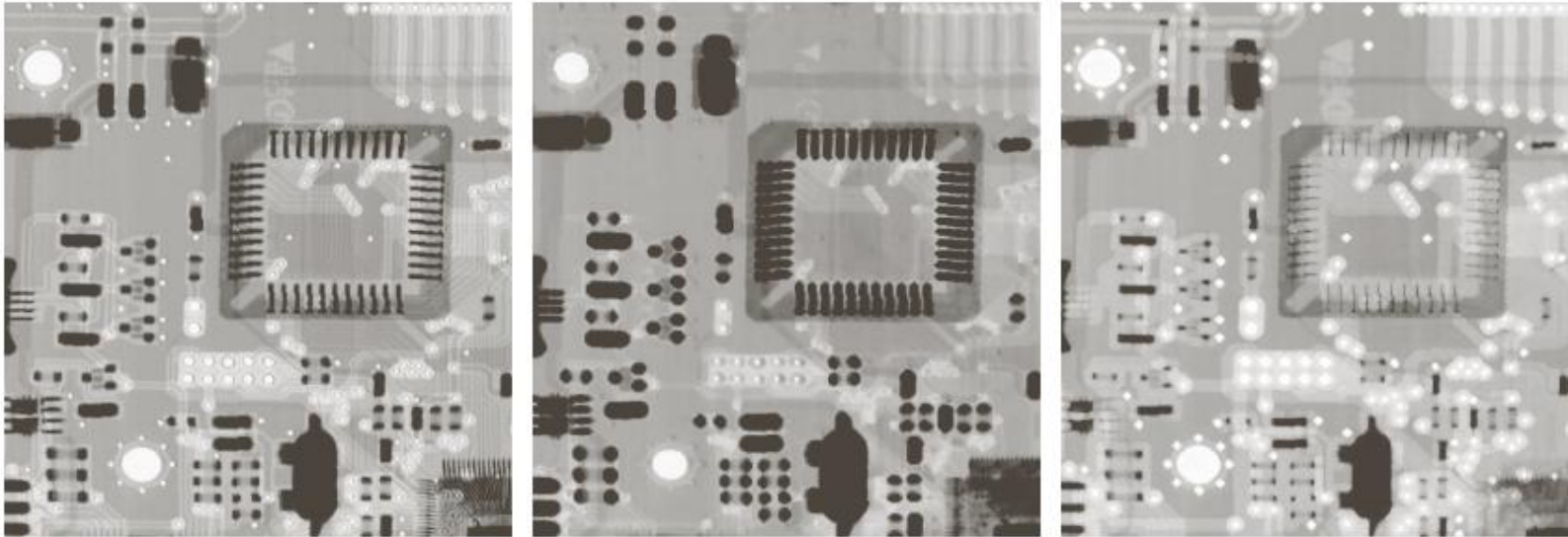


# Digital Image Processing

## Lecture # 8

### Gray Scale Image Morphology & Segmentation

# Dilation & Erosion

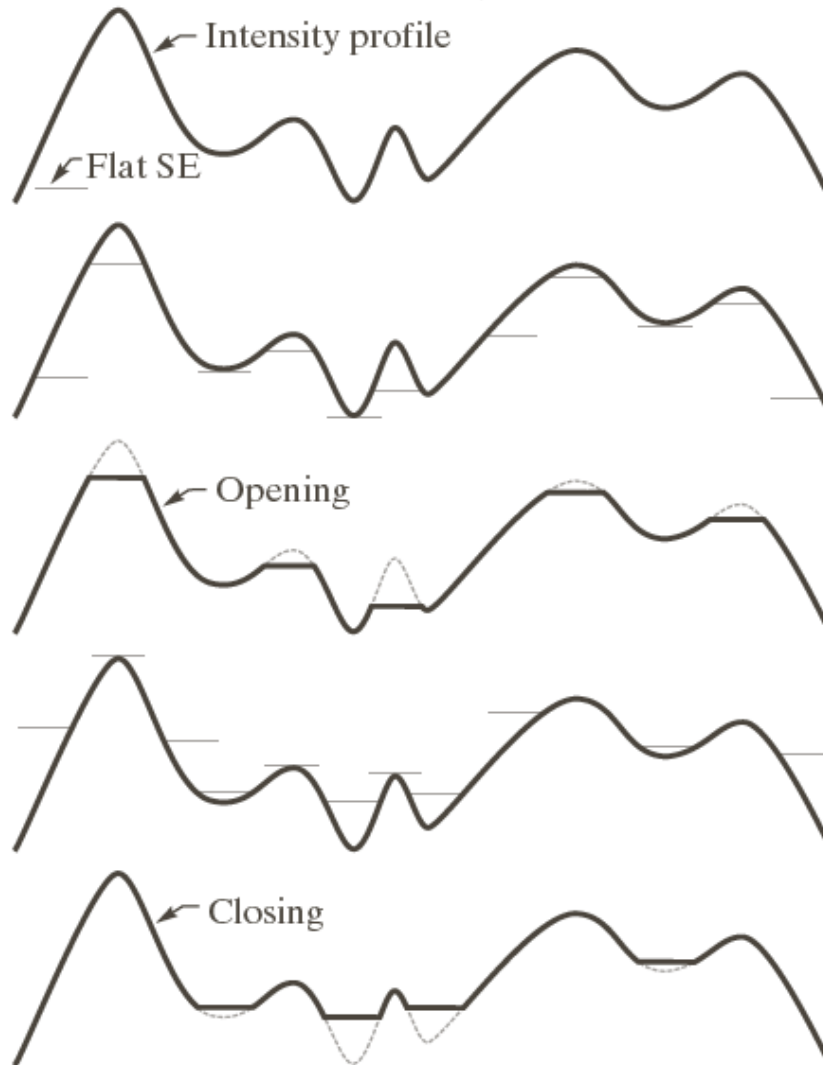


a b c

**FIGURE 9.35** (a) A gray-scale X-ray image of size  $448 \times 425$  pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

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# Opening & Closing

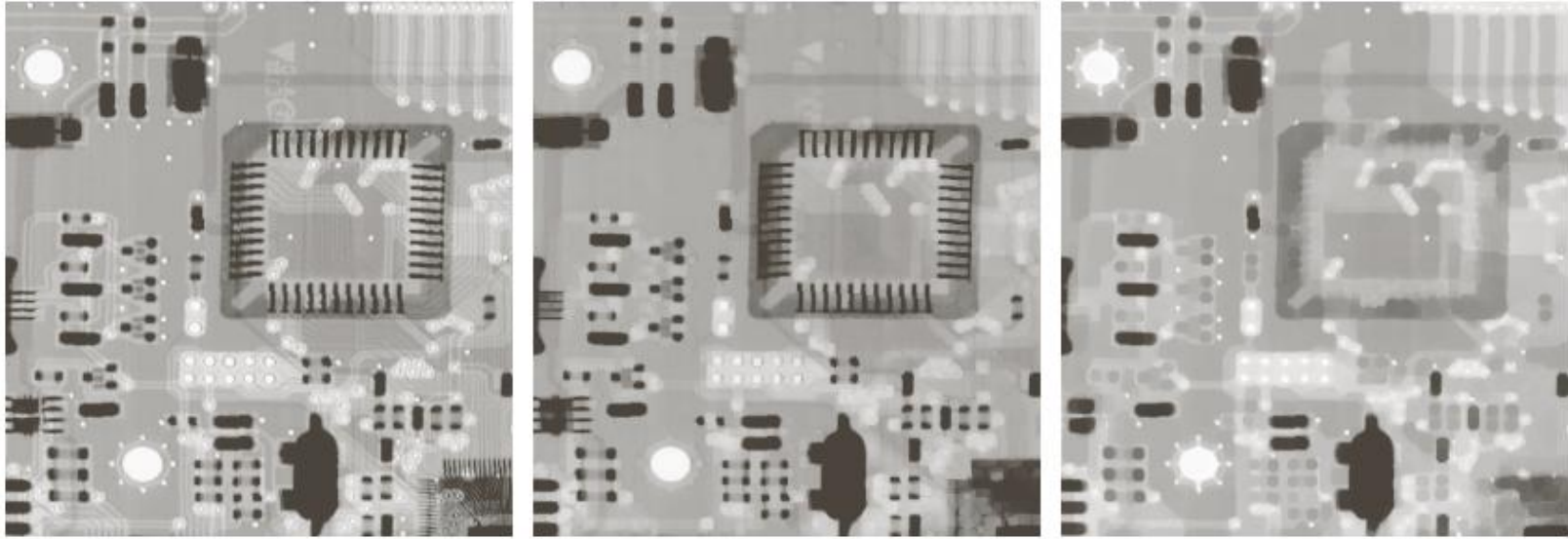


a  
b  
c  
d  
e

**FIGURE 9.36**

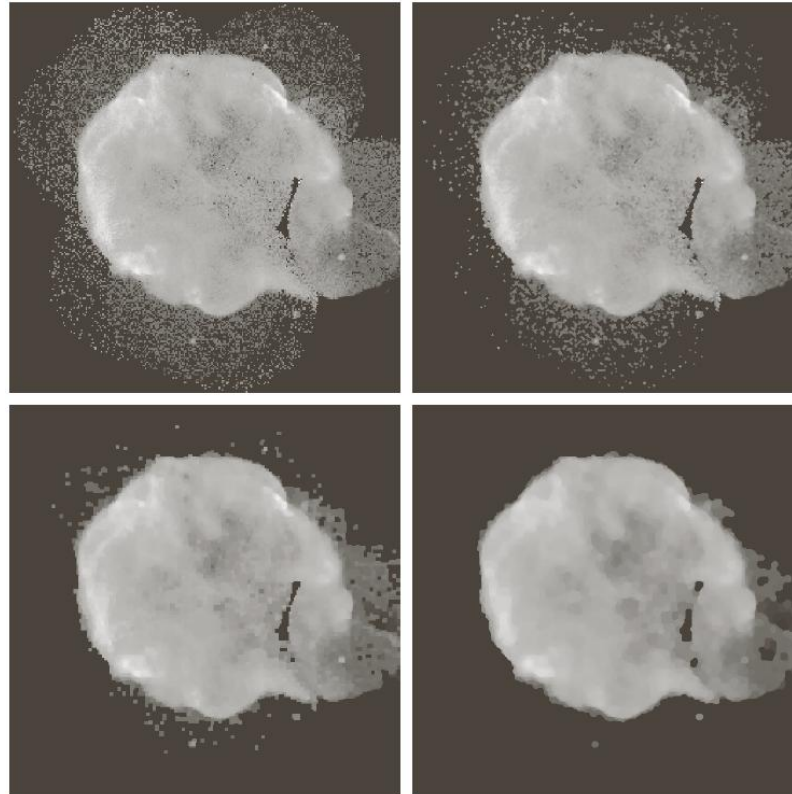
Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

# Opening & Closing



a b c

**FIGURE 9.37** (a) A gray-scale X-ray image of size  $448 \times 425$  pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.



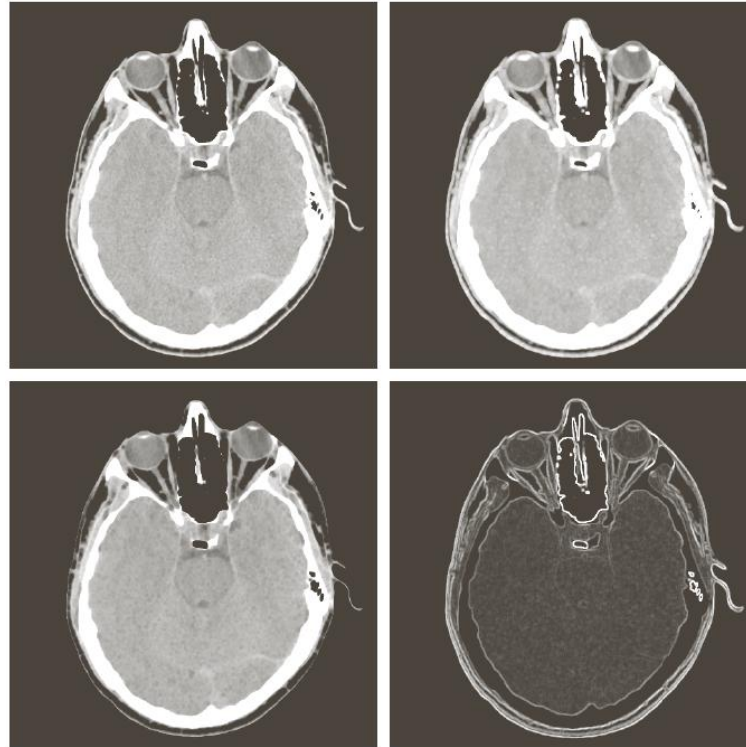
a b  
c d

**FIGURE 9.38**

(a)  $566 \times 566$  image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

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# Morphological Gradient

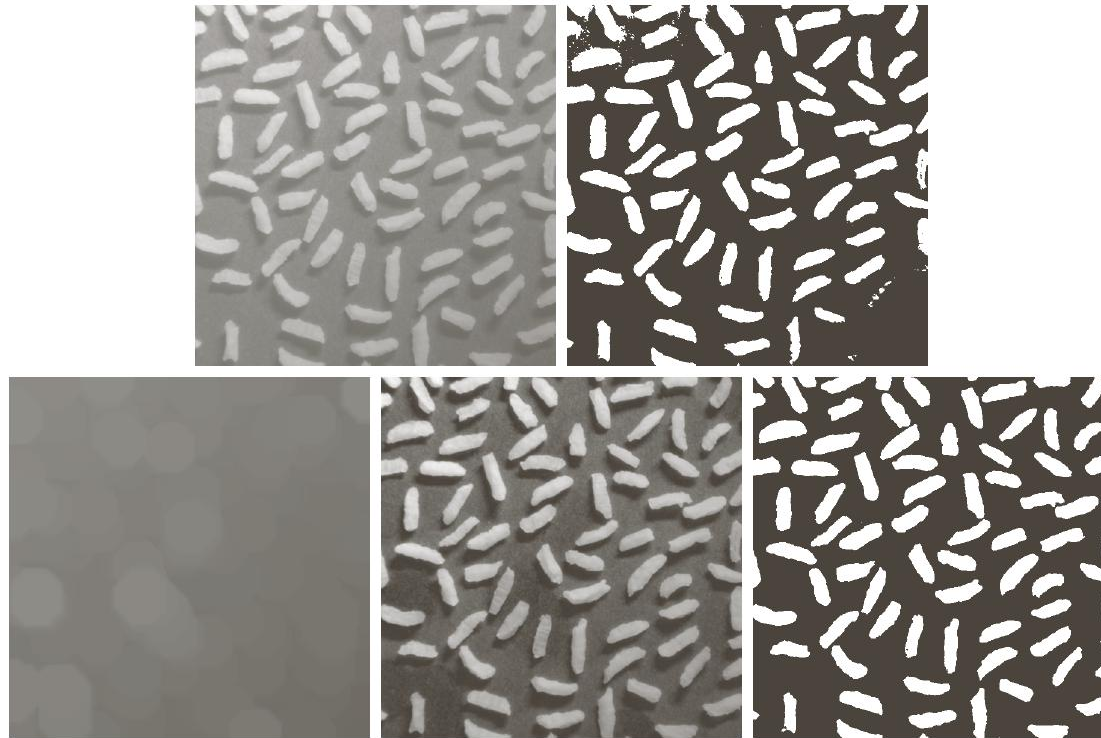


a	b
c	d

**FIGURE 9.39**

(a)  $512 \times 512$  image of a head CT scan.  
(b) Dilation.  
(c) Erosion.  
(d) Morphological gradient, computed as the difference between (b) and (c).  
(Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

# Top Hat & Bottom Hat Transformations



a b  
c d e

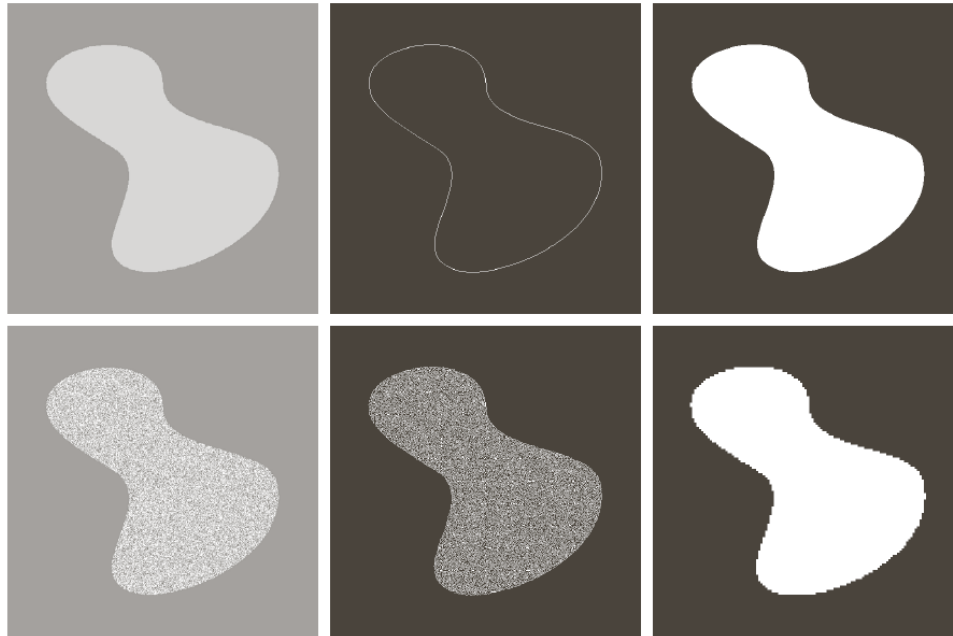
**FIGURE 9.40** Using the top-hat transformation for *shading correction*. (a) Original image of size  $600 \times 600$  pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

# Features Extraction: Edge Detection

# Features detection

- Image Segmentation: Image processing methods whose inputs are images but the outputs are attributes extracted from those images. Segmentation subdivides an image into its constituent regions or objects.
- Edges: Partitioning the image based on abrupt changes in intensity. Assumption is that boundaries of regions are sufficiently different from each other and from the background to allow boundary detection.

## Edge Based Segmentation



a	b	c
d	e	f

**FIGURE 10.1** (a) Image containing a region of constant intensity. (b) Image showing the boundary of the inner region, obtained from intensity discontinuities. (c) Result of segmenting the image into two regions. (d) Image containing a textured region. (e) Result of edge computations. Note the large number of small edges that are connected to the original boundary, making it difficult to find a unique boundary using only edge information. (f) Result of segmentation based on region properties.

# Features detection

- Edge pixels: Pixels at which the intensity of an image changes abruptly and edges are sets of connected edge pixels, e.g., a line may be viewed as edge in which the intensity of the background on either side of the line is either much higher or much lower than the intensity of the line pixels.
- Features can be
  - Edges
  - Corner Points
  - Texture

# Edge Detection

- Edge detectors can be based on the first and second derivatives which can detect abrupt intensity changes.
- Derivates of digital functions are defined in terms of differences
- See approximations on using first and second derivatives in Gonzalez section 10.2.1

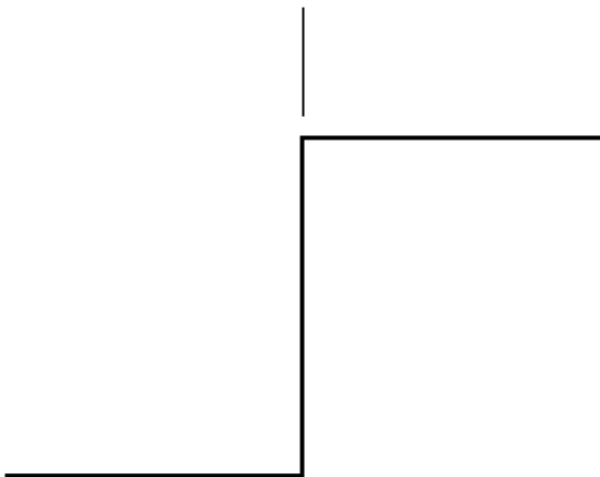


# Edge Detection

- First order derivatives produce **thick edges** while second order derivatives produce finer ones. See **ramp** edge on the figure 10.2 on previous slide
- Second order derivatives **enhance sharp changes** and fine details more aggressively than first order derivatives see the isolated point and the line in the same figure. This can be a problem if the noise is present in the image
- Second derivative changes its **sign as it** transitions into and out of a ramp or step edge. See the step edge. This “double edge” effect can be used to locate edges.
- The sign of the second derivative is also used to determine whether an edge is a transition from **light dark (-ve value)** or from dark to light (+ve value). See step edge

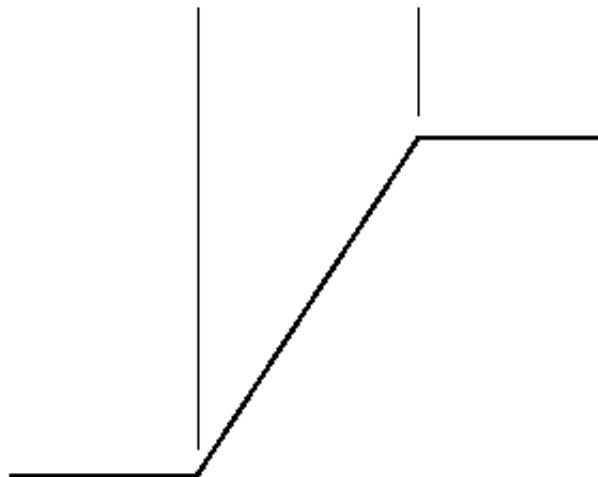
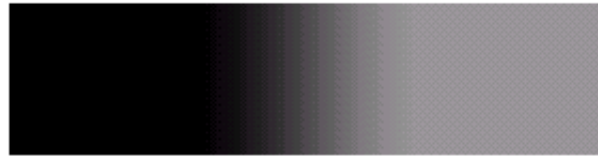
# Edge Detection

Model of an ideal digital edge



Gray-level profile of a horizontal line through the image

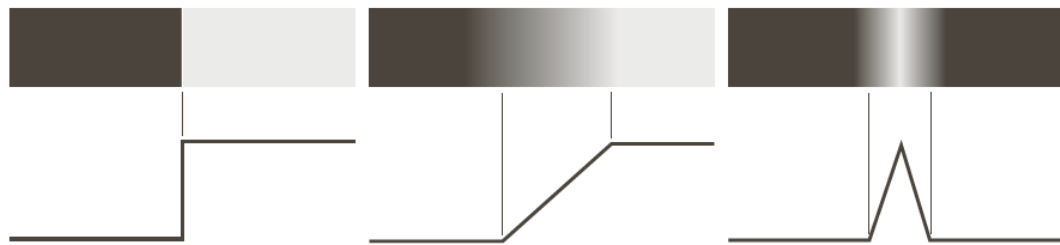
Model of a ramp digital edge



Gray-level profile of a horizontal line through the image

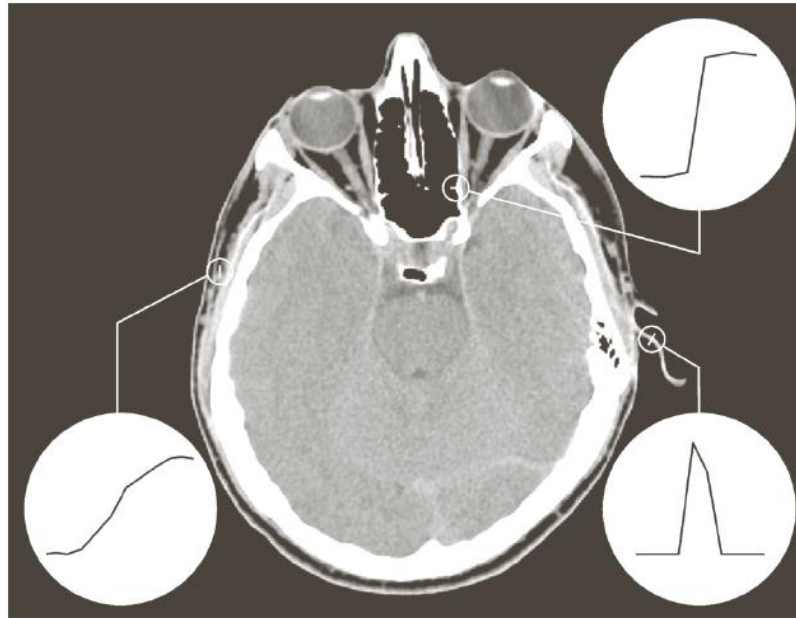
a b

**FIGURE 10.5**  
(a) Model of an ideal digital edge.  
(b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.



a b c

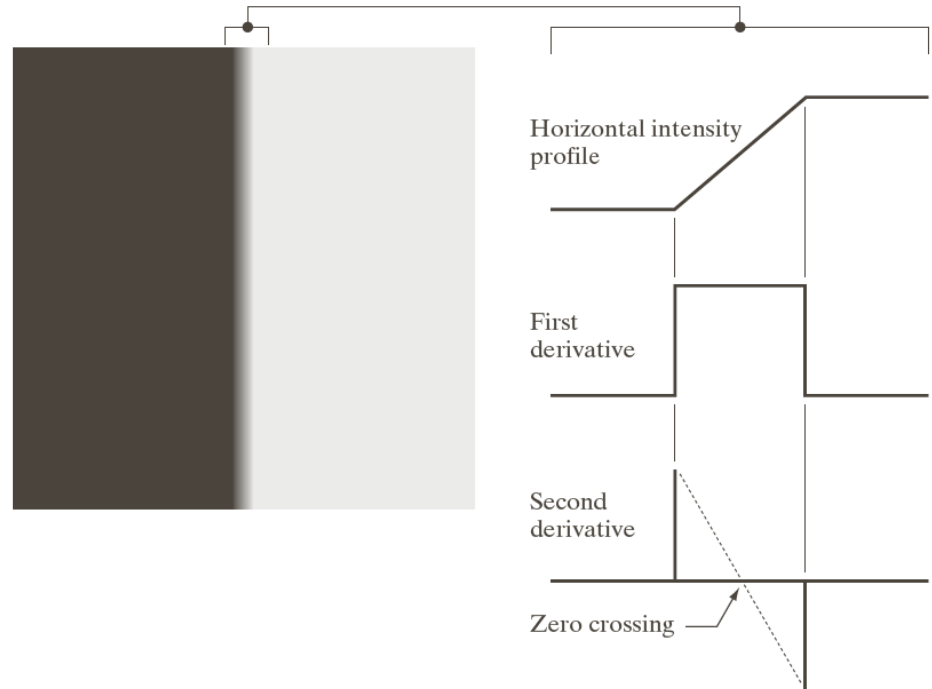
**FIGURE 10.8**  
From left to right,  
models (ideal  
representations) of  
a step, a ramp,  
and a roof edge,  
and their  
corresponding  
intensity profiles.



**FIGURE 10.9** A  $1508 \times 1970$  image showing (zoomed) actual ramp (bottom, left), step (top, right), and roof edge profiles. The profiles are from dark to light, in the areas indicated by the short line segments shown in the small circles. The ramp and “step” profiles span 9 pixels and 2 pixels, respectively. The base of the roof edge is 3 pixels. (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

# Edge Detection

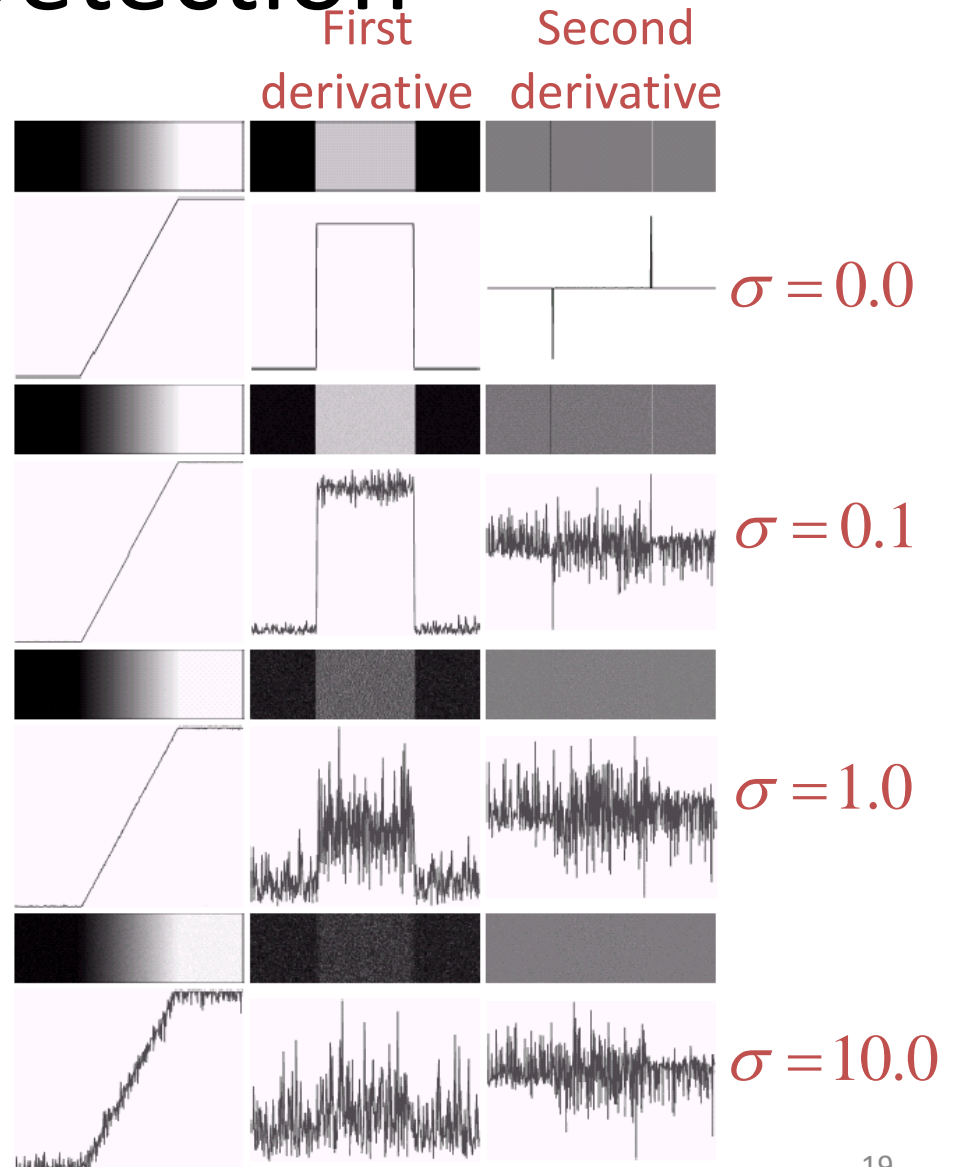
First and second derivative  
for smooth noiseless edge



- The zero crossings of the second derivative can be used for locating edges in an image.

# Edge Detection

Results of first and second derivative for edges with Gaussian noise of mean = 0.



- Smoothing step is a must before taking derivative for edge detection
- Localization of the edge

# Gradient Operators

- Most common differentiation operator is the gradient vector.

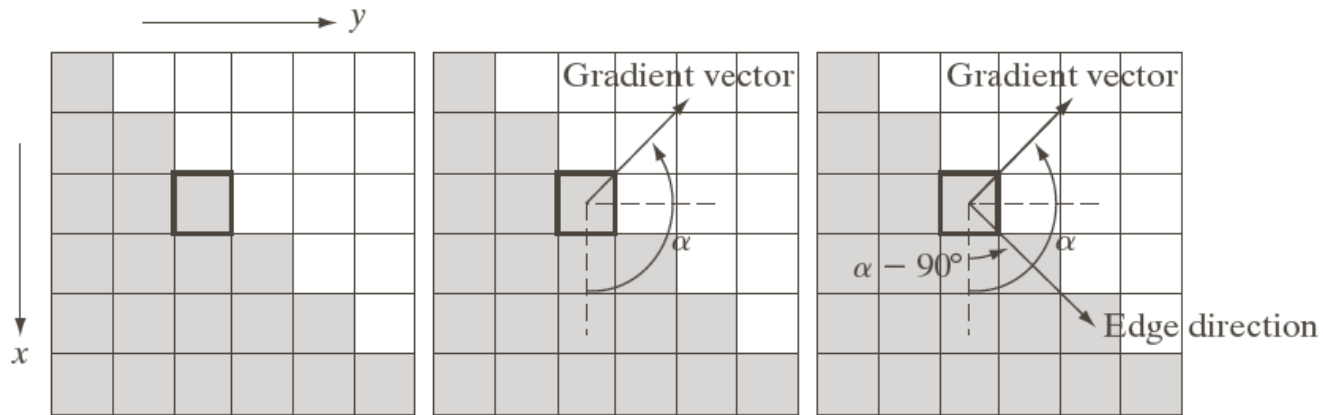
$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} G_x \\ G_y \end{bmatrix}$$

$$|\nabla f(x, y)| = \left[ G_x^2 + G_y^2 \right]^{1/2} \approx |G_x| + |G_y|$$

Magnitude:

$$\angle f(x, y) = \tan^{-1} \left[ \frac{G_y}{G_x} \right]$$

Direction:



a b c

**FIGURE 10.12** Using the gradient to determine edge strength and direction at a point. Note that the edge is perpendicular to the direction of the gradient vector at the point where the gradient is computed. Each square in the figure represents one pixel.

The direction of an edge at a point is orthogonal to the direction of the gradient vector at the point

# Gradient Operators

Some common gradient operators

- Roberts and Prewitt masks are the simplest but not robust against noise
- Sobel edge detection masks are the most common and give satisfactory results in presence of noise

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

-1	0	0	-1
0	1	1	0

Roberts

-1	-1	-1	-1	0	1
0	0	0	-1	0	1
1	1	1	-1	0	1

Prewitt

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

Sobel

$z_1$	$z_2$	$z_3$
$z_4$	$z_5$	$z_6$
$z_7$	$z_8$	$z_9$

a	
b	c
d	e
f	g

**FIGURE 10.14**  
 A  $3 \times 3$  region of an image (the  $z$ 's are intensity values) and various masks used to compute the gradient at the point labeled  $z_5$ .

-1	0	0	-1
0	1	1	0

Roberts

-1	-1	-1	-1	0	1
0	0	0	-1	0	1
1	1	1	-1	0	1

Prewitt

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

Sobel

0	1	1	-1	-1	0
-1	0	1	-1	0	1
-1	-1	0	0	1	1

a	b
c	d

**FIGURE 10.15**  
Prewitt and Sobel masks for detecting diagonal edges.

Prewitt

0	1	2	-2	-1	0
-1	0	1	-1	0	1
-2	-1	0	0	1	2

Sobel



a b  
c d

**FIGURE 10.16**

(a) Original image of size  $834 \times 1114$  pixels, with intensity values scaled to the range  $[0, 1]$ .  
(b)  $|g_x|$ , the component of the gradient in the  $x$ -direction, obtained using the Sobel mask in Fig. 10.14(f) to filter the image.  
(c)  $|g_y|$ , obtained using the mask in Fig. 10.14(g).  
(d) The gradient image,  $|g_x| + |g_y|$ .



a	b
c	d

**FIGURE 10.18**  
Same sequence as in Fig. 10.16, but with the original image smoothed using a  $5 \times 5$  averaging filter prior to edge detection.

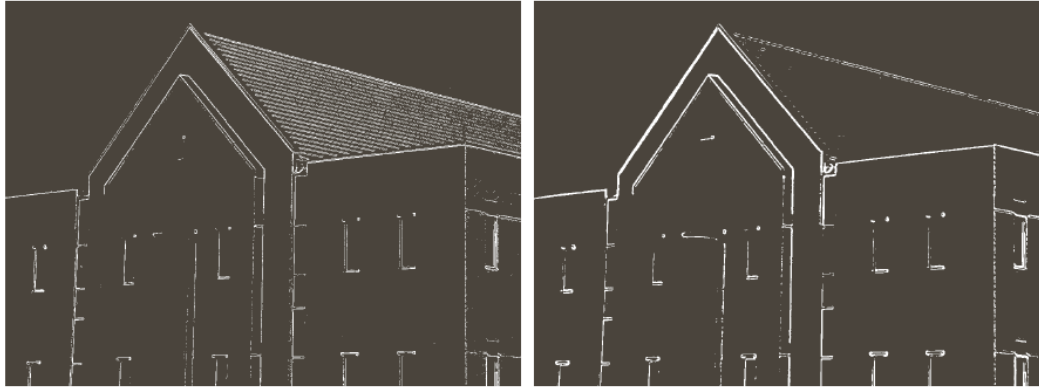
---



a b

**FIGURE 10.19**  
Diagonal edge detection.

(a) Result of using the mask in Fig. 10.15(c).  
(b) Result of using the mask in Fig. 10.15(d). The input image in both cases was Fig. 10.18(a).



a b

**FIGURE 10.20** (a) Thresholded version of the image in Fig. 10.16(d), with the threshold selected as 33% of the highest value in the image; this threshold was just high enough to eliminate most of the brick edges in the gradient image. (b) Thresholded version of the image in Fig. 10.18(d), obtained using a threshold equal to 33% of the highest value in that image.

---

# Marr-Hildreth Edge Detector

- The filter should be tunable so that large filters can be used to detect blurry edges and small operators to detect sharply focused fine detail

# Laplacian of a Gaussian (LoG)

- A filter which combines the smoothing function (Gaussian) with the Laplacian is called Laplacian of a Gaussian (LoG) filter.
- Robust against noise.
- Consider a smoothing Gaussian function:

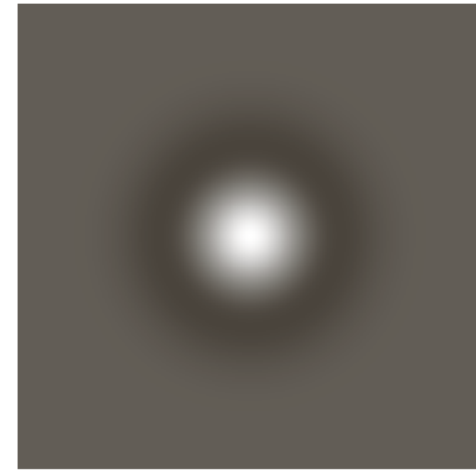
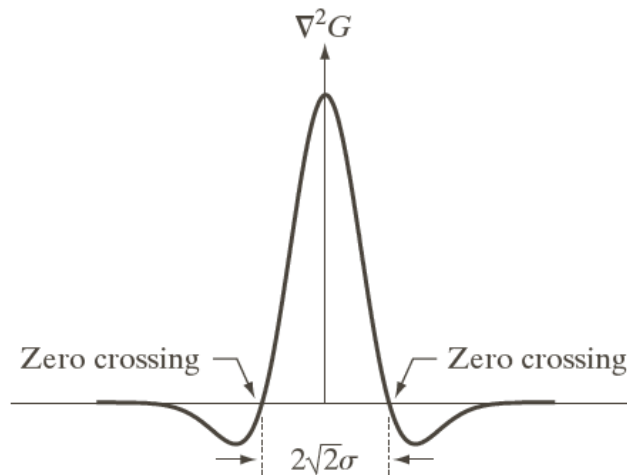
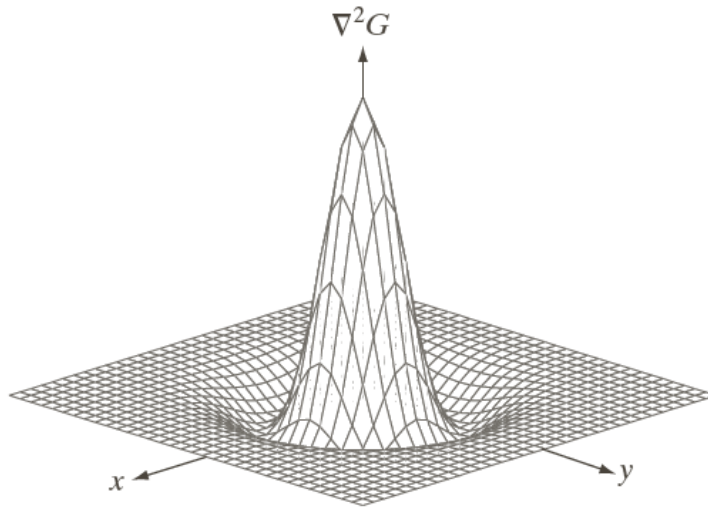
where  $r^2 = x^2 + y^2$ ,  $\sigma$ : standard deviation

$$G(r) = e^{-\frac{r^2}{2\sigma^2}}$$

- The Laplacian of this function gives the LoG function:

$$\nabla^2 G(r) = \frac{\partial^2 G(r)}{\partial r^2} = \left[ \frac{r^2 - 2\sigma^2}{\sigma^4} \right] e^{-\frac{r^2}{2\sigma^2}}$$

# Laplacian of a Gaussian (LoG) Filter



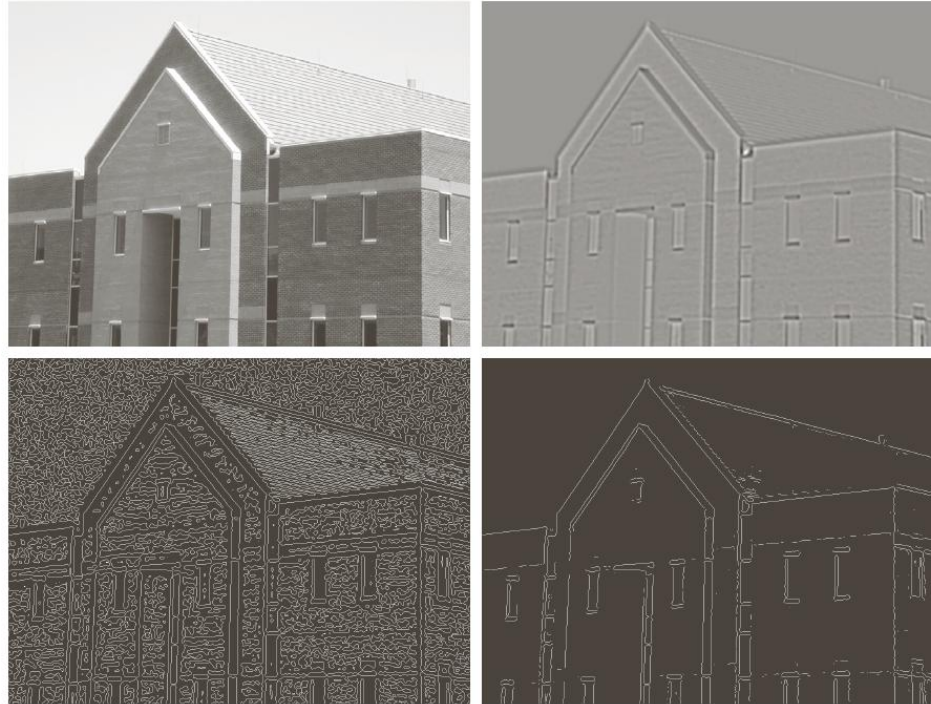
a b  
c d

**FIGURE 10.21**  
 (a) Three-dimensional plot of the *negative* of the LoG. (b) Negative of the LoG displayed as an image. (c) Cross section of (a) showing zero crossings. (d)  $5 \times 5$  mask approximation to the shape in (a). The negative of this mask would be used in practice.

0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

# Edge detection by LoG

- Due to the shape of this function it is also called Mexican hat function (or Mexican hat filters).
- Better performance against noise. Reduces the intensity of structures or noise, which are at scales much smaller than sigma.
- Choose size of Gaussian mask to be  $n \geq 6 * \text{sigma}$
- Then use a 3x3 Laplacian
- Find the zero crossings



a	b
c	d

**FIGURE 10.22**

(a) Original image of size  $834 \times 1114$  pixels, with intensity values scaled to the range  $[0, 1]$ . (b) Results of Steps 1 and 2 of the Marr-Hildreth algorithm using  $\sigma = 4$  and  $n = 25$ . (c) Zero crossings of (b) using a threshold of 0 (note the closed-loop edges). (d) Zero crossings found using a threshold equal to 4% of the maximum value of the image in (b). Note the thin edges.