

Lecture 15

**Color Processing,
Compression**

COLOR IMAGE PROCESSING

COLOR IMAGE PROCESSING

✚ Color Importance

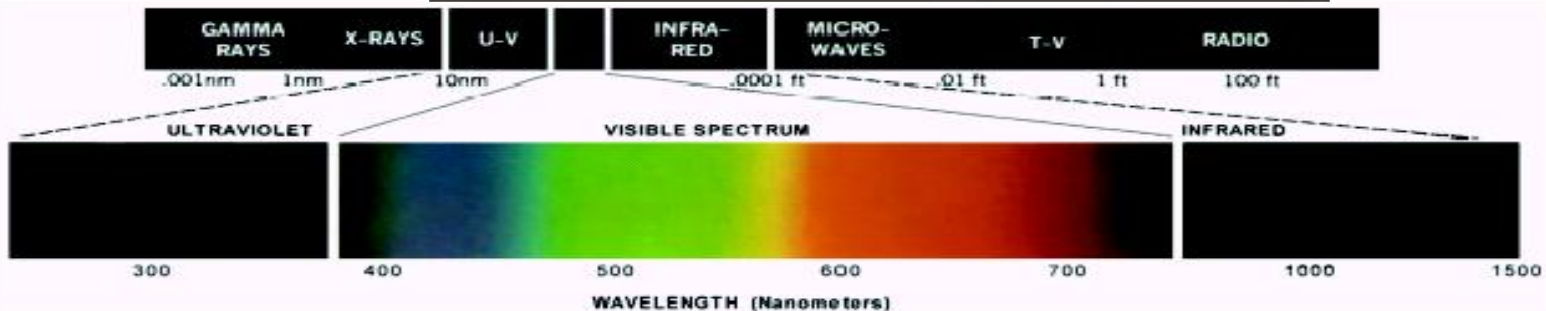
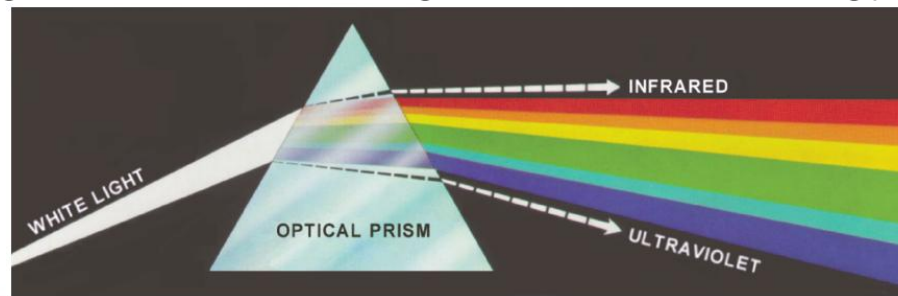
- Color is an excellent descriptor
 - Suitable for object Identification and Extraction
- Discrimination
 - Humans can distinguish thousands of color shades and intensities but few shades of gray levels

✚ Color Image Processing

- Full-Color Processing
 - Color is acquired with a full-color sensor
- Pseudo-Color Processing
 - Assigning colors to monochrome images

COLOR FUNDAMENTALS

- ✚ Colors that humans perceive in an object are determined by the nature of the light reflected from the object
- ✚ Visible light is composed of a relatively narrow band of frequencies in the electromagnetic spectrum
- ✚ A body that reflects light that is balanced in all visible wavelengths appears white to the observer
- ✚ A body that favours reflectance in a limited range of the visible spectrum exhibits some shades of color
- ✚ Green objects reflect light with wavelengths primarily in the 500 to 570 nm range while absorbing most of the energy at other wavelengths



HUMAN PERCEPTION OF COLOR

+ Retina contains receptors

- Cones

- Day vision, can perceive color tone

- Red, green, and blue cones

- Rods

- Night vision, perceive brightness only

+ Color sensation

- Luminance (brightness)

- Chrominance

- Hue (color tone)

- Saturation (color purity)

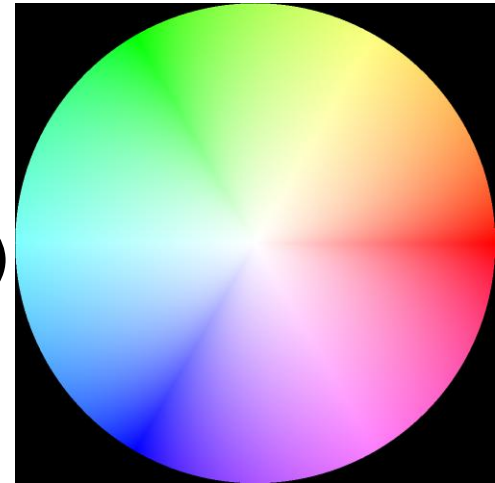
Monochromatic images

- ✚ Image processing - static images -
- ✚ Monochromatic static image - continuous image function $f(x,y)$
 - arguments - two co-ordinates (x,y)
- ✚ Digital image functions - represented by matrices
 - co-ordinates = integer numbers
 - Cartesian (horizontal x axis, vertical y axis)
 - OR (row, column) matrices
- ✚ Monochromatic image function range
 - lowest value - black
 - highest value - white
- ✚ Limited brightness values = gray levels

Chromatic images

Colour

- Represented by vector not scalar
 - Red, Green, Blue (RGB)
 - Hue, Saturation, Value (HSV)
 - luminance, chrominance (Yuv , Luv)



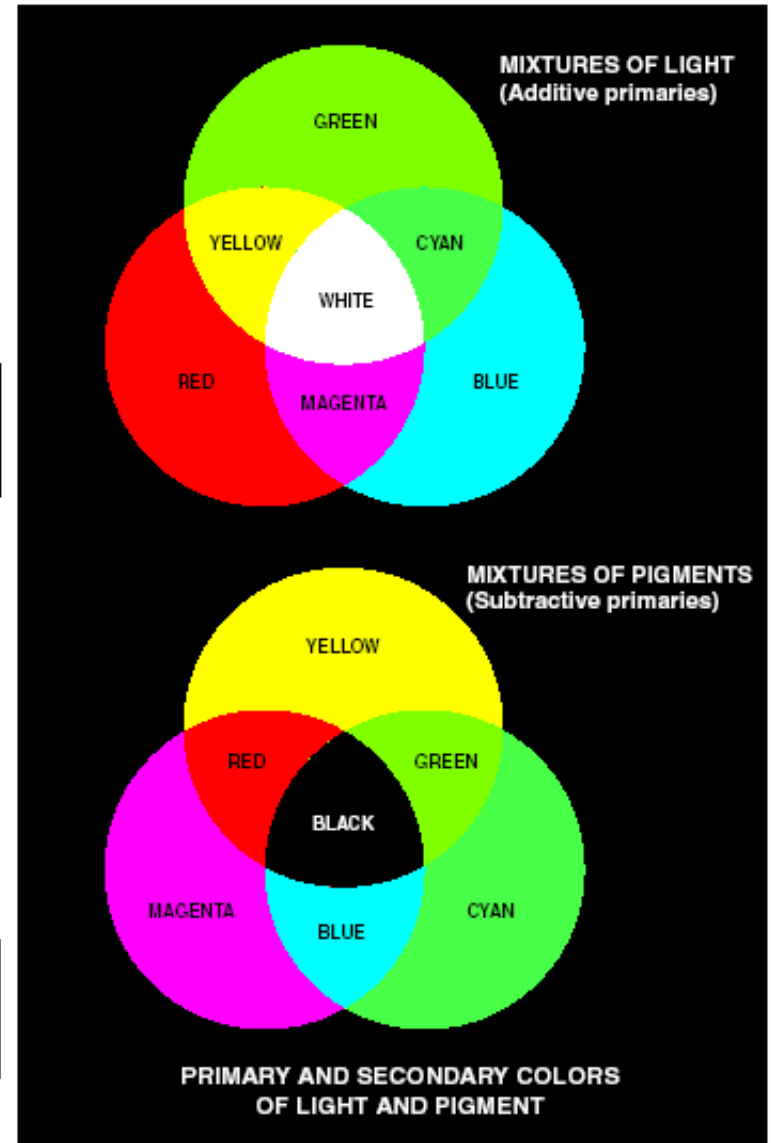
PRIMARY AND SECONDARY COLORS OF LIGHT AND PIGMENTS

- ✚ The primary colors can be added to produce the secondary colors of light
- ✚ The primary colors of light and primary colors of pigments are different

Magenta = Red + Blue
Cyan = Blue + Green
Yellow = Green + Red

- ✚ For pigments, a primary color is defined as one that absorbs a primary color of light and reflects the other two
- ✚ Therefore, the primary colors of pigments are magenta, cyan, and yellow

Magenta = White - Green
Cyan = White - Red
Yellow = White - Blue



COLOR MODELS

Color Model

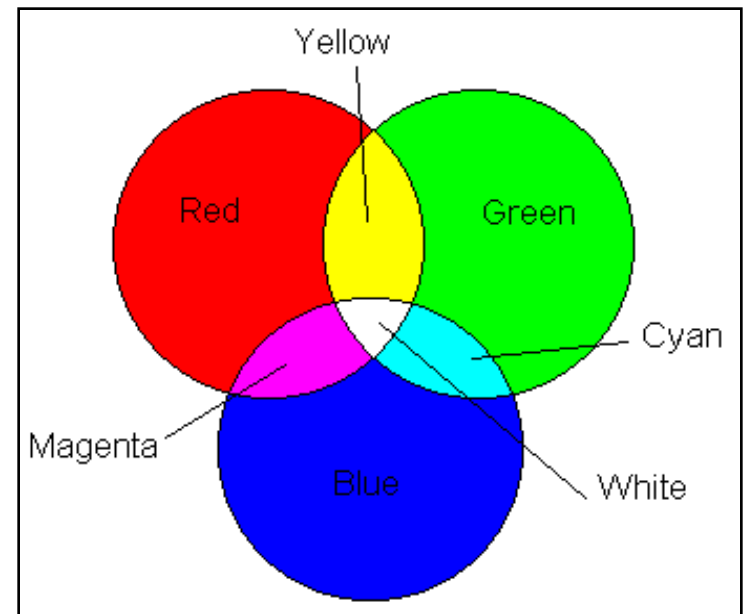
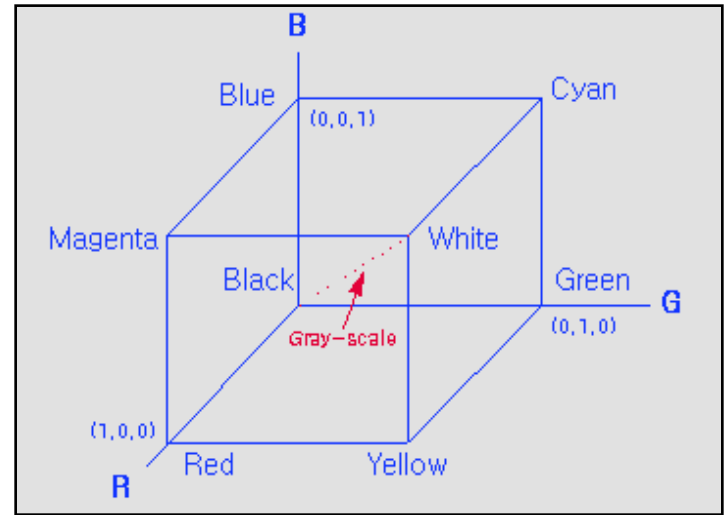
- Specify colors in a standard way
- A coordinate system that each color is represented by a single point.

Most used models:

- RGB model (Monitor/TV)
- CMY model (3-color Printers)
- HSI model (Color Image Processing and Description)

RGB COLOR MODEL

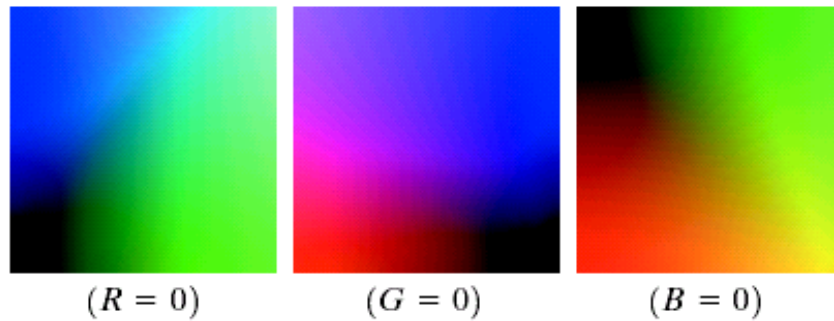
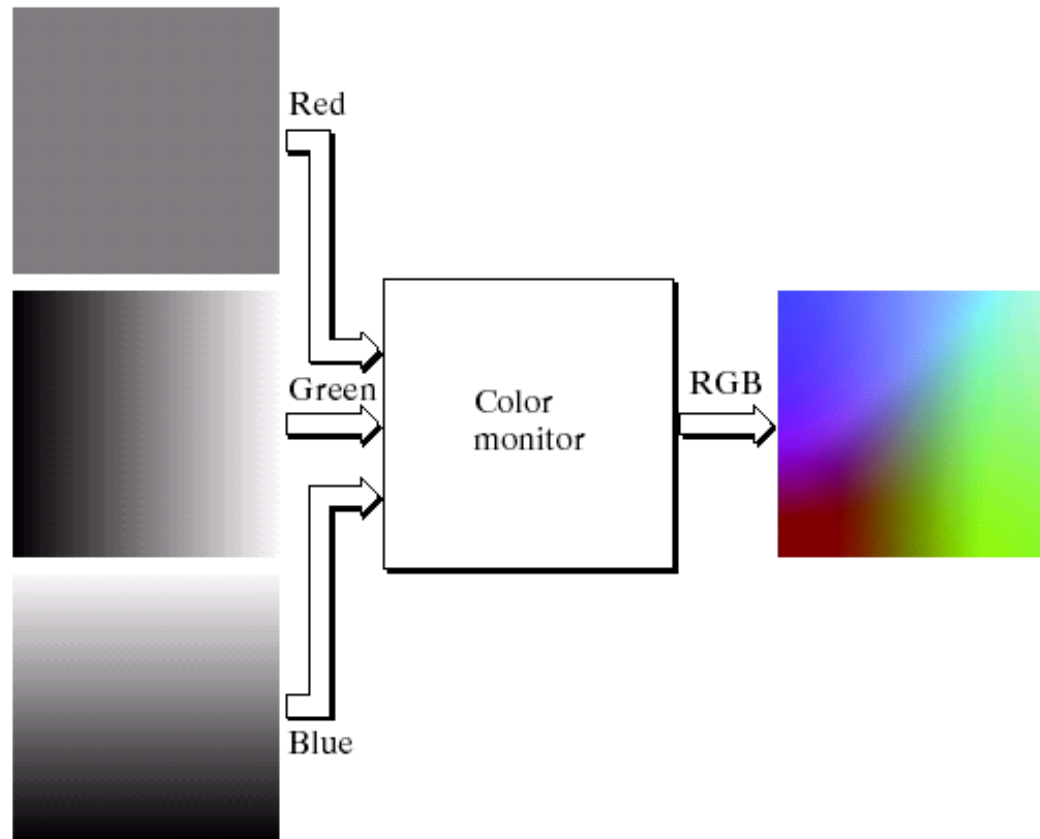
- ✚ **Pixel Depth:** The number of bits used to represent each pixel in RGB space.
- ✚ **Full-color image:** 24-bit RGB color image.
 - (R, G, B) = (8 bits, 8 bits, 8 bits)
 - Number of colors:
 $(2^8)^3 = 16,777,216$



a
b

FIGURE 6.9

(a) Generating the RGB image of the cross-sectional color plane ($127, G, B$).
(b) The three hidden surface planes in the color cube of Fig. 6.8.



COLOR IMAGE - RGB

Color Image



a b
c d

FIGURE 6.38
(a) RGB image.
(b) Red component image.
(c) Green component.
(d) Blue component.

R-Channel

G-Channel



B-Channel

CMY Model

- **Color Printer, Color Copier**
- **RGB data to CMY**

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

HSI COLOR MODEL

Human description of color is Hue, Saturation and Brightness:

Hue

- represents dominant color as perceived by an observer. It is an attribute associated with the dominant wavelength.

Saturation

- refers to the relative purity or the amount of white light mixed with a hue. The pure spectrum colors are fully saturated.
 - Pure colors are fully saturated.
 - Pink is less saturated.

Intensity

- reflects the brightness.






HSI Color Model

✚ The HSI model uses three measures to describe colors:

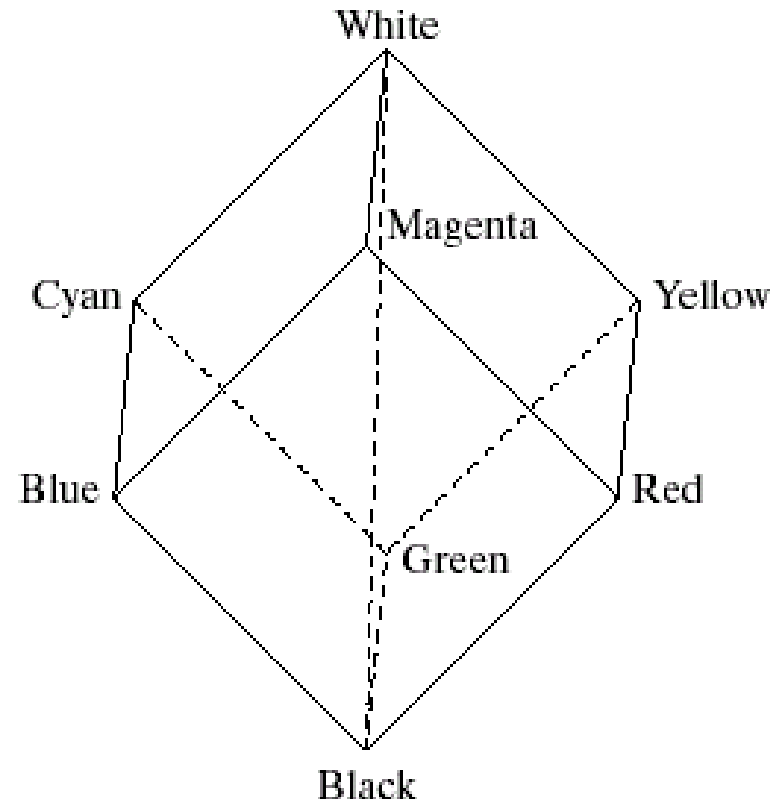
- **Hue:** A color attribute that describes a pure color (pure yellow, orange or red)
- **Saturation:** Gives a measure of how much a pure color is diluted with white light
- **Intensity:** Intensity is the same achromatic notion that we have seen in grey level images

HSI Color Model

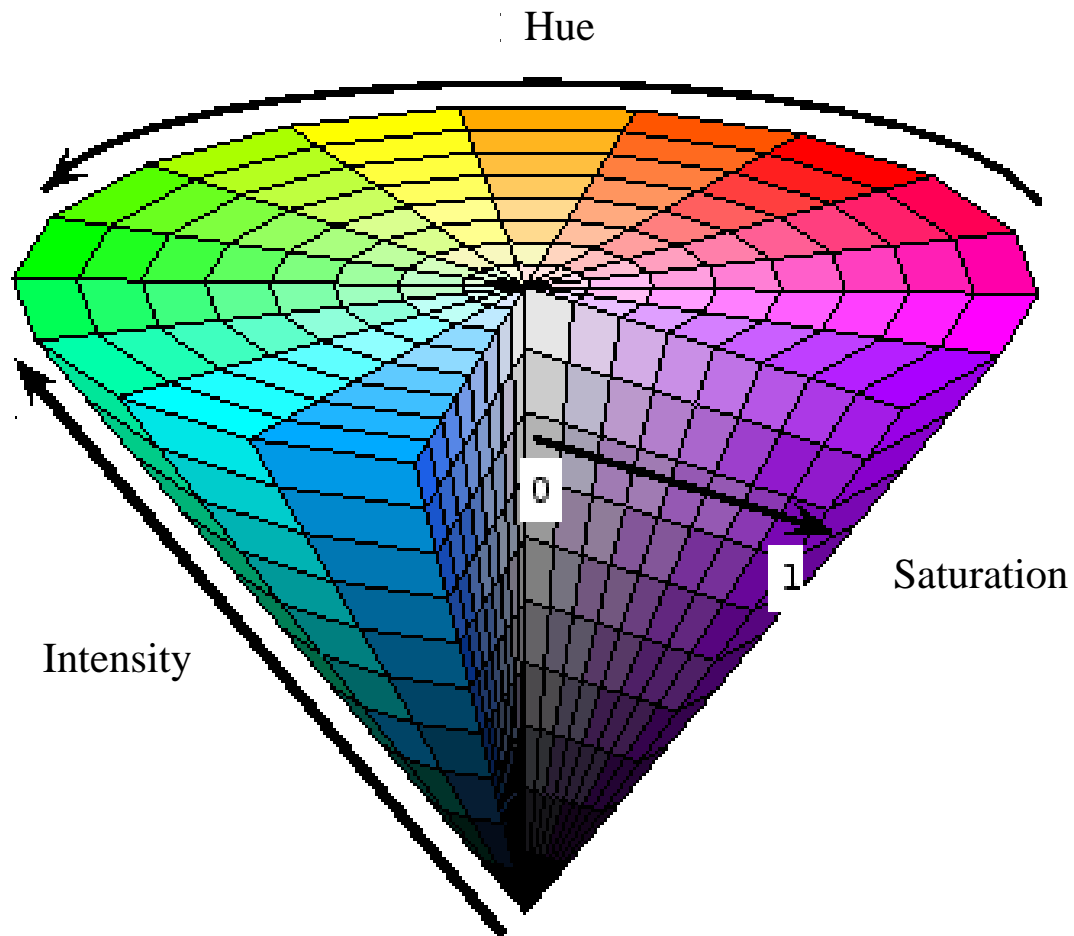
-  Intensity can be extracted from RGB images
-  Remember the diagonal on the RGB color cube that we saw previously ran from black to white
-  Now consider if we stand this cube on the black vertex and position the white vertex directly above it

HSI Color Model

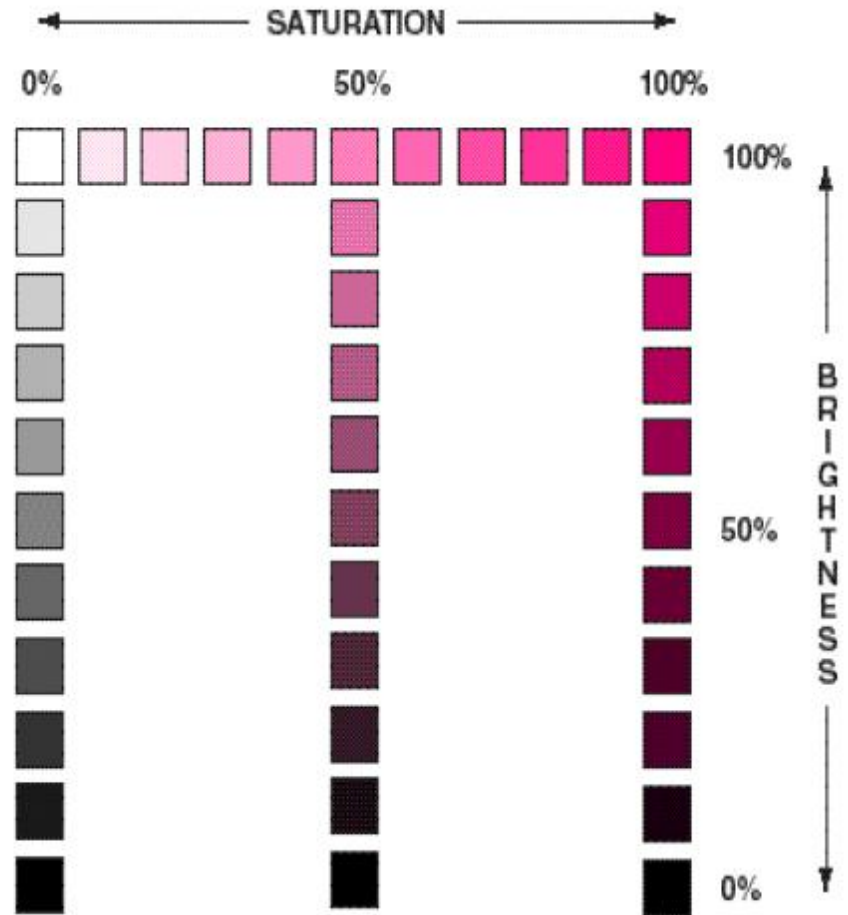
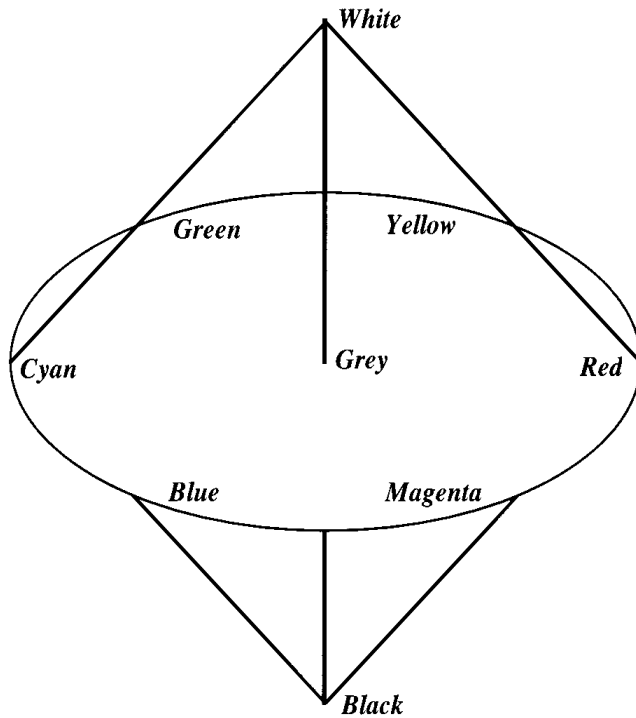
- ✚ The intensity component of any color can be determined by passing a plane *perpendicular* to the intensity axis and containing the color point
- ✚ The intersection of the plane with the intensity axis gives us the intensity component of the color



HSI Color Model

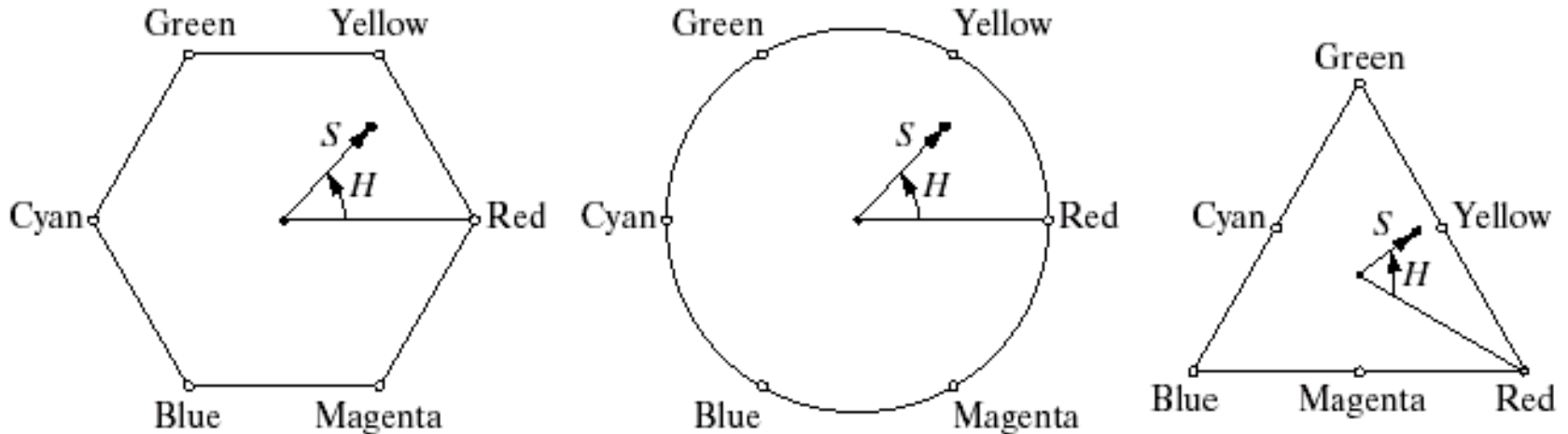


HSI COLOR MODEL- SINGLE HUE

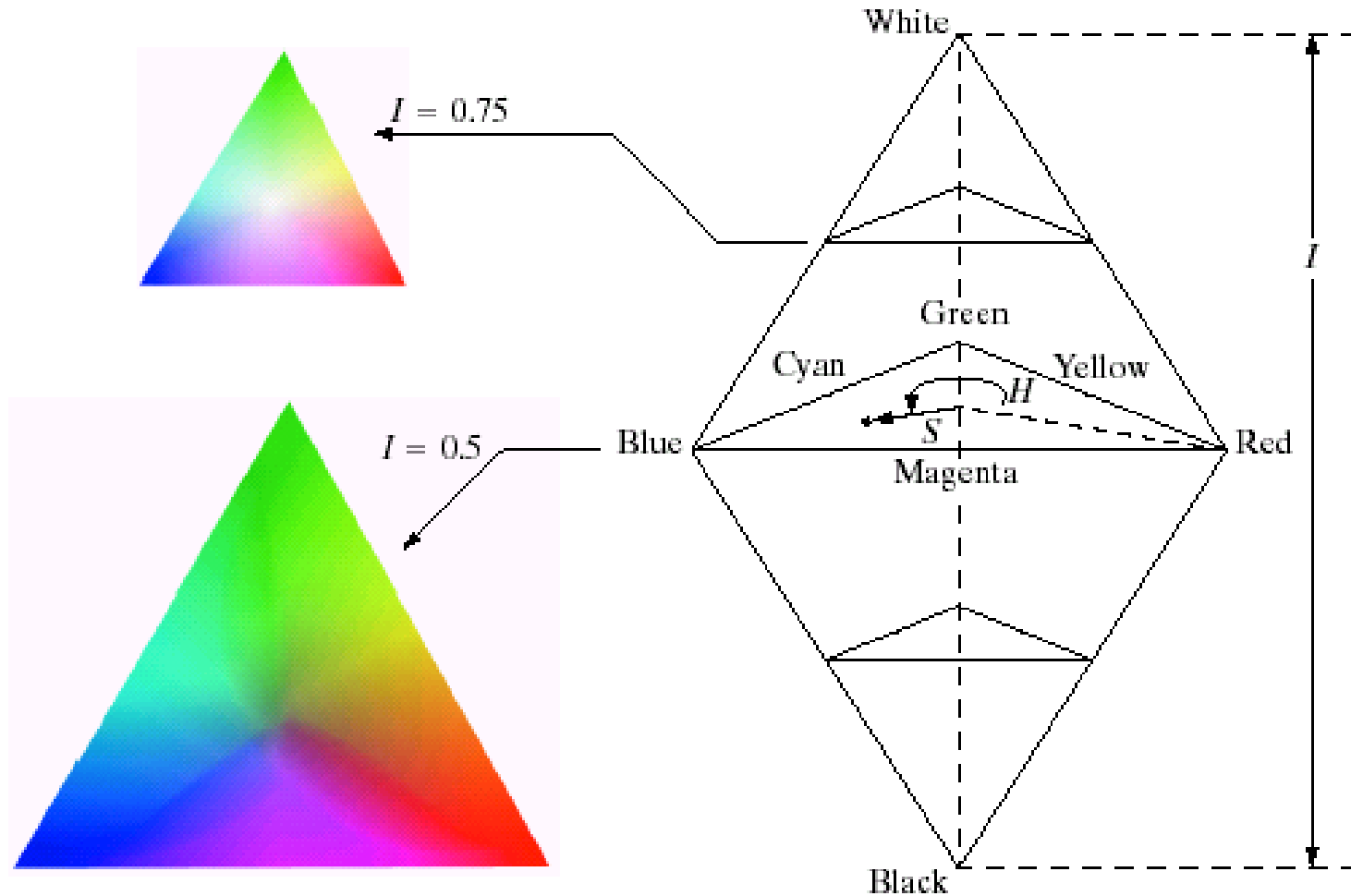


HSI Color Model

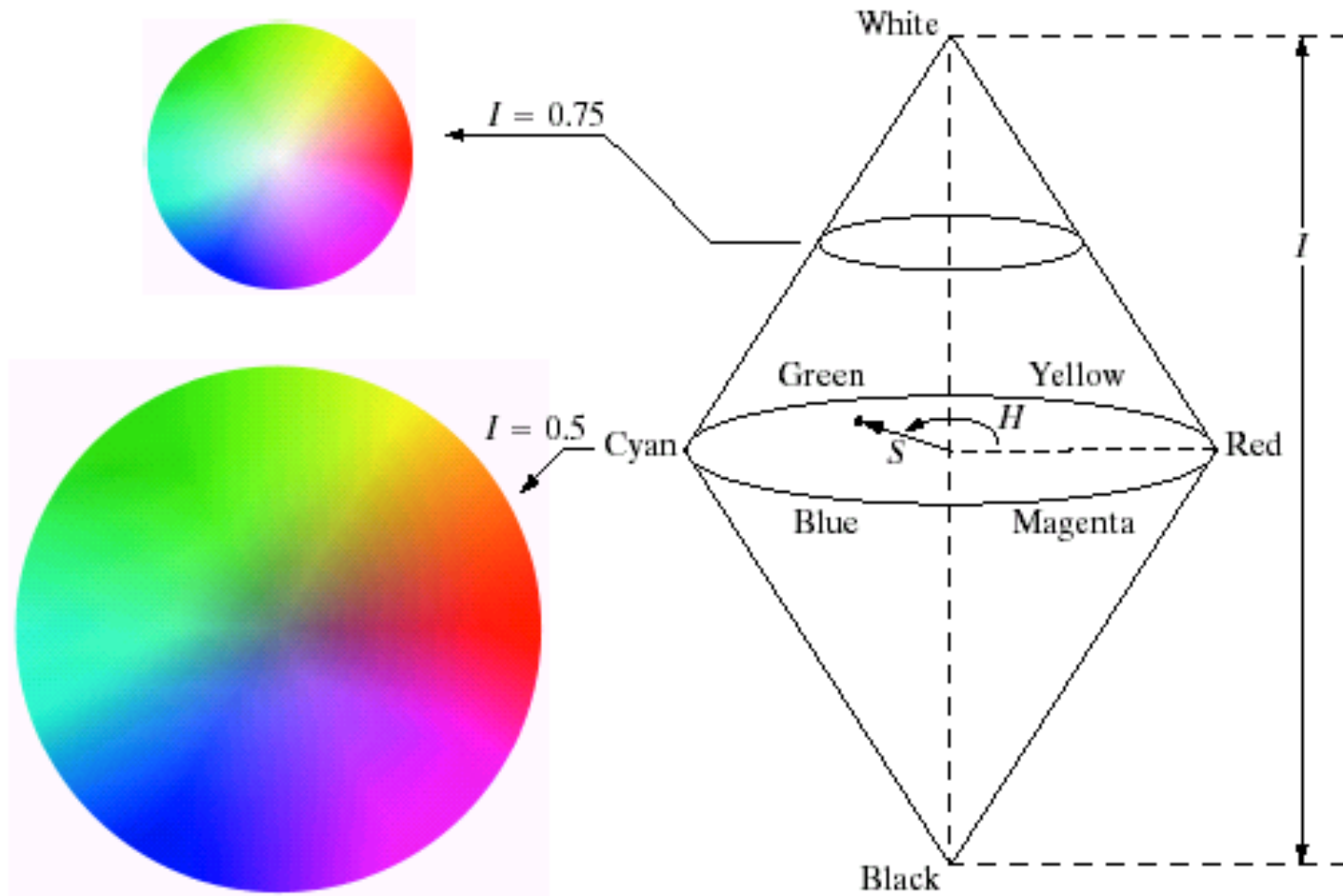
Because the only important things are the angle and the length of the saturation vector this plane is also often represented as a circle or a triangle



HSI Color Model



HSI Color Model



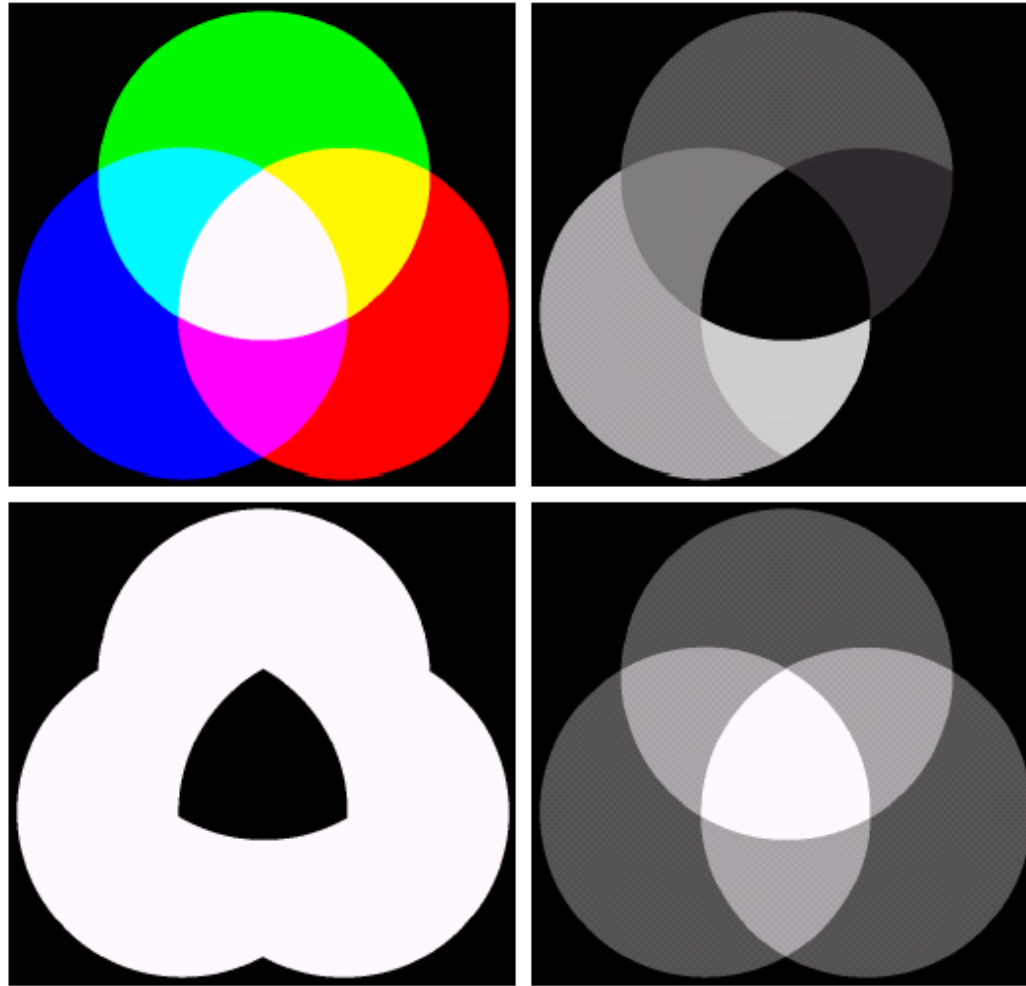
Converting from RGB to HSI

$$\theta = \cos^{-1} \left\{ \frac{\frac{1}{2} [(R - G) + (R - B)]}{\left[(R - G)^2 + (R - B)(G - B) \right]^{\frac{1}{2}}} \right\}$$

$$H = \begin{cases} \theta & \text{if } B \leq G \\ 360 - \theta & \text{if } B > G \end{cases}$$

$$S = 1 - \frac{3}{(R + G + B)} [\min(R, G, B)]$$

$$I = \frac{1}{3} (R + G + B)$$



a b
c d

FIGURE 6.16 (a) RGB image and the components of its corresponding HSI image: (b) hue, (c) saturation, and (d) intensity.

COLOR IMAGE - HSI

H-Channel

S-Channel

I-Channel

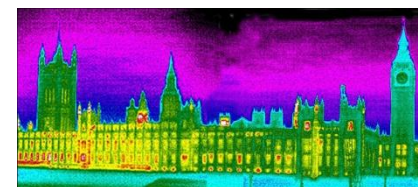
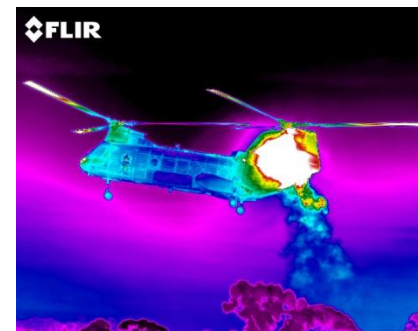
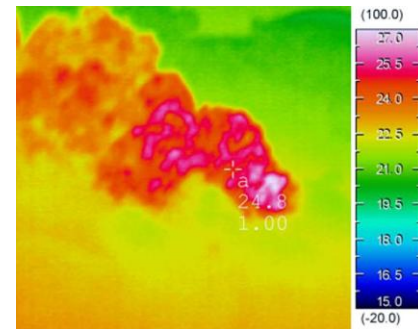


a b c

FIGURE 6.39 HSI components of the RGB color image in Fig. 6.38(a). (a) Hue. (b) Saturation. (c) Intensity.

Pseudocolor Image Processing

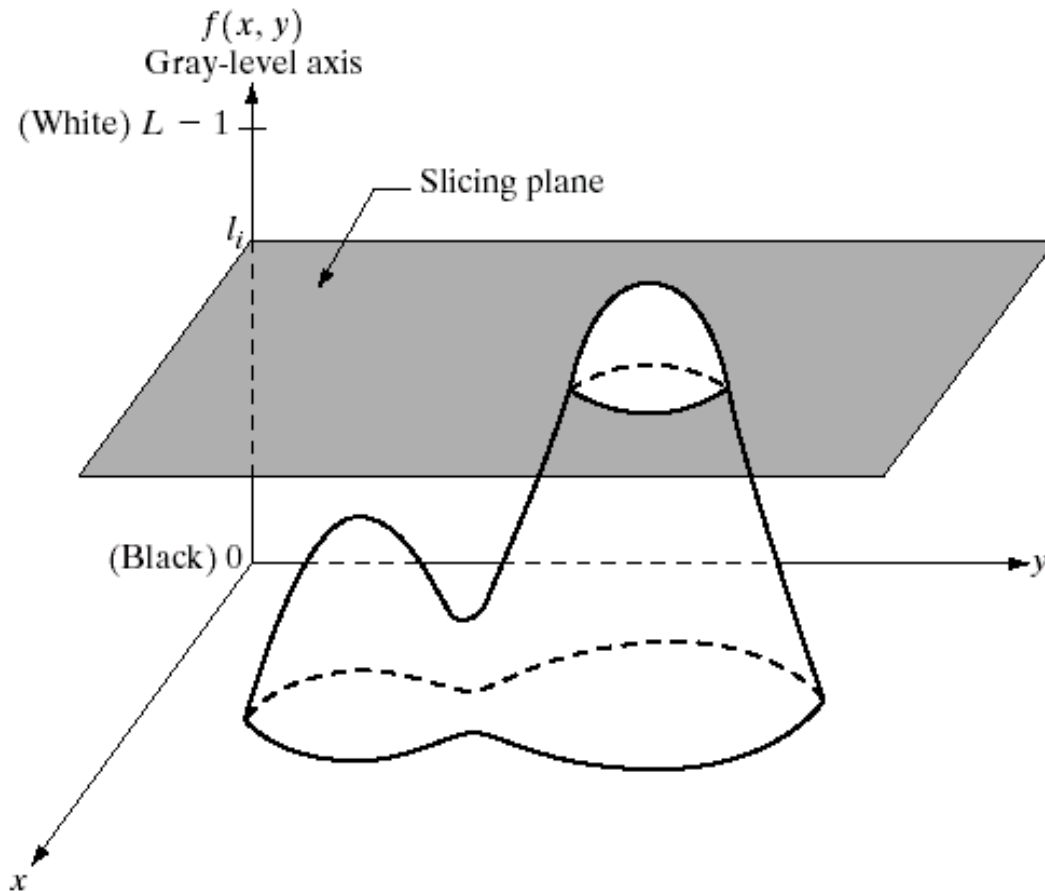
- ✚ Pseudocolor (also called false color) image processing consists of assigning colors to grey values based on a specific criterion
- ✚ The principle use of pseudocolor image processing is for human visualization



Pseudo Color Image Processing – Intensity Slicing

- ✚ Intensity slicing and color coding is one of the simplest kinds of pseudocolor image processing
- ✚ First we consider an image as a 3D function mapping spatial coordinates to intensities (that we can consider heights)
- ✚ Now consider placing planes at certain levels parallel to the coordinate plane
- ✚ If a value is one side of such a plane it is rendered in one color, and a different color if on the other side

Pseudo Color Image Processing – Intensity Slicing



Pseudo Color Image Processing – Intensity Slicing

- ✚ In general intensity slicing can be summarized as:
 - Let $[0, L-1]$ represent the grey scale
 - Let I_0 represent black [$f(x, y) = 0$] and let I_{L-1} represent white [$f(x, y) = L-1$]
 - Suppose P planes perpendicular to the intensity axis are defined at levels I_1, I_2, \dots, I_p
 - Assuming that $0 < P < L-1$ then the P planes partition the grey scale into $P + 1$ intervals V_1, V_2, \dots, V_{P+1}

Pseudo Color Image Processing – Intensity Slicing

- Grey level color assignments can then be made according to the relation:

$$f(x, y) = c_k \quad \text{if } f(x, y) \in V_k$$

- where c_k is the color associated with the k^{th} intensity level V_k defined by the partitioning planes at $l = k - 1$ and $l = k$

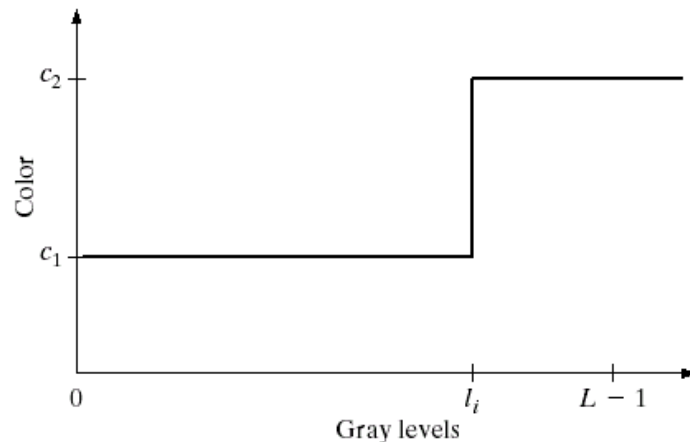
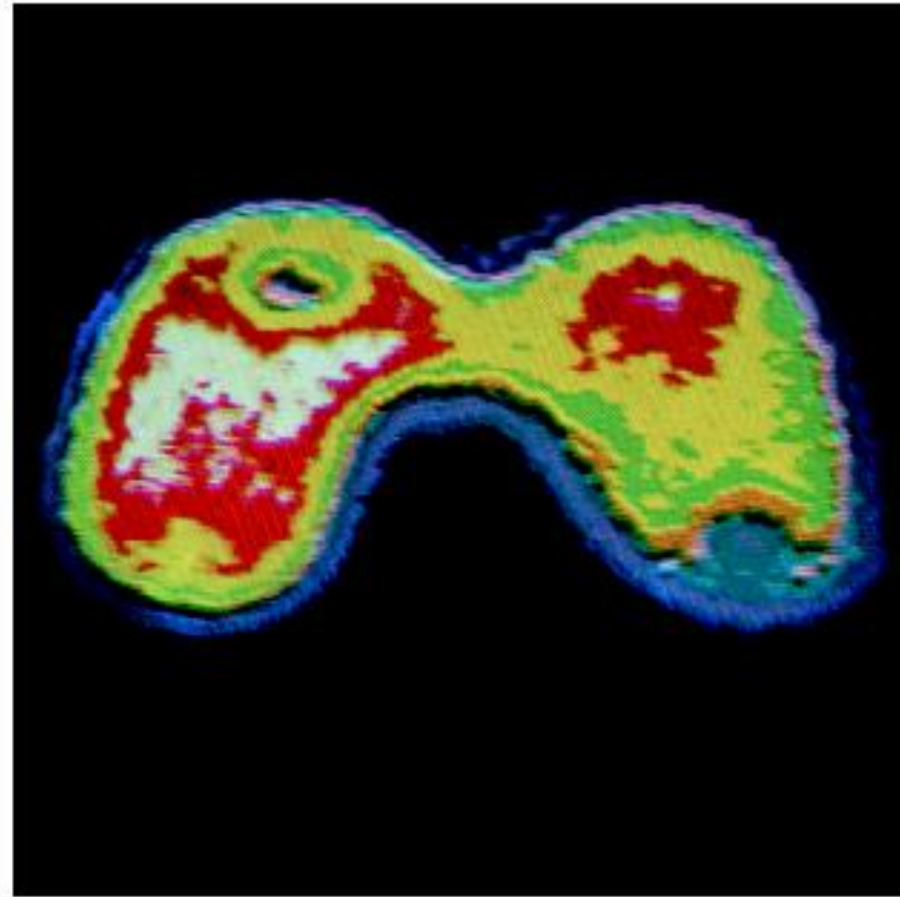
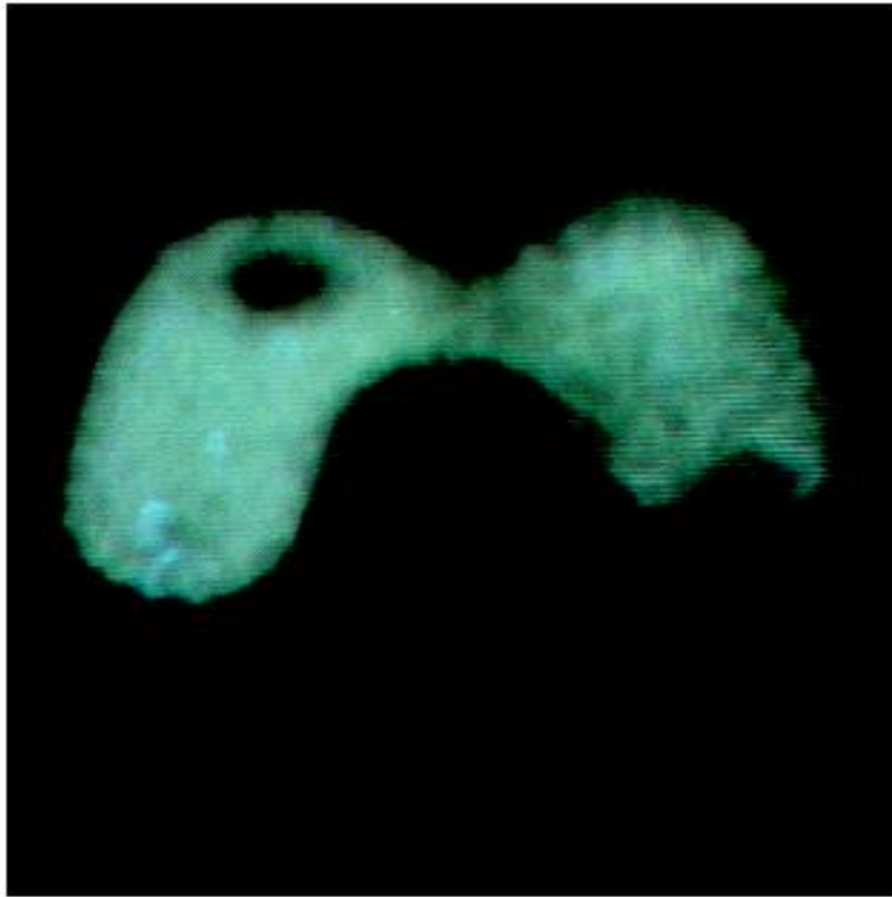
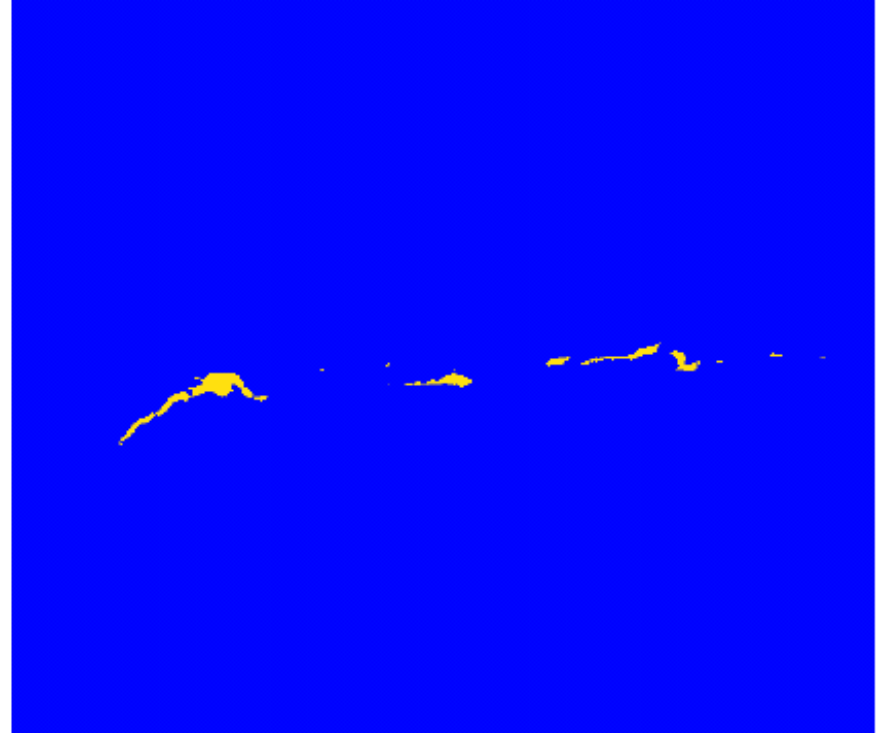


FIGURE 6.19 An alternative representation of the intensity-slicing technique.

Pseudo Color Image Processing – Intensity Slicing

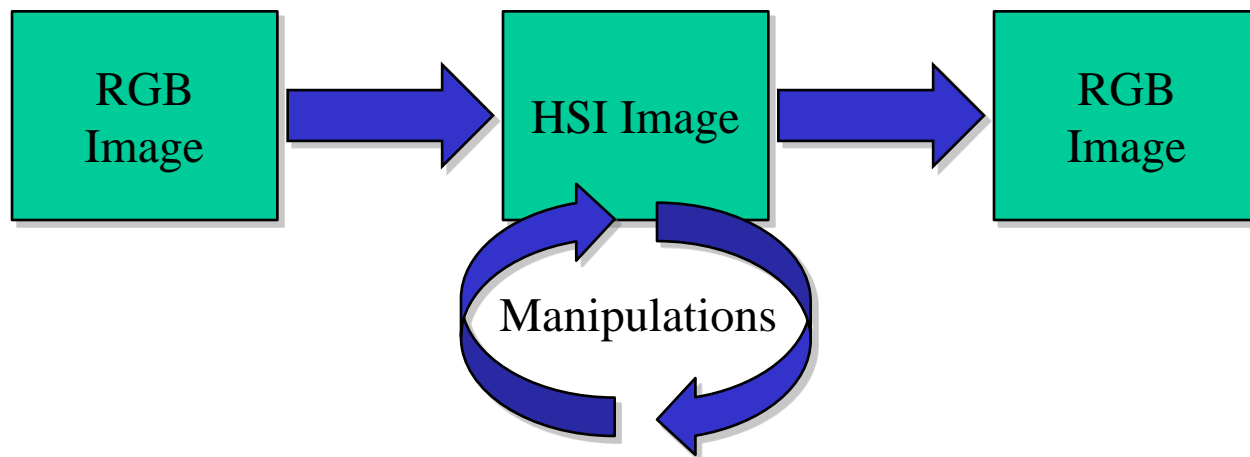


Pseudo Color Image Processing – Intensity Slicing



Manipulating Images In The HSI Model

- ✚ In order to manipulate an image under the HSI model we:
 - First convert it from RGB to HSI
 - Perform our manipulations under HSI
 - Finally convert the image back from HSI to RGB



COLOR IMAGE - SMOOTHING

- ✚ Smoothing can be viewed as a spatial filtering operation in which the coefficients of the filtering mask are all 1's
- ✚ This concept can be easily extended to the processing of full-color images
- ✚ Simply smooth each of the RGB color planes and then combine the processed planes to form a smoothed full-color result

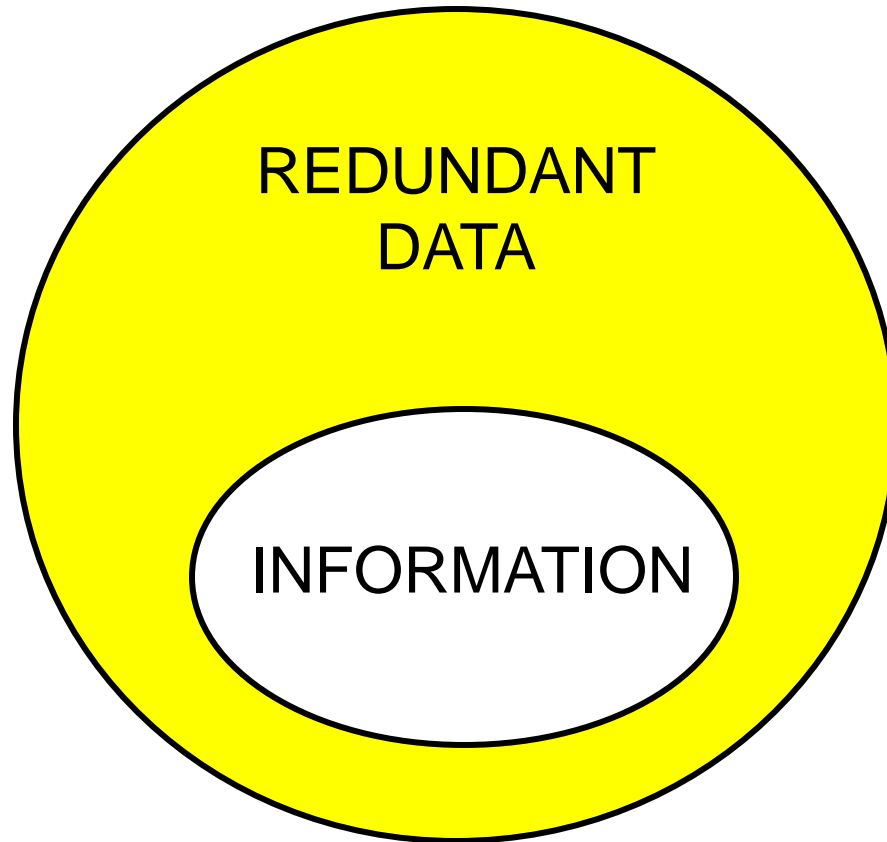
$$\hat{C}(x, y) = \frac{1}{MN} \begin{bmatrix} \sum_{(x,y) \in S_{xy}} R(x, y) \\ \sum_{(x,y) \in S_{xy}} G(x, y) \\ \sum_{(x,y) \in S_{xy}} B(x, y) \end{bmatrix}$$

IMAGE COMPRESSION

IMAGE COMPRESSION

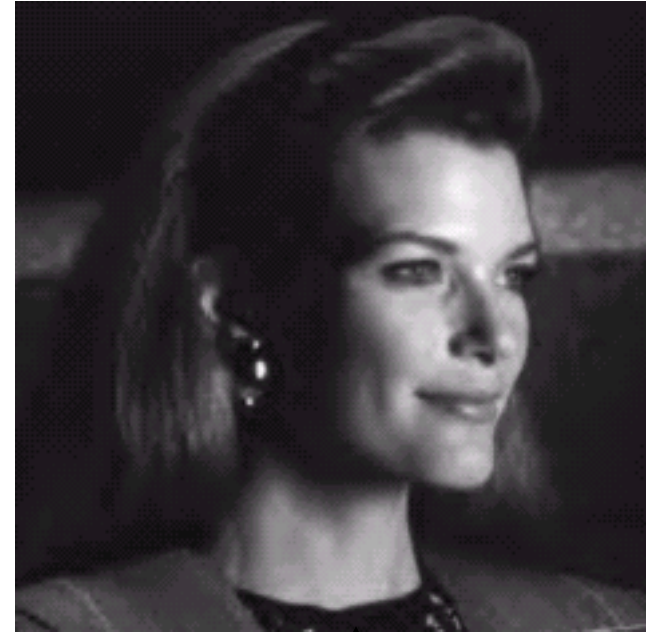
- ✚ Addresses the problem of reducing the amount of data required to represent a digital image
- ✚ The underlying basis of the reduction process is the removal of redundant data

Information vs Data



DATA = INFORMATION + REDUNDANT DATA

IMAGE COMPRESSION: CODING AND DECODING



original image
262144 Bytes

**image
encoder**

compressed bit stream
00111000001001101...
(2428 Bytes)

**image
decoder**

compression ratio (CR) = 108:1

LOSSY VS LOSSLESS COMPRESSION

+ **Lossless** (Information preserving)

- Images can be compressed and restored without any loss of information.
- Application: Medical images

+ **Lossy**

- Perfect recovery is not possible but provides a large data compression.
- Example: TV signals, teleconferencing

FUNDAMENTALS

✚ Raw image: A set of **n1** bits

✚ Compressed image: A set of **n2** bits.

✚ Compression ratio:
$$C_R = \frac{n_1}{n_2}$$

✚ Relative Data Redundancy of first set:
$$R_D = 1 - \frac{1}{C_R}$$

✚ Example: $n_1 = 100\text{KB}$ and $n_2 = 10\text{Kb}$, then $CR = 10$, and $RD = 90\%$

✚ Special cases:

■ $n_1 \gg n_2 \rightarrow CR \approx \infty, RD \approx 1$

■ $n_1 \approx n_2 \rightarrow CR \approx 1, RD \approx 0$

DATA REDUNDANCY

- ✚ Three basic data redundancies:
 - Coding redundancy
 - Spatial and Temporal redundancy
 - Irrelevant Information

CODING REDUNDANCY

✚ Type of coding (# of bits for each gray level)

✚ Image histogram:

- r_k : Represents the gray levels of an image
- $pr(r_k)$: Probability of occurrence of r_k

$$p_r(r_k) = \frac{n_k}{n} \quad k = 0, 1, 2, \dots, L-1$$

- $l(r_k)$: Number of bits used to represent each r_k (after compression)
- L_{avg} : Average # of bits required to represent each pixel:

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k) p_r(r_k)$$

CODING REDUNDANCY

- ✚ It makes sense to assign fewer bits to those r_k for which $p_r(r_k)$ are large in order to reduce the sum.
- ✚ This achieves data compression and results in a variable length code.
- ✚ More probable gray levels will have fewer # of bits.
- ✚ Basic type is variable length coding

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k) p_r(r_k)$$

VARIABLE LENGTH CODING

r_k	$p_r(r_k)$	code1	$l_1(r_k)$	code2	$l_2(r_k)$
$r_0=0$	0.19	000	3	11	2
$r_1=1/7$	0.25	001	3	01	2
$r_2=2/7$	0.21	010	3	10	2
$r_3=3/7$	0.16	011	3	001	3
$r_4=4/7$	0.08	100	3	0001	4
$r_5=5/7$	0.06	101	3	00001	5
$r_6=6/7$	0.03	110	3	000001	6
$r_7=1$	0.02	111	3	000000	6

VARIABLE LENGTH CODING

+ Computing L_{avg}

$$L_{avg} = \sum_{k=0}^7 l_2(r_k) p_r(r_k)$$

$$= 2(0.19) + 2(0.05) + 2(0.21) + 3(0.16) + 4(0.08) \\ + 5(0.06) + 6(0.03) + 6(0.02)$$

$$= 2.7 \text{ bits}$$

$$+ C_R = 3/2.7 = 1.11$$

$$+ R_D = 1 - 1/1.11 = 0.099 = 9.9\%$$

IMAGE COMPRESSION MODEL

- ✚ **Encoder:** Create a set of symbols from image data.
 - **Source Encoder:** Reduce input redundancy.
 - **Channel Encoder:** Increase noise resistance
- ✚ **Channel:** Transmission path.
- ✚ **Decoder:** Construct output image from transmitted symbols
 - **Source Decoder:** Reverse of Source encoder
 - **Channel Decoder:** Error detection/correction.

IMAGE COMPRESSION MODEL

- ✚ **Encoder:** Create a set of symbols from image data.
 - **Source Encoder:** Reduce input redundancy.
 - **Channel Encoder:** Increase noise immunity (noise robustness)
- ✚ **Channel:** Transmission path.
- ✚ **Decoder:** Construct output image from transmitted symbols
 - **Source Decoder:** Reverse of Source encoder
 - **Channel Decoder:** Error detection/correction.

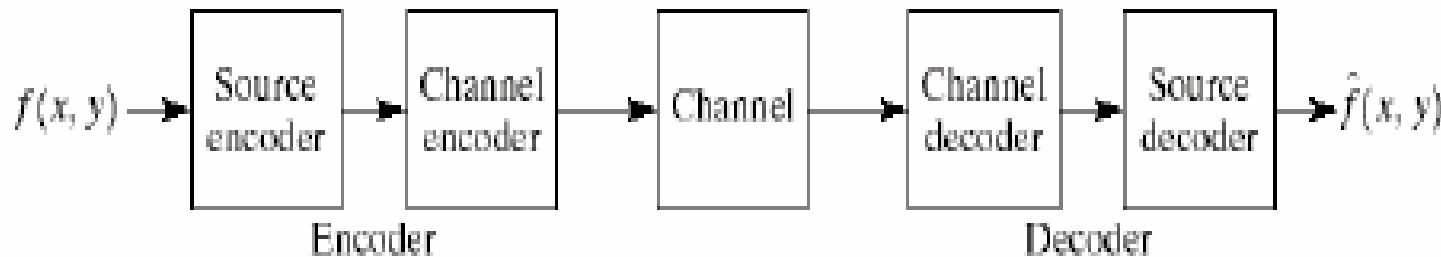


FIGURE 8.5 A general compression system model.

LOSSLESS GRAY-SCALE IMAGE COMPRESSION

VARIABLE WORD LENGTH CODING: EXAMPLE

✚ A 4x4 4bits/pixel original image is given by

Default Code Book

0: 0000
1: 0001
2: 0010
3: 0011
4: 0100
5: 0101
6: 0110
7: 0111
8: 1000
9: 1001
10: 1010
11: 1011
12: 1100
13: 1101
14: 1110
15: 1111

2	8	6	6
6	8	8	8
8	8	10	10
9	10	10	14

↓ encode

0010	1000	0110	0110
0110	1000	1000	1000
1000	1000	1010	1010
1001	1010	1010	1110

Bit rate = 4bits/pixel

Total # of bits used to represent the image:

$$4 \times 16 = 64 \text{ bits}$$

VARIABLE WORD LENGTH CODING: EXAMPLE

✚ Encode the original image with a **CODE BOOK** given left

Huffman Code Book

0: 0000000
 1: 0000001
 2: 0001
 3: 0000010
 4: 0000011
 5: 0000100
 6: 01
 7: 0000101
 8: 10
 9: 00100
 10: 11
 11: 0000110
 12: 0000111
 13: 001010
 14: 0011
 15: 001011

2	8	6	6
6	8	8	8
8	8	10	10
9	10	10	14

encode

0001	10	01	01
01	10	10	10
10	10	11	11
00100	11	11	0011

Total # of bits used to represent the image:

$$4+2+2+2+2+2+2+2+2+2+2+2+5+2+2+4 = 39 \text{ bits}$$

$$\text{Bit rate} = 39/16 = 2.4375 \text{ bits/pixel}$$

$$\text{CR} = 64/39 = 1.6410$$

HUFFMAN CODING

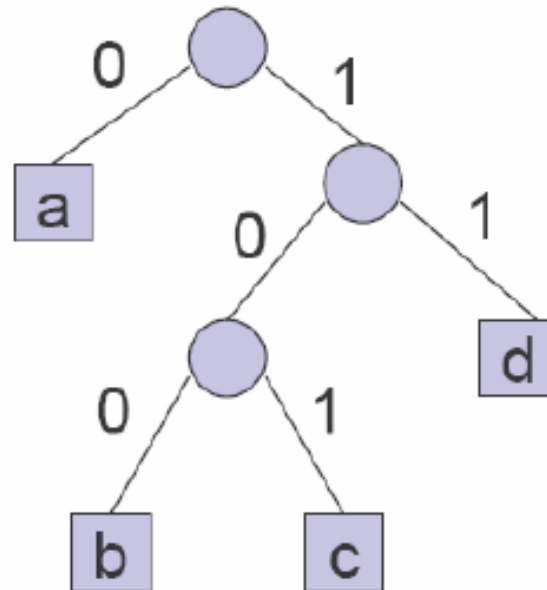
- ✚ Uses frequencies (Probability) of symbols in a string to build a variable rate prefix code.
- ✚ Each symbol is mapped to a binary string.
- ✚ More frequent symbols have shorter codes.
- ✚ No code is a prefix of another. (Uniquely decodable)

HUFFMAN CODING

✚ Example:

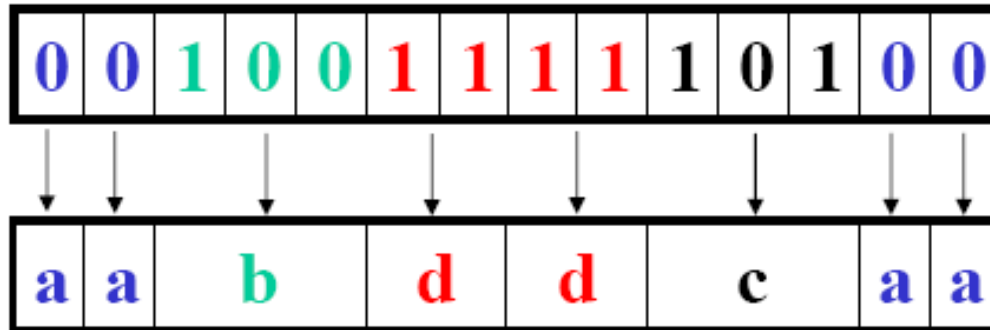
■ We have four symbols a, b, c, and d.

a 0
b 100
c 101
d 11



HUFFMAN CODING

- ✚ A source string: **aabddcaa**
 - Fixed Length Coding: 16 bits (ordinary coding)
 - 00 00 01 11 11 10 00 00
 - ✚ Variable length coding: 14 bits (Huffman coding)
 - 0 0 100 11 11 101 0 0
 - ✚ Uniquely Decodable:



HUFFMAN CODING

- ✚ Step-1
 - Arrange probability in decreasing order and consider them as tree leaves
- ✚ Step-2
 - Merge two nodes with smallest prob. to a new node and sum up prob.
 - Arbitrarily assign 1 and 0 to each pair of merging branch
- ✚ Step-3
 - Repeat until no more than one node left.
 - Read out codeword sequentially from root to leaf

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	
a_1	0.1	0.1	0.2	0.3	0.4
a_4	0.1	0.1			
a_3	0.06	0.1	0.1	0.1	0.1
a_5	0.04				

FIGURE 8.11
Huffman source reductions.

HUFFMAN CODING

FIGURE 8.12
Huffman code
assignment
procedure.

Original source			Source reduction				
Sym.	Prob.	Code	1	2	3	4	
a_2	0.4	1	0.4	1	0.4	1	0.6 0 0.4 1
a_6	0.3	00	0.3	00	0.3	00	
a_1	0.1	011	0.1	011	0.2	010	0.3 01
a_4	0.1	0100	0.1	0100	0.1	011	
a_3	0.06	01010	0.1	0101			
a_5	0.04	01011					

$$L_{avg} = 2.2 \text{ bits/symbol}$$

Example Huffman encoding

✚ **A = 0**

B = 100

C = 1010

D = 1011

R = 11

✚ **ABRACADABRA =**

01001101010010110100110

✚ **This is eleven letters in 23 bits**

✚ **A fixed-width encoding would require
3 bits for five different letters, or 33
bits for 11 letters**

✚ **Notice that the encoded bit string *can*
be decoded!**

Why it works

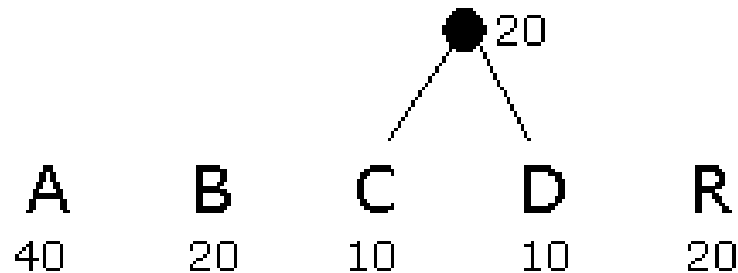
- ✚ In this example, **A** was the most common letter
- ✚ In **ABRACADABRA**:
 - 5 **A**s code for **A** is 1 bit long
 - 2 **R**s code for **R** is 2 bits long
 - 2 **B**s code for **B** is 3 bits long
 - 1 **C** code for **C** is 4 bits long
 - 1 **D** code for **D** is 4 bits long

Creating a Huffman encoding

- ✚ For each encoding unit (letter, in this example), associate a frequency (number of times it occurs)
 - You can also use a percentage or a probability
- ✚ Create a binary tree whose children are the encoding units with the smallest frequencies
 - The frequency of the root is the sum of the frequencies of the leaves
- ✚ Repeat this procedure until all the encoding units are in the binary tree

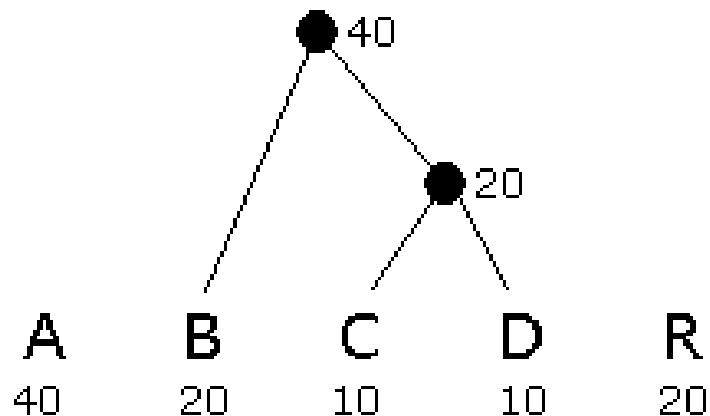
Example, step I

- ✚ Assume that relative frequencies are:
 - A: 40
 - B: 20
 - C: 10
 - D: 10
 - R: 20
- ✚ (I chose simpler numbers than the real frequencies)
- ✚ Smallest number are 10 and 10 (C and D), so connect those



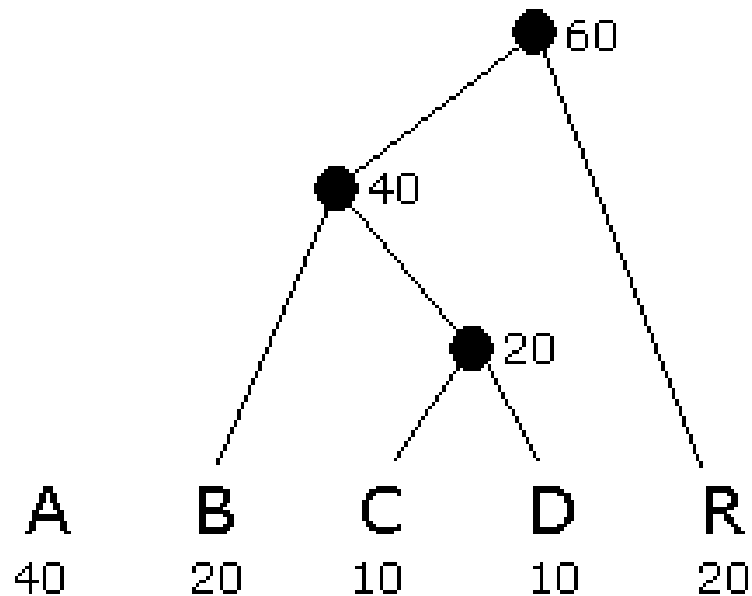
Example, step II

- ✚ c and D have already been used, and the new node above them (call it c+D) has value 20
- ✚ The smallest values are B, c+D, and R, all of which have value 20
 - Connect any two of these



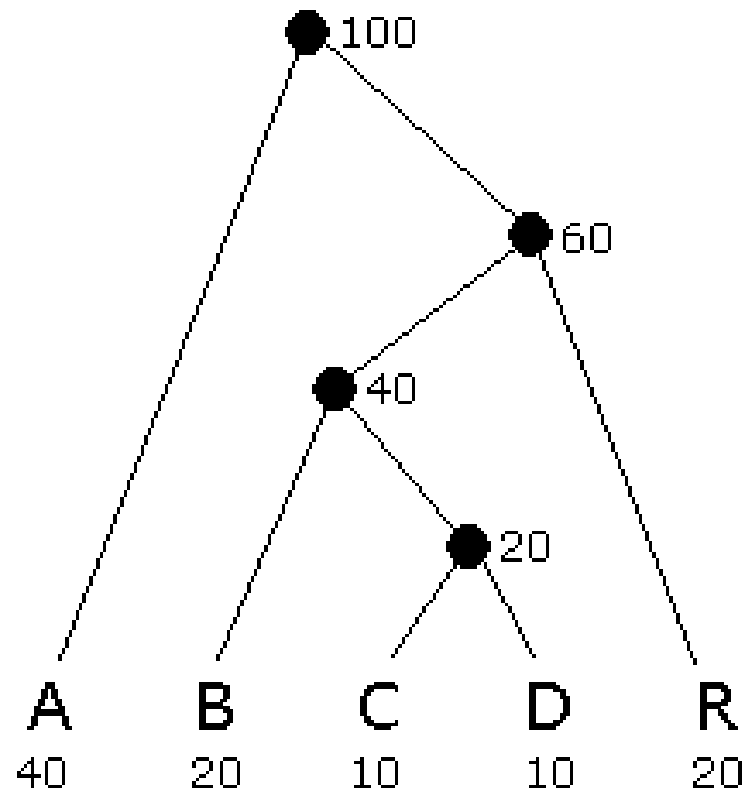
Example, step III

- ✚ The smallest values is R, while A and B+C+D all have value 40
- ✚ Connect R to either of the others



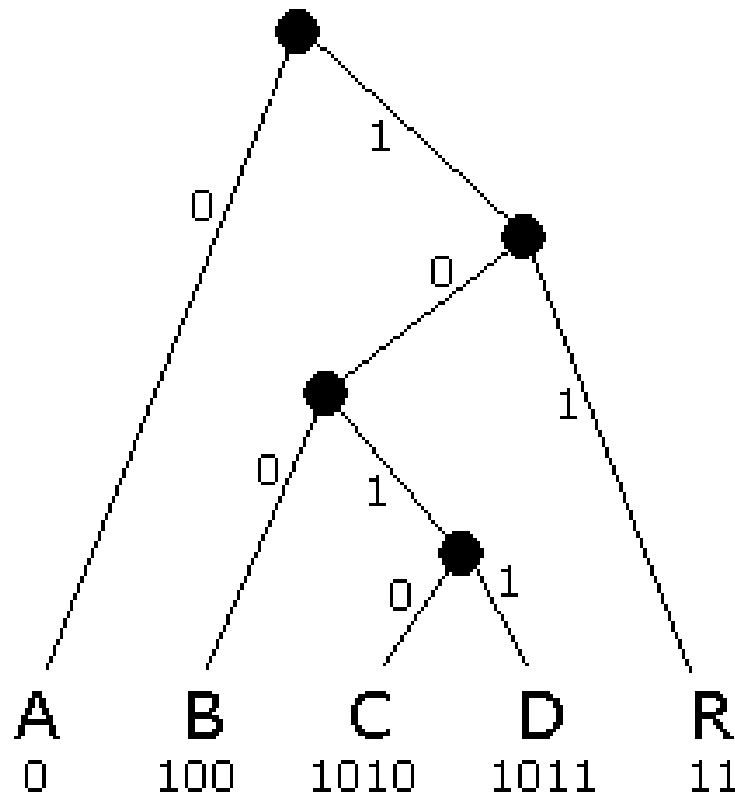
Example, step IV

✚ Connect the final two nodes



Example, step V

- Assign 0 to left branches, 1 to right branches
- Each encoding is a path from the root



- A = 0
B = 100
C = 1010
D = 1011
R = 11
- Each path terminates at a leaf
- Do you see why encoded strings are decodable?

OBJECTIVE CRITERIA

- ✚ The error $e(x,y)$ can be defined as

$$e(x, y) = \hat{f}(x, y) - f(x, y)$$

- ✚ Total error between the two images

$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x, y) - f(x, y)]$$

- ✚ Root-mean-square-error

$$e_{rms} = \left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x, y) - f(x, y)]^2 \right]^{1/2}$$

- ✚ Mean-square signal-to-noise ratio of the output image

$$SNR_{ms} = \frac{\left[\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x, y)]^2 \right]}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\hat{f}(x, y) - f(x, y)]^2}$$