This achieves data compression and results in a variable length code.
- More probable gray levels will have fewer # of bits.
- Basic type is variable length coding

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)\, p_r(r_k)$$

VARIABLE LENGTH CODING

r_k	$p_r(r_k)$	code1	$l_1(r_k)$	code2	$l_2(r_k)$
$r_0=0$	0.19	000	3	11	2
$r_1=1/7$	0.25	001	3	01	2
$r_2=2/7$	0.21	010	3	10	2
$r_3=3/7$	0.16	011	3	001	3
$r_4=4/7$	0.08	100	3	0001	4
$r_5=5/7$	0.06	101	3	00001	5
$r_6=6/7$	0.03	110	3	000001	6
$r_7=1$	0.02	111	3	000000	6

VARIABLE LENGTH CODING

- Computing L_{avg}

$$L_{avg} = \sum_{k=0}^{7} l_2(r_k)\, p_r(r_k)$$

=2(0.19)+2(0.05)+2(0.21)+3(0.16)+4(0.08)+5(0.06)+6(0.03)+6(0.02)

=2.7 bits

- C_R=3/2.7=1.11
- R_D=1-1/1.11=0.099=9.9%

LOSSLESS GRAY-SCALE IMAGE COMPRESSION

VARIABLE WORD LENGTH CODING: EXAMPLE

- **A 4x4 4bits/pixel original image is given by**

Default Code Book

0: 0000
1: 0001
2: 0010
3: 0011
4: 0100
5: 0101
6: 0110
7: 0111
8: 1000
9: 1001
10: 1010
11: 1011
12: 1100
13: 1101
14: 1110
15: 1111

2	8	6	6
6	8	8	8
8	8	10	10
9	10	10	14

encode

0010	1000	0110	0110
0110	1000	1000	1000
1000	1000	1010	1010
1001	1010	1010	1110

Bit rate = 4bits/pixel

Total # of bits used to represent the image:

4x16 = 64 bits

VARIABLE WORD LENGTH CODING: EXAMPLE

- **Encode the original image with a CODE BOOK given left**

Huffman Code Book

0: 0000000
1: 0000001
2: 0001
3: 0000010
4: 0000011
5: 0000100
6: 01
7: 0000101
8: 10
9: 00100
10: 11
11: 0000110
12: 0000111
13: 001010
14: 0011
15: 001011

2	8	6	6
6	8	8	8
8	8	10	10
9	10	10	14

encode

0001	10	01	01
01	10	10	10
10	10	11	11
00100	11	11	0011

Total # of bits used to represent the image:

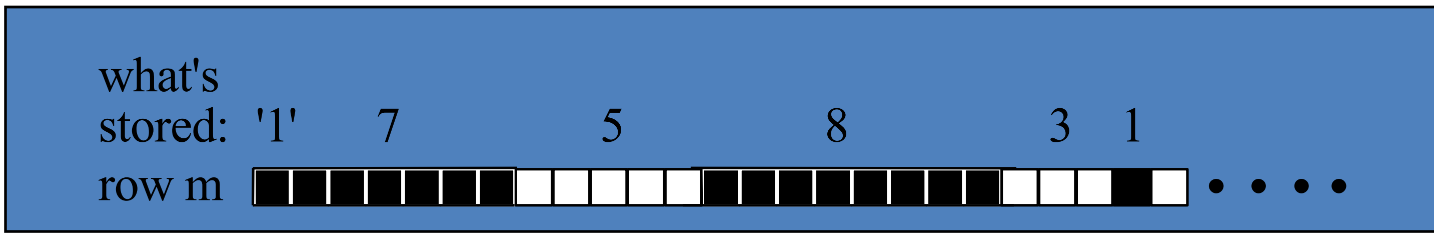
4+2+2+2+2+2+2+2+2+
2+2+2+5+2+2+4
= 39 bits

Bit rate = 39/16
= 2.4375 bits/pixel

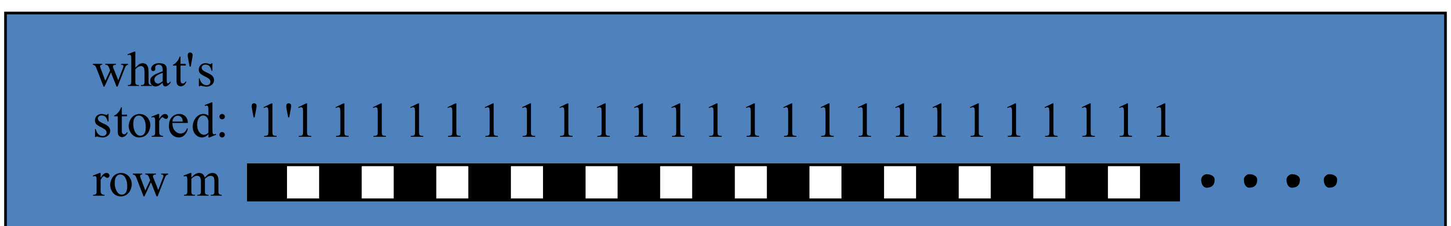
CR = 64/39 = 1.6410

RUN LENGTH CODING

- Run Length
  - **The length of consecutively identical symbols**
- Run length Coding Example



- When Does it Work?
  - **Images containing many runs of 1's and 0's**
- When Does it Not Work?



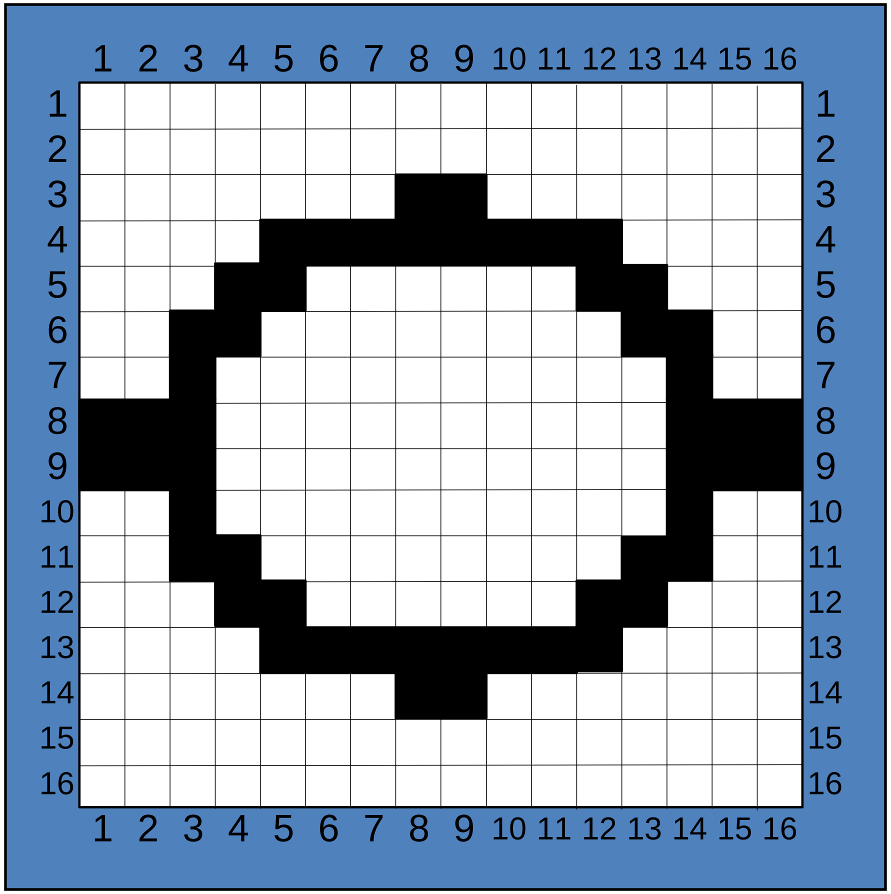
RUN LENGTH CODING

- Decoding Example

A binary image is encoded using run length code row by row, where "0" represents white, and "1" represents black. The code is given by

Row 1: "0", 16
Row 2: "0", 16
Row 3: "0", 7, 2, 7
Row 4: "0", 4, 8, 4
Row 5: "0", 3, 2, 6, 3, 2
Row 6: "0", 2, 2, 8, 2, 2
Row 7: "0", 2, 1, 10, 1, 2
Row 8: "1", 3, 10, 3
Row 9: "1", 3, 10, 3
Row 10: "0", 2, 1, 10, 1, 2
Row 11: "0", 2, 2, 8, 2, 2
Row 12: "0", 3, 2, 6, 3, 2
Row 13: "0", 4, 8, 4
Row 14: "0", 7, 2, 7
Row 15: "0", 16
Row 16: "0", 16

decode →



Entropy

Entropy calculation for a two symbol alphabet.

Example 1: A p_A=0.5
B p_B=0.5

$$H(A,B) = -p_A \log_2 p_A - p_B \log_2 p_B =$$
$$= -0.5\log_2 0.5 - 0.5\log_2 0.5 = 1$$

It requires one bit per symbol on the average to represent the data.

Example 2: A p_A=0.8
B p_B=0.2

$$H(A,B) = -p_A \log_2 p_A - p_B \log_2 p_B =$$
$$= -0.8\log_2 0.8 - 0.2\log_2 0.2 \cong 0.7219$$

It requires less than one bit per symbol on the average to represent the data.

How can we code this ?

HUFFMAN CODING

- Uses frequencies (Probability) of symbols in a string to build a variable rate prefix code.
- Each symbol is mapped to a binary string.
- More frequent symbols have shorter codes.
- No code is a prefix of another. (Uniquely decodable)

HUFFMAN CODING

- Each symbol is assigned a variable-length code, depending on its frequency. The higher its frequency, the shorter the codeword
- Number of bits for each codeword is an integral number
- A prefix code
- A variable-length code
- Huffman code is the **optimal** prefix and variable-length code, **given** the symbols' probabilities of occurrence
- Codewords are generated by building a Huffman Tree

HUFFMAN CODING

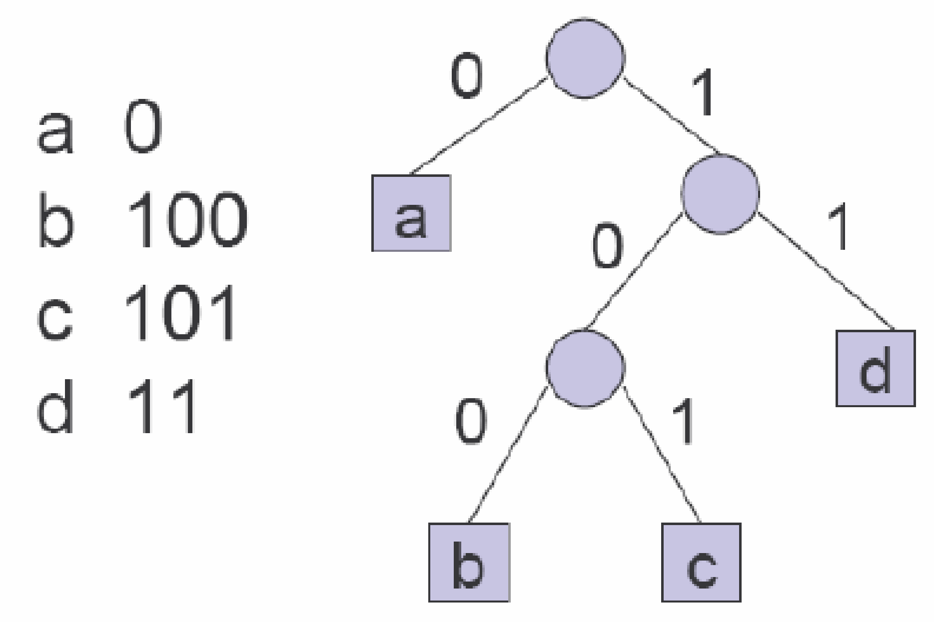
- **Prefix code** - No codeword is a prefix of any other codeword.

  A = 0;   B = 10;   C = 110; D = 111

- **Uniquely decodable code** - Has only one possible source string producing it.
  - Unambigously decoded
  - Examples:
    - Prefix code - the end of a codeword is immediately recognized without ambiguity: 010011001110 → 0 | 10 | 0 | 110 | 0 | 111 | 0
    - Fixed-length code

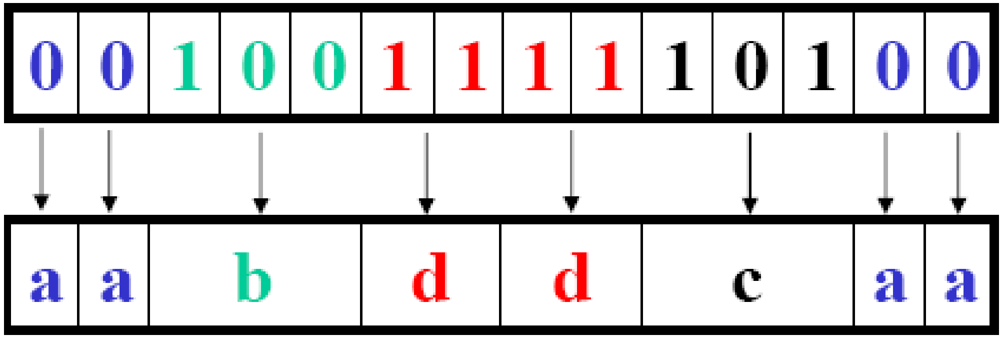
HUFFMAN CODING

- Example:
  - We have four symbols a, b, c, and d.



HUFFMAN CODING

- A source string: **aabddcaa**
  - Fixed Length Coding: 16 bits (ordinary coding)
    - **00 00 01 11 11 10 00 00**
- Variable length coding: 14 bits (Huffman coding)
  - **0 0 100 11 11 101 0 0**
- Uniquely Decodable:



HUFFMAN CODING

- Step-1
  - Arrange probability in decreasing order and consider them as tree leaves
- Step-2
  - Merge two nodes with smallest prob. to a new node and sum up prob.
  - Arbitrarily assign 1 and 0 to each pair of merging branch
- Step-3
  - Repeat until no more than one node left.
  - Read out codeword sequentially from root to leaf

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				

FIGURE 8.7 Huffman source reductions.

HUFFMAN CODING

Original source			Source reduction							
Symbol	Probability	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								

FIGURE 8.8
Huffman code assignment procedure.

OBJECTIVE CRITERIA

- The error e(x,y) can be defined as

$$e(x,y) = \hat{f}(x,y) - f(x,y)$$

- Total error between the two images

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\hat{f}(x,y) - f(x,y)\right]$$

- Root-mean-square-error

$$e_{rms} = \left[\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\hat{f}(x,y) - f(x,y)\right]^2\right]^{1/2}$$

- Mean-square signal-to-noise ratio of the output image

$$SNR_{ms} = \frac{\left[\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\hat{f}(x,y)\right]^2\right]}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\hat{f}(x,y) - f(x,y)\right]^2}$$

Texture Analysis

Texture

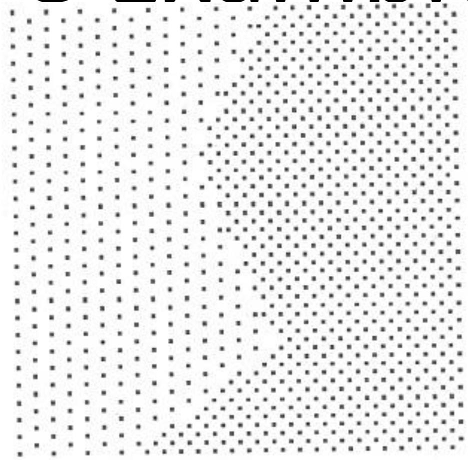
- Organized patterns of quite regular subelements called textons.
- Texture is a property of sufficiently large regions

Applications:

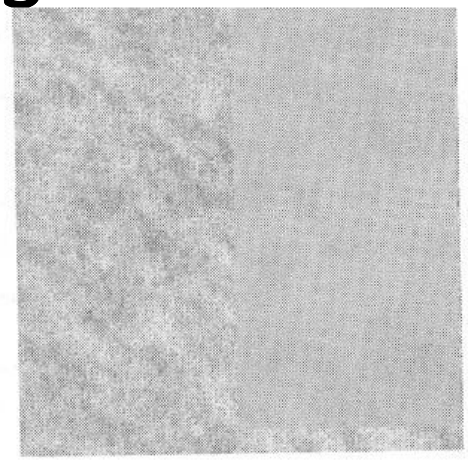
  - Texture based segmentation
  - Texture synthesis
  - Texture analysis and texture based matching
  - Shape (surface orientation) from texture

Texture Examples

Test image T1
(a)



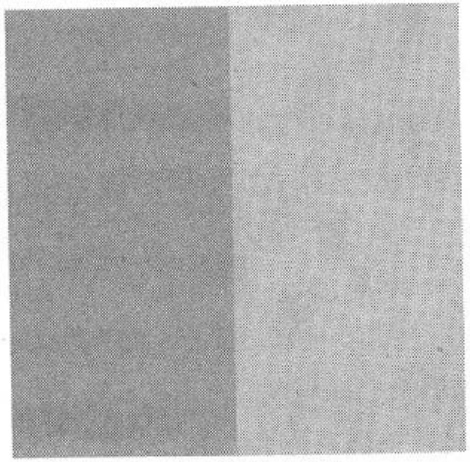
Test image T2
(b)



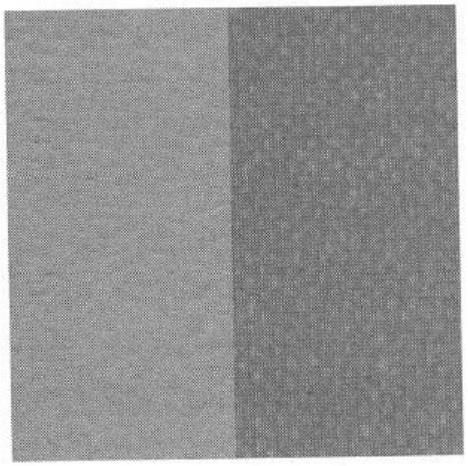
Test image T3
(c)

(a,b): Artificial textures

(c,d,e): Naturally occurring textures



Test image T4
(d)



Test image T5
(e)

Representing textures

Statistical

yields characterization of textures as smooth, coarse grainy, etc.

Spectral

are based on Fourier spectrum and are primarily used to detect the global periodicity in an image by identifying high energy narrow peaks in the spectrum.

Statistical approaches

- Based on the histogram measures of image
- Based on the Grey Level Co-occurrence Matrix (GLCM) and related measurement

Histogram based texture description

Using statistical moments of grey level histogram of the image or region

Let $p(z_i)$ is the histogram of the grey levels z_i of an image

The nth moment about the mean is given by:

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n p(z_i)$$

Where mean is $$m = \sum_{i=0}^{L-1} z_i p(z_i)$$

The 0th moment = 1, and the 1st moment = 0,

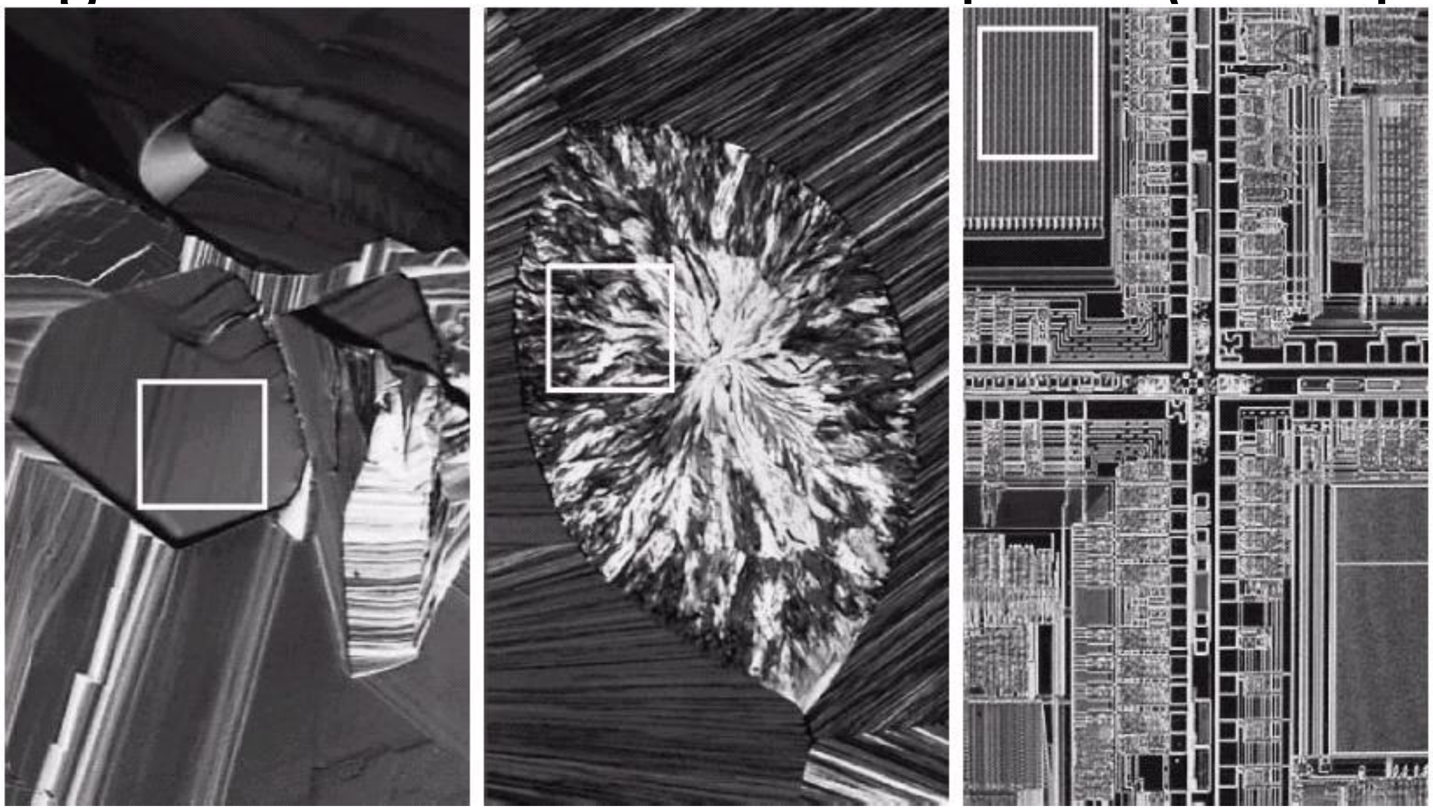
The variance is the second moment and is given by

$$\sigma^2(z) = \mu_2(z) = \sum_{i=0}^{L-1} (z_i - m)^2 p(z_i)$$

Histogram based texture description

- For texture description the following parameters are useful
  - Variance and related measures: descriptor of relative smoothness, use normalized variance $$R = 1 - \frac{1}{1+\sigma^2(z)}$$
  - Skewness of histogram $$\mu_3(z) = \sum_{i=0}^{L-1} (z_i - m)^3 p(z_i)$$
  - Relative flatness of histogram $$\mu_4(z) = \sum_{i=0}^{L-1} (z_i - m)^4 p(z_i)$$
  - Uniformity $$U = \sum_{i=0}^{L-1} p^2(z_i)$$
  - Average Entropy $$e = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$$

Histogram based texture description (example)



Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

GLCMs

- For texture description the following parameters of GLCM are measured and analyzed

  Maximum probability

$$\max_{i,j}(c_{ij})$$

  Contrast

$$\sum_i \sum_j (i-j)^2 c_{ij}$$

  Uniformity

$$\sum_i \sum_j c_{ij}^2$$

  Entropy

$$-\sum_i \sum_j c_{ij} \log_2 c_{ij}$$

FIGURE 11.29
How to generate a co-occurrence matrix.

1	1	7	5	3	2
5	1	6	1	2	5
8	8	6	8	1	2
4	3	4	5	5	1
8	7	8	7	6	2
7	8	6	2	6	2

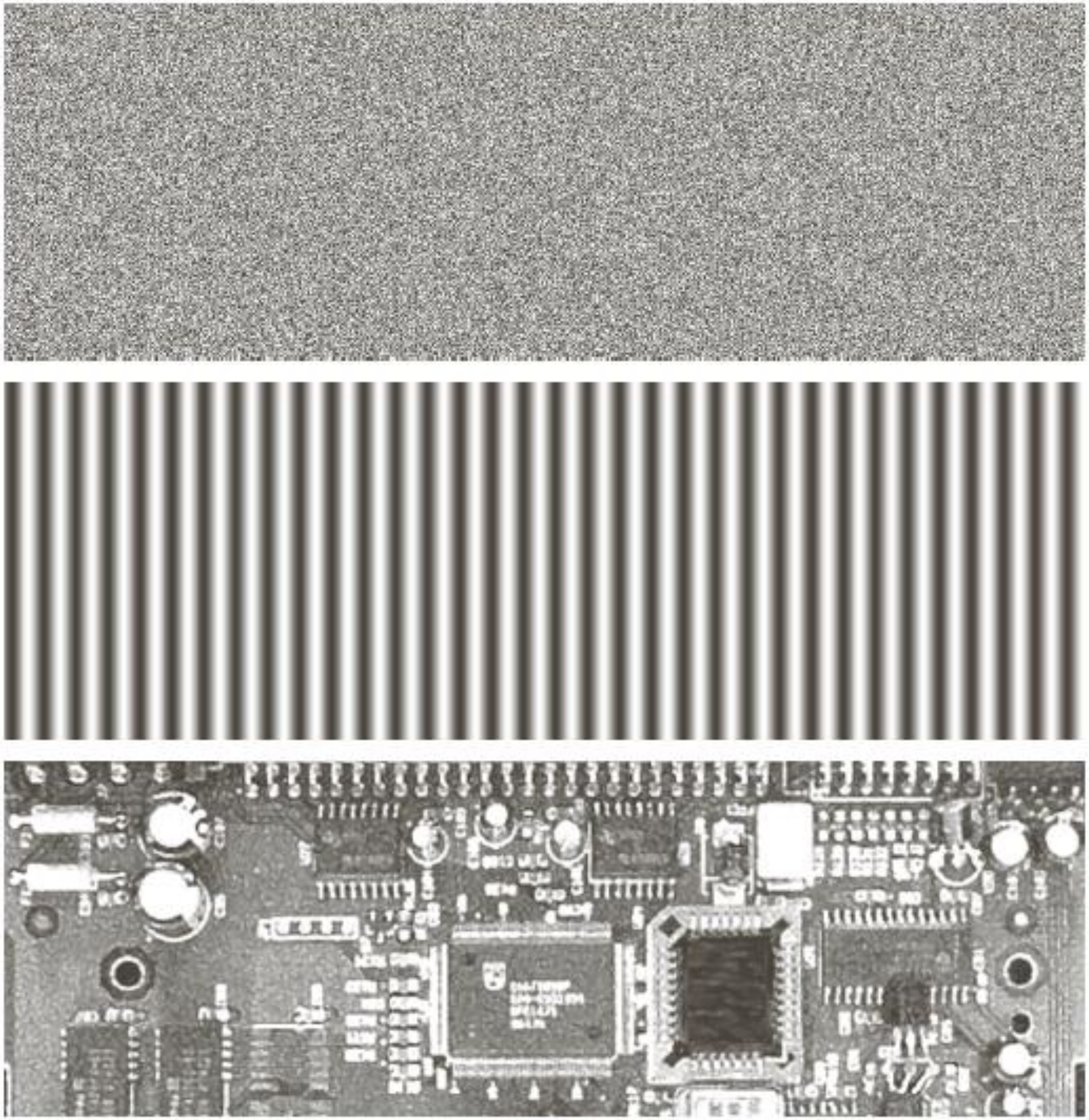
Image f

	1	2	3	4	5	6	7	8
1	1	2	0	0	0	1	1	0
2	0	0	0	0	1	1	0	0
3	0	1	0	1	0	0	0	0
4	0	0	1	0	1	0	0	0
5	2	0	1	0	1	0	0	0
6	1	3	0	0	0	0	0	1
7	0	0	0	0	1	1	0	2
8	1	0	0	0	0	2	2	1

Co-occurrence matrix **G**

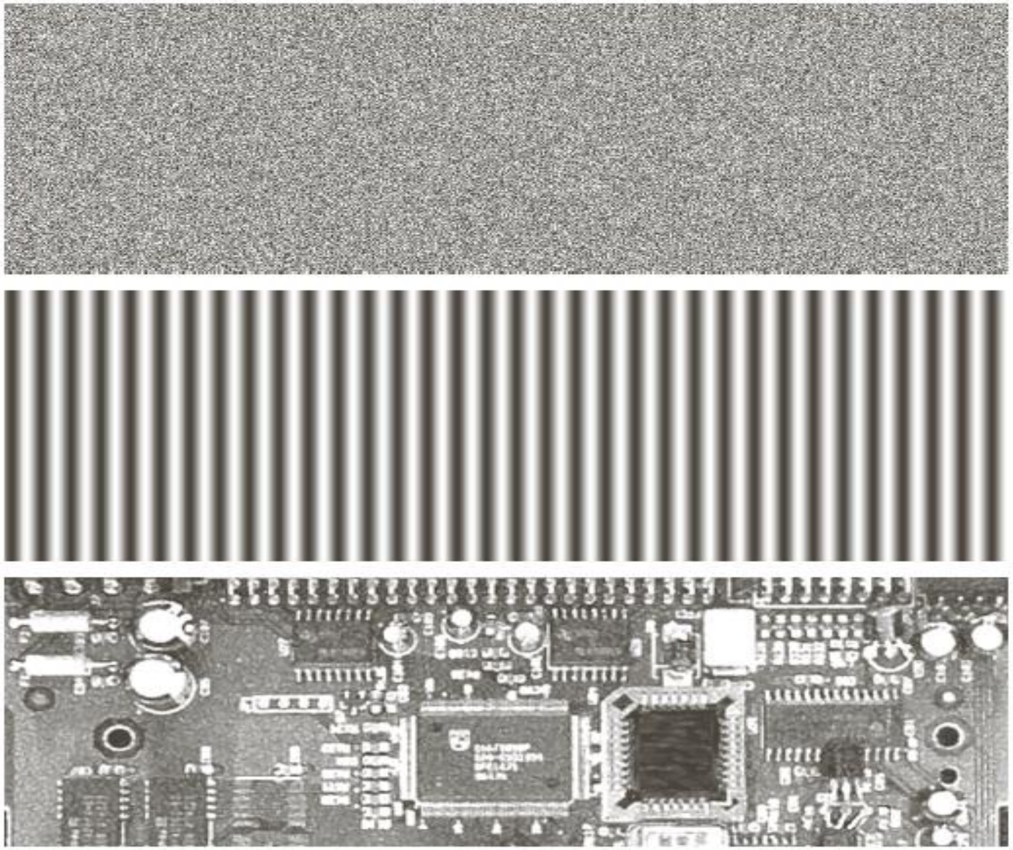
Descriptor	Explanation	Formula
Maximum probability	Measures the strongest response of **G**. The range of values is [0, 1].	$\max_{i,j}(p_{ij})$
Correlation	A measure of how correlated a pixel is to its neighbor over the entire image. Range of values is 1 to −1, corresponding to perfect positive and perfect negative correlations. This measure is not defined if either standard deviation is zero.	$\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{(i - m_r)(j - m_c)p_{ij}}{\sigma_r\sigma_c}$ $\sigma_r \neq 0;\ \sigma_c \neq 0$
Contrast	A measure of intensity contrast between a pixel and its neighbor over the entire image. The range of values is 0 (when **G** is constant) to $(K - 1)^2$.	$\sum_{i=1}^{K}\sum_{j=1}^{K}(i - j)^2 p_{ij}$
Uniformity (also called Energy)	A measure of uniformity in the range [0, 1]. Uniformity is 1 for a constant image.	$\sum_{i=1}^{K}\sum_{j=1}^{K} p_{ij}^2$
Homogeneity	Measures the spatial closeness of the distribution of elements in **G** to the diagonal. The range of values is [0, 1], with the maximum being achieved when **G** is a diagonal matrix.	$\sum_{i=1}^{K}\sum_{i=1}^{K}\frac{p_{ij}}{1 + \lvert i - j \rvert}$
Entropy	Measures the randomness of the elements of **G**. The entropy is 0 when all p_{ij}'s are 0 and is maximum when all p_{ij}'s are equal. The maximum value is $2\log_2 K$. (See Eq. (11.3-9) regarding entropy).	$-\sum_{i=1}^{K}\sum_{i=1}^{K} p_{ij}\log_2 p_{ij}$

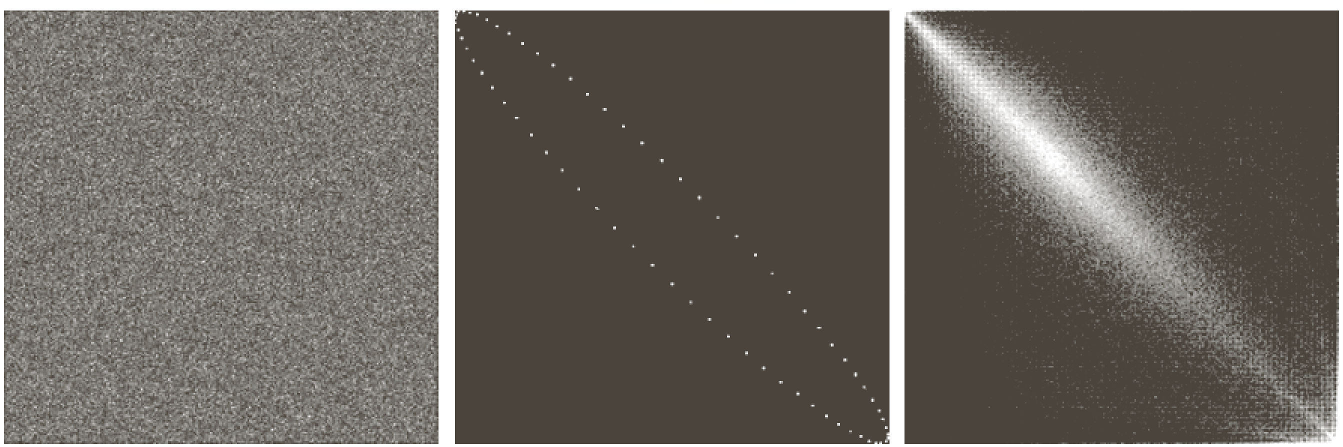
TABLE 11.3 Descriptors used for characterizing co-occurrence matrices of size $K \times K$. The term p_{ij} is the ijth term of **G** divided by the sum of the elements of **G**.



a
b
c

FIGURE 11.30 Images whose pixels have (a) random, (b) periodic, and (c) mixed texture patterns. Each image is of size 263×800 pixels.





a b c

FIGURE 11.31 256 × 256 co-occurrence matrices, $\mathbf{G}_1$, $\mathbf{G}_2$, and $\mathbf{G}_3$, corresponding from left to right to the images in Fig. 11.30.

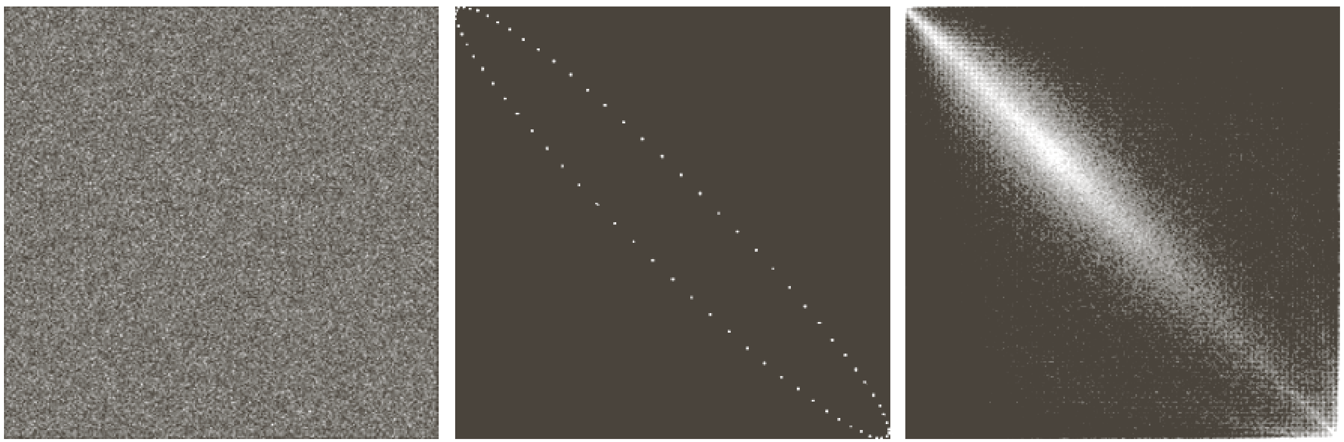


TABLE 11.4 Descriptors evaluated using the co-occurrence matrices displayed in Fig. 11.31.

Normalized Co-occurrence Matrix	Descriptor					
	Max Probability	Correlation	Contrast	Uniformity	Homogeneity	Entropy
$\mathbf{G}_1/n_1$	0.00006	−0.0005	10838	0.00002	0.0366	15.75
$\mathbf{G}_2/n_2$	0.01500	0.9650	570	0.01230	0.0824	6.43
$\mathbf{G}_3/n_3$	0.06860	0.8798	1356	0.00480	0.2048	13.58

Readings from Book (3^{rd} Edn.)

- Image Compression (Chapter-8)
- Texture (Chapter-11)

Reading Assignment:

- Table-11.3, 11.4

Acknowledgements

- Digital Image Processing”, Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002

Material in these slides has been taken from, the following resources